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## On the evolutionary interplay between environmental CSR and emission tax

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# On the Evolutionary Interplay between Environmental CSR and Emission Tax\*

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## Abstract

This paper analyses the steady-state industry configuration of an oligopoly composed of profit-seeking (PS) and environmentally socially responsible (ECSR) firms in an evolutionary setting. In the industry, an emission tax is levied and firms may invest in emission abatement technology to reduce the tax burden. Our main findings show that, despite the commitment toward emission abatement, an individual ECSR firm may pollute more than its PS counterpart, with ill-fated effects on the environment. By contrast, the introduction of an emission tax puts competitive pressure on emission abatement to ECSR firms, by inducing profit-seeking firms to invest in emission abatement. The industry configuration that minimises the environmental damage (and maximises social welfare) is mixed, with a small but relevant share of ECSR firms, combined with the adoption of the tax on emissions.

**JEL Codes:** C73, H23, L13, L21, M14

**Keywords:** Mixed oligopoly markets, emission reduction investment, evolutionary dynamics

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# 1 Introduction

The fact that human emissions of carbon dioxide and other polluting gases are a major driver of climate change is not a matter of debate any longer. In response to this challenge, many firms worldwide are nowadays willing to reduce polluting emissions in their production processes. Together with the social impact, now also the environmental concern entered the business agenda. The KPMG Survey of Sustainability (2020) reports 96% of the world's biggest companies introduced programs of social and environmental responsibility, as well as 80% of "N100", a worldwide sample of 5,200 companies (KPMG, 2020).

In the past years, discussions on the relevance of Environmental Corporate Social Responsibility (ECSR) flourished in the academic debate on industrial organisation, particularly to understand the strategic role of these practices. The general question is whether ECSR activities are a tool to reach some strategic advantage in the interaction with competitors or the government. How come investing in emissions abatement rather than improving production processes or product quality should be good for business? And how firms adopting ECSR practices may survive competition against profit-seeking (PS) competitors? A possible reason for a firm to willingly bear pollution abatement costs without any regulatory enforcement is that consumers have a preference for environmentally friendly products (Liu et al., 2015).

Alternatively, a commonly considered explanation is the fact that ECSR combine environmental concern with social concern, which boosts production and compensate the increase in cost abatement (Lambertini and Tampieri, 2015, Lambertini et al., 2016, Hirose et al., 2017, Nie et al., 2018 and Li and Wang, 2021, among others). With the right conditions in terms of market size, ECSR commitment and cost of emission reduction technology, a generally accepted result is that firms may indeed implement ECSR activities to obtain higher profits than their PS competitors.

Another relevant question is how ECSR practices interact with the widespread adoption of environmental regulation, particularly on emissions tax. Only the surface of this issue has been tackled in the literature: recent contributions considered the interaction between CSR practices (Xu and Lee, 2018, Leal et al., 2018,) and the introduction of a tax on emissions, by setting aside environmental concerns. Fukuda and Ouchida (2020) first considered the effects of an optimal

tax on emissions issued by a time-consistent government and an ECSR monopolist. Their results show the conditions under which ECSR practices might increase the monopolist's emissions. [Xu et al. \(2022\)](#) and [Xu and Lee \(2022\)](#) consider an optimal emission tax in a duopoly, the former by comparing Bertrand with Cournot competition, the latter by comparing cooperative versus noncooperative ECSR activities between firms.

Starting from these contributions, one question that has been left unanswered is how the interplay between ECSR practices and an emissions tax influences the endogenous long-run configuration of the industry. This is the scope of the present paper.

We evaluate the endogenous industry structure of a continuous-time, evolutionary mixed oligopoly in which production is polluting. In the industry, PS firms compete with ECSR firms in quantities. Introducing ECSR practices entails that a firm is both socially and environmentally concerned, resulting in an objective function that takes into account a share of consumer surplus and internalises part of their polluting emissions ([Lambertini and Tampieri, 2015](#), [Lambertini et al., 2016](#), [Hirose et al., 2017](#), [Nie et al., 2018](#), [Fukuda and Ouchida, 2020](#) and [Li and Wang, 2021](#), among others).

In every instant, firms first choose simultaneously whether to adopt an ECSR or PS statute by comparing the expected profits obtained by the two objectives; then, they fix their production quantity to maximise the chosen objective function. To determine the long run market configuration, we rely on the method established by [Droste et al. \(2002\)](#) and recently employed in a similar setting by [Kopel and Lamantia \(2018\)](#). In every instant, a number of firms are drawn from a population of firms to play the oligopoly game. Thus, the share of ECSR and PS firms in the industry evolves based on the comparison of expected profits, even though production quantities are determined to maximise each firm's objective function.

We consider the implementation of an exogenous emission tax to fight pollution. To ease the tax burden, PS and ECSR firms may in turn invest in emission reduction technology. The focus of this work is on how the interaction between ECSR practices and the emission tax influences the long run evolutionary equilibrium.

In line with the existing literature ([Fukuda and Ouchida, 2020](#)), our results show that an ECSR firm may pollute more than a PS firm, despite its commitment to emission reduction. Albeit this result might seem surprising *prima facie*, it is quite simple to explain: to compete with

PS firms, an ECSR firm must counterbalance its environmental concern with social concern, which implies corporate policies that increase demand and in turn the equilibrium quantity produced by the ECSR firm compared to the PS firm. Note that this intuition is robust to different modelling of ECSR, namely, those according to which the cost of environmental concern is compensated by the presence of green consumers who prefer the ECSR to the PS product. Indeed, even in this case the emission reduction practices boost demand, with the possible effect of an increase in emissions compared to PS firms.

The first conclusion, hence, seems that the presence of ECSR firms in the industry is bad for the environment. However, things change with the introduction of an emission tax. Indeed, the tax pushes also PS firms to invest in emission reduction technology. Through competition then, ECSR firms have an incentive to further lower their emissions. The overall outcome is that the environmental damage reaches its minimum value and social welfare its maximum value when a small but relevant share of ECSR stays in the industry in the long run equilibrium configuration. The results are striking: the adoption of ECSR practices is not beneficial *per se*, but for the competitive pressure that is put on the industry, provided that an emission tax is in place.

The general conclusion is that, while environmental regulation and voluntary emission reduction practices have been seen as alternative ways to pursue containment of environmental damage, they appear to be complement features: their combination yields the best outcome for both the environment and the society as a whole.

Together with the abovementioned contributions in the literature on strategic ECSR and its interaction with an emission tax, the paper is also related to the literature on the endogenous market structure in mixed oligopolies with CSR (and ECSR) firms. This topic has been studied in [Lambertini and Tampieri \(2015\)](#) and [Gioffré et al. \(2021\)](#) in static settings. In evolutionary frameworks, it has been analysed in [Kopel et al. \(2014\)](#) and [Kopel and Lamantia \(2018\)](#). The present analysis is mainly linked to the latter two papers, in which firms compete *à la* Cournot and can choose whether to adopt a CSR or a PS behaviour. Compared with these two contributions, we consider a continuous rather than discrete time setting, and ECSR firms rather than CSR firms, which implement a fraction of consumer surplus but not the environmental externality in their objective function. In addition, we take into account the possibility of investment in emission abatement and we study the effects of the implementation of the tax on emissions.

The remainder of the paper is organized as follows. Section 2 outlines the framework, while Section 3 and 4 develop the static and evolutionary equilibrium, respectively. The results are discussed in Section 5, in which first we illustrate the long run equilibrium market configuration (Section 5.1) and then we analyse the market and welfare effects of an introduction of an emission tax and its variation and of different internalisation of polluting emission from the ECSR firm (Section 5.2). Concluding remarks are in Section 6, while all the proofs can be found in the Appendix.

## 2 The model

Consider an industry composed of  $N \geq 2$  firms that produce a unique homogeneous good and compete in quantities. Of these  $N$  firms,  $m \in \{0, 1, 2, \dots, N\}$  are profit seeking (PS) and  $N - m$  are environmentally socially concerned (ECSR). The inverse demand of the good produced by the firms is given by the following linear function:

$$p = a - mq_P - (N - m)q_E, \quad (1)$$

where  $p$  is the unit price of the good,  $a > 0$  is the market reservation price, while quantities  $q$  of PS and ECSR firms are denoted by subscripts  $P$  and  $E$ , respectively.

The market is subject to emission taxation so, to reduce the tax burden, firms can invest to abate emissions. The abatement technology is the same for both types of firms. Emissions are defined as production quantities minus abatement investments:  $e = q - z$ . Denoting as  $i \in \{P, E\}$  a generic firm, its profit function is

$$\pi_i = (p - c)q_i - \frac{\theta}{2}z_i^2 - (q_i - z_i)\tau, \quad (2)$$

where  $c > 0$  is the production cost,  $\theta > 0$  captures the efficiency of the abatement technology,  $z_i$  represents the abatement investment and  $\tau$  is the unit tax on emissions.

The representative ECSR firm maximises profits plus a fraction of consumer surplus, and internalises its own share of emissions. The objective function is the following (see, for further

details, [Lambertini et al., 2016](#)):

$$O_E = \pi_E - \alpha(q_E - z_E) + \beta CS. \quad (3)$$

In (3),  $\alpha \in [0, 1]$  represents the fraction of emissions that is voluntarily internalised by an ECSR in its production process, while  $\beta \in [0, 1]$  is the ECSR firms sensitivity to social concern, represented by social surplus  $CS$ , which is denoted as

$$CS = \frac{[mq_P + (N - m)q_E]^2}{2}. \quad (4)$$

### 3 Static equilibrium

In this section we outline the market equilibrium in every instant for any possible industry configuration. The optimisation problem of the representative PS firm  $i$ :

$$\begin{aligned} \max \pi_i &= [a - q_i - (m - 1)q_P - (N - m)q_E - c]q_i - \frac{\theta}{2}z_i^2 - (q_i - z_i)\tau, \\ \text{s.t. } q_i &\geq 0, z_i \geq 0, q_i - z_i \geq 0. \end{aligned} \quad (5)$$

Similarly, the optimisation problem of the representative ECSR firm  $j$  is

$$\begin{aligned} \max O_j &= [a - q_j - (N - m - 1)q_E - mq_P - c]q_j - \frac{\theta}{2}z_j^2 - (q_j - z_j)(\tau + \alpha) \\ &\quad + \frac{\beta}{2}[mq_P + q_j + (N - m - 1)q_E]^2, \\ \text{s.t. } q_j &\geq 0, z_j \geq 0, q_j - z_j \geq 0. \end{aligned} \quad (6)$$

For notational simplicity we set  $\mu = a - c$ , where  $\mu$  measures market size. We assume that  $\mu - \tau > 0$  and  $\mu - \alpha > 0$  always occur, implying that the market size is sufficiently large so that PS and ECSR firms are both able to bear the cost of emissions and the internalisation of pollution. The following proposition summarises the equilibrium outcomes for interior solutions.

**Proposition 1** *The optimal values of quantities and abatement of PS and ECSR firms are:*

$$\begin{aligned}
q_P^* &= \frac{\mu - \tau + (N - m)\alpha - (\mu - \tau)(N - m)\beta}{N + 1 - (N - m)\beta}, \\
q_E^* &= \frac{\mu - \tau - (m + 1)\alpha + (\mu - \tau)\beta m}{N + 1 - (N - m)\beta}, \\
z_P^* &= \frac{\tau}{\theta}, \\
z_E^* &= \frac{\tau + \alpha}{\theta}.
\end{aligned} \tag{7}$$

The conditions derived in the following corollary allow to restrict the analysis to interior solutions.

**Corollary 2** *Condition  $\beta \in (\hat{\beta}_E, \hat{\beta}_P)$  guarantees both positive quantities and emissions of both PS and ECSR firms, where*

$$\hat{\beta}_P := \frac{(\mu - \tau + \alpha N)\theta - (N + 1)\tau}{[(\mu - \tau)\theta - \tau]N}, \tag{8}$$

$$\hat{\beta}_E := \max \left\{ \frac{(\tau + \alpha)(N + 1) - (\mu - \tau - \alpha)\theta}{(\tau + \alpha)N}, \frac{(\tau + \alpha)(N + 1) - [\mu - \tau - (N + 1)\alpha]\theta}{(\mu - \tau)\theta N} \right\}. \tag{9}$$

From Proposition 1, we can rewrite optimal profits as:

$$\begin{aligned}
\pi_P^*(m) &= [q_P^*(m)]^2 + \frac{\tau^2}{2\theta}, \\
\pi_E^*(m) &= \{[1 + \alpha - (N - m)\beta]q_E^*(m) - \beta m q_P^*(m)\}q_E^*(m) + \frac{\tau^2 - \alpha^2}{2\theta}.
\end{aligned} \tag{10}$$

## 4 Evolutionary competition

We are now in a position to endogenise a firm's choice of being of PS or ECSR type according to expected profits. We adopt the evolutionary setting introduced by [Droste et al. \(2002\)](#) for a duopoly and more recently developed in an oligopoly by [De Giovanni and Lamantia \(2016\)](#), [Hommes et al. \(2018\)](#), [Kopel and Lamantia \(2018\)](#), [Lamantia et al. \(2018\)](#), and [Tichý et al. \(2020\)](#), among others. The approach is the following: in each instant,  $N$  firms are randomly selected to play the one-shot game described in the previous section. Firms are Nash players in the sense that before choosing quantities, they observe the composition of the  $N - 1$  rivals.



This means that each firm computes the value of the profit functions for every possible market composition. We denote  $x \in [0, 1]$  as the probability that the firm adopts the profit seeking strategy. Therefore, the probability  $x$  can be interpreted as the share of PS firms on the market and  $1 - x$  as the share of ECSR firms. The expected profits of a PS firm is:

$$\mathbb{E}(\pi_P^N(x)) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \pi_P^*(k+1), \quad (11)$$

where

$$\pi_P^*(k+1) = [a - (k+1)q_P^*(k+1) - (N-1-k)q_E^*(k+1) - c]q_P^*(k+1) - \frac{\theta}{2}(z_P^*)^2 - (q_P^*(k+1) - z_E^*)\tau.$$

Analogously, the expected profit of a ECSR firm is:

$$\mathbb{E}(\pi_E^N(x)) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \pi_E^*(k), \quad (12)$$

where

$$\pi_E^*(k) = [a - kq_P^*(k) - (N-k)q_E^*(k) - c]q_E^*(k) - \frac{\theta}{2}(z_E^*)^2 - (q_E^*(k) - z_E^*)\tau.$$

The time evolution of the share  $x$  is given by the following replicator dynamics in continuous time (Weibull, 1995 and, more recently, Antoci et al., 2008 and Antoci et al., 2021, among others):<sup>1</sup>

$$\dot{x} = x(1-x)[\mathbb{E}(\pi_P^N(x)) - \mathbb{E}(\pi_E^N(x))]. \quad (13)$$

## 5 Steady state analysis

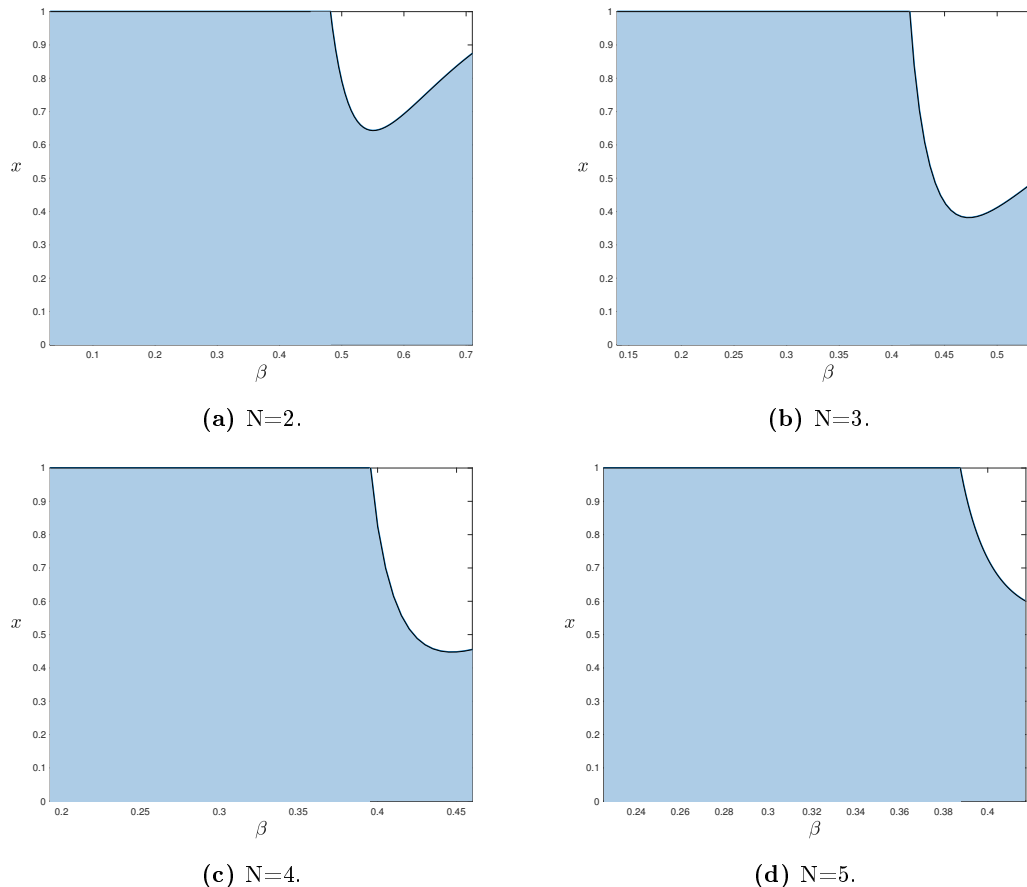
In this section we illustrate our main results. Given the analytical complexity of the problem at hand, we base our discussion on numerical examples. In the appendix, we verify the global stability of our equilibrium configuration.

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<sup>1</sup>A similar approach in discrete time has been adopted by Bischi et al. (2018) and Dieci et al. (2018).

## 5.1 Industry configuration and competition

We begin by showing the long run market configuration  $x$  based on different sensitivity to social concern, for given numbers of firms  $N$ . In practice, we outline bifurcation diagrams to determine the share of PS firms  $[1 - x(\beta)]$  changes to changes in the level of social concern  $\beta$ . This exercise also allows us to compare our results with other contributions in the literature, such as [Kopel and Lamantia \(2018\)](#).



**Fig. 1.** Dominance of strategies. Parameter values:  $a = 1$ ,  $c = 0.3$ ,  $\alpha = 0.15$ ,  $\tau = 0.1$ ,  $\theta = 4$ .

Figure 1 shows the different market configurations for  $N = 2$ ,  $N = 3$ ,  $N = 4$  and  $N = 5$  where  $a = 1$ ,  $c = 0.3$ ,  $\alpha = 0.15$ ,  $\tau = 0.1$ ,  $\theta = 4$ . The share of S-firms under evolutionary competition, for different  $\beta$ , is represented in the region above the line. Our results show that PS firms are generally dominant: in particular the relation between competition and the long run existence

of a share of ECSR firms is curvilinear. Finally, a market configuration composed of all ECSR firms never appears. These considerations are robust to extensive robustness checks.

Our results are qualitatively similar to the findings of [Kopel and Lamantia \(2018\)](#). Compared to our results, they find larger regions in which socially concerned firms are present, and also parameter combinations that allow market configurations of only socially concerned firms. Thus the presence of environmental concern in the CSR statute, as well as emission regulation and innovation abatement reinforce the persistence of firms focused only on profits.

## 5.2 The interaction between the emission tax and ECSR practices

This section develops the main results of the paper. It shows the effects of variations of tax rate  $\tau$  and ECSR internalisation of emissions  $\alpha$  on the long run equilibrium features, namely, market configuration, the level of polluting emissions and social welfare.

From this analysis we are able to illustrate the interaction between ECSR practices and the introduction of emission tax and why both must be present to reach the best outcome for the environment and social welfare. For completeness, in the Appendix we evaluate how the steady state equilibrium changes due to variations in the cost of technology  $\theta$ .

Social welfare is defined as the sum of industry profits, consumer surplus and tax revenue  $T$  minus environmental damage  $D$ :

$$W = m\pi_P^* - (N - m)\pi_E^* + CS + T - D. \quad (14)$$

In (14), tax revenue amounts to

$$T = \tau [m(q_P^* - z_P^*) + (N - m)(q_E^* - z_E^*)], \quad (15)$$

while the environmental damage is a quadratic function of polluting emissions,  $D = d(Ne)^2$ , where  $d = 1$  in normalised to 1. In what follows, we will describe the effects of a variation in  $\tau$  and  $\alpha$  on overall social welfare and its components separately, namely, consumer surplus, total industry profits and environmental damage. For brevity, we do not include tax revenue, which is available upon request.

Finally, note that changes in emissions and welfare may be appreciated together with changes in the market configuration, which may provide an intuition for some of the results.

### 5.2.1 Variation of the emission tax

Figure 2 shows the results of changes in the emission tax rate. The panels may be read starting from the situation in which no tax is implemented ( $\tau = 0$ ), and then see how its introduction affects the steady state.

An increase in  $\tau$  decreases the share of ECSR firms. This result may be explained by the fact that ECSR firms bear an extra cost compared to PS firms, namely, the internalisation of pollution with unit cost  $\alpha$ . The increase in  $\tau$  makes ECSR firms less and less competitive. Therefore, ECSR practices and environmental regulation appear to be alternative components of the long run equilibrium.

Also, the increase in the tax rate pushes ECSR firms to invest more in innovation abatement, which leads to lower emissions with higher  $\tau$ . By contrast, a PS firm's emissions are U-shaped with respect to the tax rate.

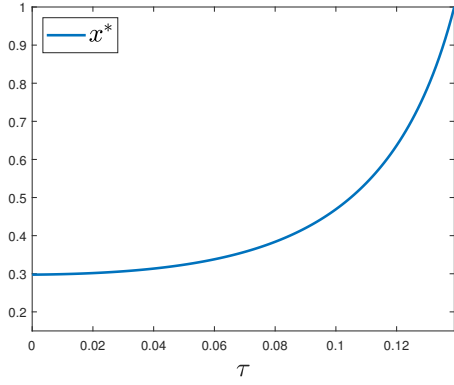
Consumer surplus decreases with  $\tau$ , this due to both the lowering production of the industry and the decrease in the share of ECSR firms, which promote it in their objective function.

An interesting result is that the increase in the tax rate may prompt an increase in total industry profits. This does not depend on the reduction in the share of ECSR firms: we may indeed show analytically that the same outcome applies in an industry with all PS firms.<sup>2</sup> Instead, the result may be explained by relying on the so-called “Porter Hypothesis”. This hypothesis claims that environmental regulation may push pollution control investment to the point to counterbalance the cost of compliance. Accordingly, environmental regulation could be beneficial not only for the environment but also for industry profits (Porter, 1991).

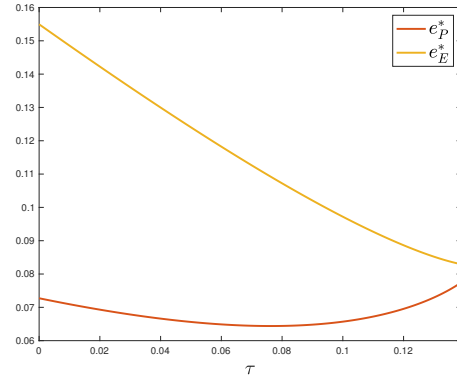
In particular, the intuition may be found in the possible combinations between the cost of innovation and the tax rate, which make it convenient to invest in abatement so that to reduce more than proportionally the tax burden. Similar results may be found, on a static setting, in Lambertini et al. (2020).

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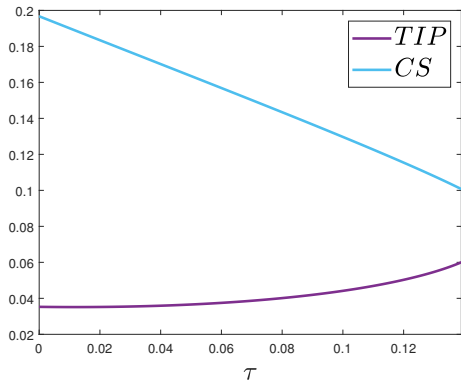
<sup>2</sup>Computations are available upon request.



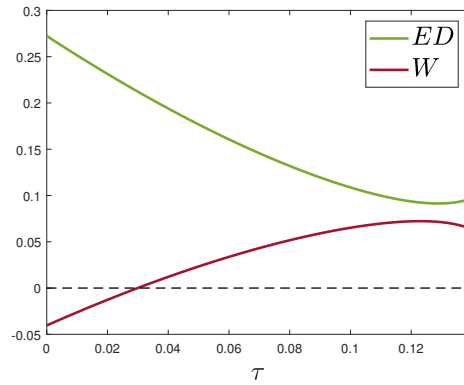
(a) Long run share of profit-seeking firms.



(b) Emissions of profit-seeking and CSR firms.



(c) Total Industry Profits and Consumer Surplus.



(d) Environmental Damage and Welfare.

**Fig. 2.** Variation in the emission tax,  $N=4$ ,  $\beta = 0.43$  and  $\alpha = 0.15$ .

We finally turn to environmental damage and social welfare. Our numerical exercise shows that the maximum of social welfare can be reached to a point in which the market configuration exhibits a prevalence of PS firms, about 80% of the market. Interestingly, the maximum point of social welfare corresponds to the minimum point of environmental damage, showing the relevance of this component to determine the former.

The most counterintuitive result of Figure 2 is the fact that ESCR firms' emissions are higher than PS firms' for all values in the simulation. In fact, this comes not as a surprise: Fukuda and Ouchida (2020) has shown this potential effect of ECSR practices in a static monopoly.

The result is explained by the social concern component of ESCR firms, which pushes their production at a higher level than that of PS firms in their competitive interaction and more

than counterbalances ECSR commitment toward abatement emissions (Lambertini and Tampieri, 2015, Lambertini et al., 2016). It is important to note that, without social concern, an ECSR firm may not survive the competitive pressure against PS firms. However, a higher level of emissions of ECSR compared to PS firms induces to question their very scope.

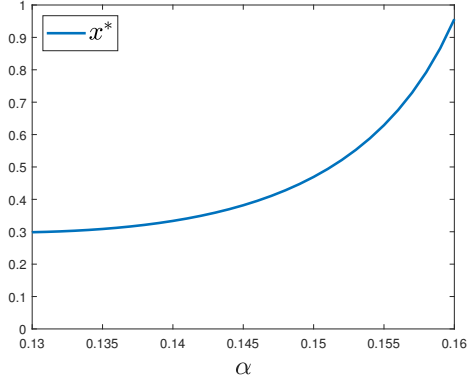
This result may be particularly appreciated when  $\tau = 0$ , which implies no abatement investment from PS firms,  $z_P^* = 0$  (see Proposition 1): social concern pushes the production of ECSR firms, resulting in the highest level of consumer surplus, ECSR emissions and overall environmental damage. In addition, ECSR firms grab a consistently higher share of the market. However, when the emission tax enters the story, PS firms' abatement investment becomes positive for PS firms and turns into a competitive instrument, by pushing ECSR firms to invest more. In the light of the initial configuration, the overall result is unexpected: the number of ECSR firm is reduced, but the minimisation of environmental damage, as well as the maximisation of social welfare, requires that a relatively small (but not irrelevant) share of them remains present in the market, despite their individual performance is worse in terms of environmental impact.

### 5.2.2 Internalisation of emissions

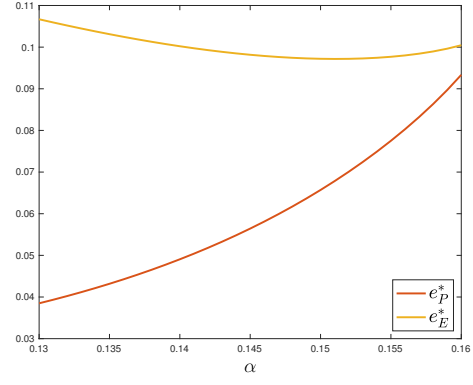
Here we investigate how a variation of the voluntary internalisation of polluting emissions by ECSR firms  $\alpha$  affects the features of the steady state equilibrium. This analysis is helpful to understand how the introduction of the emission tax influences the strategic interaction between ECSR and PS firms, as it modifies the relative incentives to undertake investments in emission abatement.

Begin by noting that  $\alpha$  may be interpreted as a further unit tax on emissions that an ECSR firm freely chooses to bear to reduce its environmental impact. Hence, it comes not as a surprise that the results in the first panel of Figure 3 are qualitatively similar to those in the first panel of Figure 2: an increase in the internalisation of emissions makes it increasingly costly to sustain ECSR practices, with the effect to reducing their competitiveness against firms and reducing their share in the market.

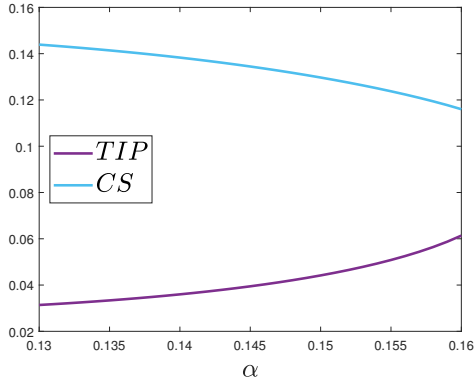
In addition, the lower competitiveness of ECSR firms due to the increase in  $\alpha$  induces PS firms to invest less in emission abatement. This point explains the importance of the presence of ECSR firms for the effectiveness of environmental regulation. They pollute more, but also they



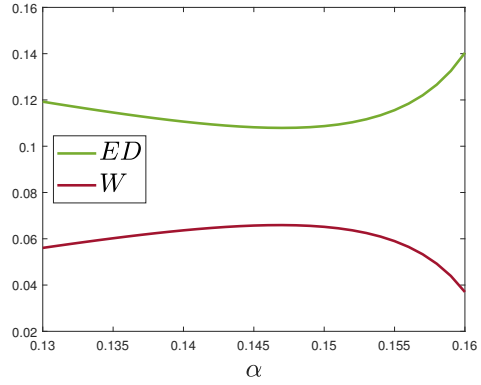
(a) Long run share of profit-seeking firms.



(b) Emissions of profit-seeking and CSR firms.



(c) Total Industry Profits and Consumer Surplus.



(d) Environmental Damage and Welfare.

**Fig. 3.** CSR internalisation of emissions,  $\beta = 0.43$ ,  $\theta = 4$ ,  $\tau = 0.1$ ,  $N = 4$ .

induce PS firms to abate more. Paradoxically, this is particularly true when the commitment towards the internalisation of emissions is not too high. The results of the fourth panel in terms of environmental damage and social welfare confirm this intuition.

## 6 Concluding remarks

We have examined the long run equilibrium configuration of a mixed oligopoly composed of PS and ECSR firms, when an emission tax is in place and firms may invest in emission reduction technology. We have shown that ECSR firms may pollute more than their PS counterparts, despite their environmental concern. Yet, provided the adoption of the emission tax, their presence in the market force spurs competition through emission abatement, resulting in minimisation of

the environmental damage and maximisation of social welfare when a small but relevant share of ECSR is active in the market.

An interesting side result is the support for the Porter Hypothesis: the adoption of an environmental policy may increase firms' profits. This is particularly true when the tax burden is sufficiently high, so that emission reduction investment is relatively more convenient. Related to the industry configuration, the increase in profits due to an increase in the emission tax goes along with a decrease in the share of ECSR firms.

Some features of the framework deserve discussion. First, given the growing environmental concern among consumers, it could be argued that the assumption of homogeneity between goods produced by PS and ECSR firms is too strong. In particular, one might expect a different willingness to pay for goods processed by firms of a different type. The assumption indeed aims at simplifying the framework, which is already rich in the number of parameters. Yet, by relaxing it we would just reinforce the existing results, so that our analysis can be thought of as a limit case that embodies all degrees of environmental concern among consumers.

Second, our findings are completely developed through numerical simulations. Yet, the results are robust to several numerical checks. These further results are available upon request.

Third, unlike other contributions in the literature ([Fukuda and Ouchida, 2020](#), [Xu et al., 2022](#) and [Xu and Lee, 2022](#)), we have focused on exogenous rather than optimal taxation. Treating the tax as a parameter has allowed us to evaluate the effects of its changes over the equilibrium configuration and compare them in different scenarios.

To conclude, the main message to regulators is that social welfare is increased by a combination of emission tax and the presence of a small share of ECSR firms. Environmental regulation and voluntary emission abatement practices seem complement features to reach the environmental optimum.



# Appendix

## Proof of Proposition 1

The Lagrangian function associated to problem (5) is the following:

$$\mathcal{L}_i = \pi_i + \lambda_1 q_i + \lambda_2 z_i + \lambda_3 (q_i - z_i),$$

from which we obtain the optimality conditions:

$$\left\{ \begin{array}{l} a - 2q_i - (m-1)q_P - (N-m)q_E - c - \tau + \lambda_1 + \lambda_3 = 0, \\ -\theta z_i + \tau + \lambda_2 - \lambda_3 = 0, \\ \lambda_1 q_i = 0, \lambda_1 \geq 0, \\ \lambda_2 z_i = 0, \lambda_2 \geq 0, \\ \lambda_3 (q_i - z_i), \lambda_3 \geq 0, \\ q_i \geq 0, z_i \geq 0, q_i - z_i \geq 0. \end{array} \right. \quad (16)$$

Invoking symmetry, system (16) amounts to

$$\left\{ \begin{array}{l} a - (m+1)q_P - (N-m)q_E - c - \tau + \lambda_1 + \lambda_3 = 0, \\ -\theta z_P + \tau + \lambda_2 - \lambda_3 = 0, \\ \lambda_1 q_P = 0, \lambda_1 \geq 0, \\ \lambda_2 z_P = 0, \lambda_2 \geq 0, \\ \lambda_3 (q_P - z_P), \lambda_3 \geq 0, \\ q_P \geq 0, z_P \geq 0, q_P - z_P \geq 0. \end{array} \right. \quad (17)$$

Similarly, the Lagrangian function associated to maximization problem (6) is the following:

$$\mathcal{L}_j = O_j + \lambda_4 q_j + \lambda_5 z_j + \lambda_6 (q_j - z_j),$$

from which we obtain the optimality conditions:

$$\left\{ \begin{array}{l} a - 2q_j - (N - m - 1)q_E - mq_P - c - \tau - \alpha + \beta[mq_P + q_j + (N - m - 1)q_E] + \lambda_4 + \lambda_6 = 0, \\ -\theta z_j + \tau + \alpha + \lambda_5 - \lambda_6 = 0, \\ \lambda_4 q_j = 0, \lambda_4 \geq 0, \\ \lambda_5 z_j = 0, \lambda_5 \geq 0, \\ \lambda_6 (q_j - z_j), \lambda_6 \geq 0, \\ q_j \geq 0, z_j \geq 0, q_j - z_j \geq 0. \end{array} \right. \quad (18)$$

Invoking symmetry, system (18) amounts to

$$\left\{ \begin{array}{l} a - (N - m + 1)q_E - mq_P - c - \tau - \alpha + \beta[mq_P + (N - m)q_E] + \lambda_4 + \lambda_6 = 0, \\ -\theta z_E + \tau + \alpha + \lambda_5 - \lambda_6 = 0, \\ \lambda_4 q_E = 0, \lambda_4 \geq 0, \\ \lambda_5 z_E = 0, \lambda_5 \geq 0, \\ \lambda_6 (q_E - z_E), \lambda_6 \geq 0, \\ q_E \geq 0, z_E \geq 0, q_E - z_E \geq 0. \end{array} \right. \quad (19)$$

Solving the system composed of the two optimality conditions we obtain:

$$\begin{aligned}
q_P &= \begin{cases} \frac{\mu - \tau + (N-m)\alpha - (\mu - \tau)(N-m)\beta}{N - (N-m)\beta + 1}, & \text{if } \beta < \frac{[\mu - \tau + (N-m)\alpha]\theta - (N+1)\tau}{[(\mu - \tau)\theta - \tau](N-m)} \\ \frac{(\mu)[1 - (N-m)\beta]}{N - (N-m)(\beta - \theta + \beta\theta) + 1 + \theta}, & \text{if } \beta \geq \frac{[\mu - \tau + (N-m)\alpha]\theta - (N+1)\tau}{[(\mu - \tau)\theta - \tau](N-m)} \end{cases} \\
z_P &= \begin{cases} \frac{\tau}{\theta}, & \text{if } \beta < \frac{[\mu - \tau + (N-m)\alpha]\theta - (N+1)\tau}{[(\mu - \tau)\theta - \tau](N-m)} \\ \frac{(\mu)[1 - (N-m)\beta]}{N - (N-m)(\beta - \theta + \beta\theta) + 1 + \theta}, & \text{if } \beta \geq \frac{[\mu - \tau + (N-m)\alpha]\theta - (N+1)\tau}{[(\mu - \tau)\theta - \tau](N-m)} \end{cases} \\
q_E &= \begin{cases} \frac{\mu - \tau - (m+1)\alpha + (\mu - \tau)\beta m}{N + 1 - (N-m)\beta}, & \text{if } \beta > \frac{(\tau + \alpha)(N+1) - [\mu - \tau - (m+1)\alpha]\theta}{(\mu - \tau)\theta m + (\tau + \alpha)(N-m)} \\ \frac{(\mu)[\beta m + \theta + 1]}{N - (N-m)(\beta - \theta + \beta\theta) + 1 + \theta}, & \text{if } \beta \leq \frac{(\tau + \alpha)(N+1) - [\mu - \tau - (m+1)\alpha]\theta}{(\mu - \tau)\theta m + (\tau + \alpha)(N-m)} \end{cases} \\
z_E &= \begin{cases} \frac{\tau + \alpha}{\theta}, & \text{if } \beta > \frac{(\tau + \alpha)(N+1) - [\mu - \tau - (m+1)\alpha]\theta}{(\mu - \tau)\theta m + (\tau + \alpha)(N-m)} \\ \frac{(\mu)[\beta m + \theta + 1]}{N - (N-m)(\beta - \theta + \beta\theta) + 1 + \theta}, & \text{if } \beta \leq \frac{(\tau + \alpha)(N+1) - [\mu - \tau - (m+1)\alpha]\theta}{(\mu - \tau)\theta m + (\tau + \alpha)(N-m)} \end{cases}
\end{aligned}$$

Assuming

$$\beta \in \left( \frac{(\tau + \alpha)(N+1) - [\mu - \tau - (m+1)\alpha]\theta}{(\mu - \tau)\theta m + (\tau + \alpha)(N-m)}, \frac{[\mu - \tau + (N-m)\alpha]\theta - (N+1)\tau}{[(\mu - \tau)\theta - \tau](N-m)} \right), \quad (20)$$

ensures interior solutions.  $\square$

## Proof of Corollary 2

From (20), the upper bound that ensures positive quantities and emissions in equilibrium for all the possible market configurations, i.e., for every  $m$ , is

$$\hat{\beta}_P := \frac{(\mu - \tau + \alpha N)\theta - (N+1)\tau}{[(\mu - \tau)\theta - \tau]N}. \quad (21)$$

Hence, condition  $\beta < \hat{\beta}_P$  is sufficient to guarantee both positive quantities and emissions of profit seeking firms for all possible market composition. Moreover, let

$$\hat{\alpha}_P^{\min} := \max \left\{ 0, \frac{(N+1)\tau - (\mu - \tau)\theta}{\theta N} \right\} \quad \text{and} \quad \hat{\alpha}_P^{\max} := \frac{\tau + (N-1)(\mu - \tau)\theta}{\theta N}.$$

Assuming  $(\mu - \tau)\theta - \tau > 0$ , it holds  $\hat{\beta}_P \in (0, 1]$  for

$$\alpha \in (\hat{\alpha}_P^{\min}, \hat{\alpha}_P^{\max}]. \quad (22)$$

For the same reasoning above, starting from (20), the lower bound that ensures positive quantities and emissions in equilibrium for all the possible market configurations, i.e., for every  $m$ , is

$$\hat{\beta}_E := \max \left\{ \frac{(\tau + \alpha)(N + 1) - (\mu - \tau - \alpha)\theta}{(\tau + \alpha)N}, \frac{(\tau + \alpha)(N + 1) - [\mu - \tau - (N + 1)\alpha]\theta}{(\mu - \tau)\theta N} \right\}. \quad (23)$$

Condition  $\beta > \hat{\beta}_E$  is sufficient to guarantee both positive quantities and emissions of CSR firms for all possible market composition. Moreover, let

$$\begin{aligned} \hat{\alpha}_{E,1}^{\min} &:= \max \left\{ 0, \frac{(\mu - \tau)\theta - (N + 1)\tau}{N + 1 + \theta} \right\}, \\ \hat{\alpha}_{E,2}^{\min} &:= \max \left\{ 0, \frac{(\mu - \tau)\theta - (N + 1)\tau}{(N + 1)(1 + \theta)} \right\}, \\ \hat{\alpha}_E^{\max} &:= \frac{(\mu - \tau)\theta - \tau}{1 + \theta}. \end{aligned}$$

Then

$$\frac{(\tau + \alpha)(N + 1) - (\mu - \tau - \alpha)\theta}{(\tau + \alpha)N} \in (0, 1],$$

for  $\alpha \in (\hat{\alpha}_{E,1}^{\min}, \hat{\alpha}_E^{\max}]$ . Differently,

$$\frac{(\tau + \alpha)(N + 1) - [\mu - \tau - (N + 1)\alpha]\theta}{(\mu - \tau)\theta N} \in (0, 1],$$

for  $\alpha \in (\hat{\alpha}_{E,2}^{\min}, \hat{\alpha}_E^{\max}]$ .

## Stability of the market configuration

In this Appendix, we check for the stability properties of the equilibrium market configuration. In the present dynamical system, the stability properties of its stationary states are simple: in one-dimensional autonomous system, an equilibrium is stable if the derivative of the differential

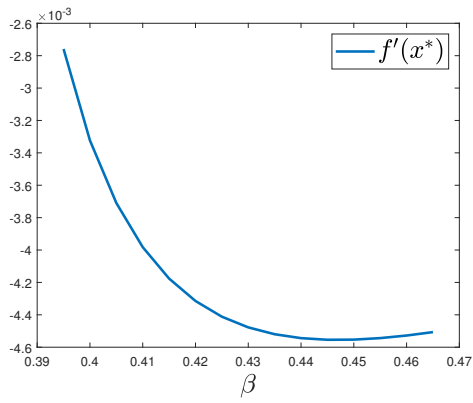
equation, evaluated at the equilibrium point, with respect to the variable itself is negative.

Denoting  $\dot{x} = f(x)$ , in our case the equilibrium  $x^* \in (0, 1)$  is stable if

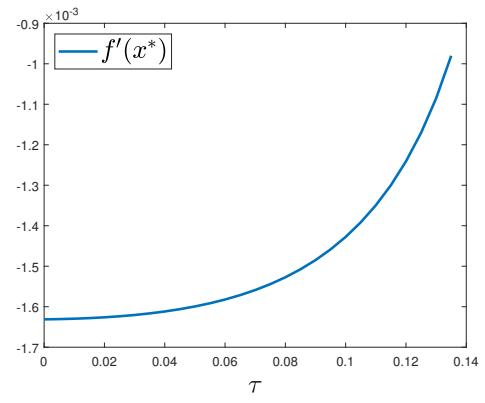
$$f'(x^*) < 0. \quad (24)$$

If condition (24) is satisfied, then the equilibrium point  $x^* \in (0, 1)$  is globally attractive. This means that, whatever the initial conditions, all the trajectories converge to  $x^*$ .

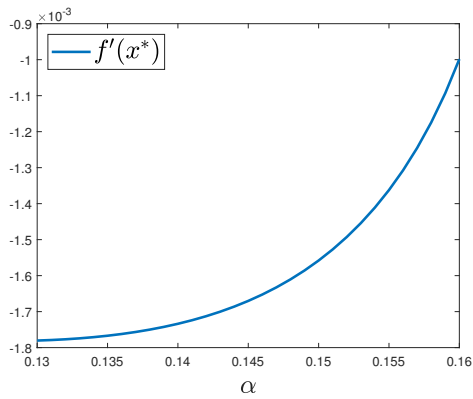
In Figure 4, we check the negativity of condition (24) with respect to variations to different parameters in an oligopoly with  $N = 4$  firms. The results are robust to all the parameter values considered.



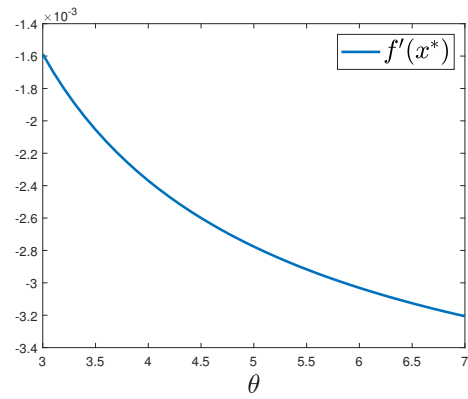
(a)  $\alpha = 0.15, \tau = 0.1, \theta = 4.$



(b)  $\alpha = 0.15, \beta = 0.43, \theta = 4.$



(c)  $\beta = 0.43, \tau = 0.1, \theta = 4.$

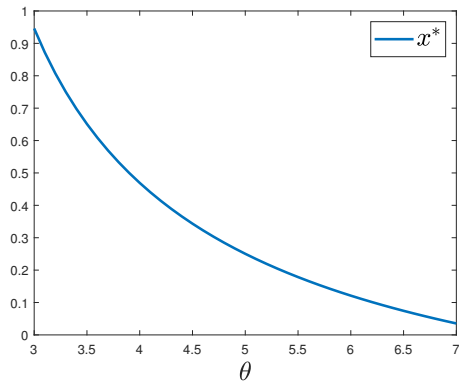


(d)  $\beta = 0.43, \tau = 0.1, \alpha = 0.15.$

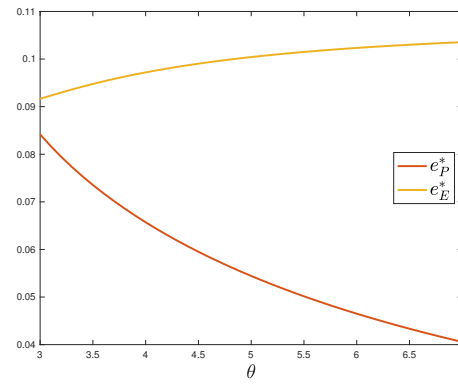
**Fig. 4.** Market configuration stability,  $N = 4$ .

## Cost of innovation

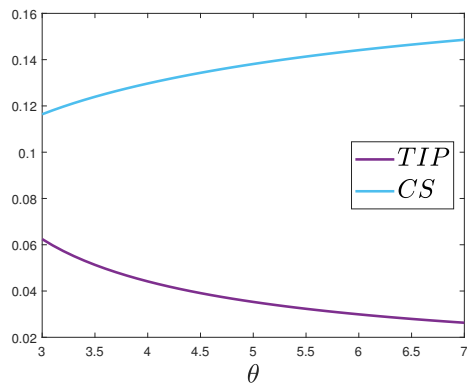
In this appendix we show the changes in equilibrium following a variation in the cost of innovation.



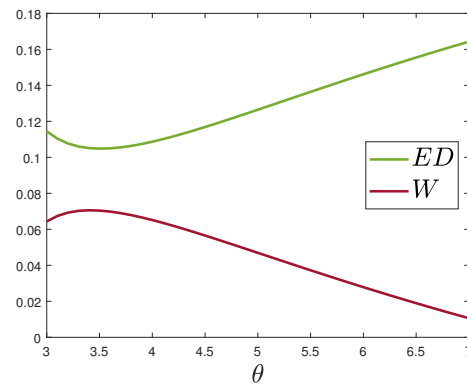
(a) Long run share of profit-seeking firms.



(b) Emissions of profit-seeking and CSR firms.



(c) Total Industry Profits and Consumer Surplus.



(d) Environmental Damage and Welfare.

**Fig. 5.** The cost of innovation,  $\alpha = 0.15$ ,  $\beta = 0.43$ ,  $\tau = 0.1$ ,  $N = 4$ .

## References

- Antoci, A., Borghesi, S., Iannucci, G., Sodini, M., 2021. Should I stay or should I go? carbon leakage and ets in an evolutionary model. *Energy Economics* 103, 105561.
- Antoci, A., Gay, A., Landi, M., Sacco, P. L., 2008. Global analysis of an expectations augmented evolutionary dynamics. *Journal of Economic Dynamics and Control* 32 (12), 3877–3894.
- Bischi, G. I., Merlone, U., Pruscini, E., 2018. Evolutionary dynamics in club goods binary games. *Journal of Economic Dynamics and Control* 91, 104–119.
- De Giovanni, D., Lamantia, F., 2016. Control delegation, information and beliefs in evolutionary oligopolies. *Journal of Evolutionary Economics* 26 (5), 1089–1116.
- Dieci, R., Schmitt, N., Westerhoff, F., 2018. Interactions between stock, bond and housing markets. *Journal of Economic Dynamics and Control* 91, 43–70.
- Droste, E., Hommes, C., Tuinstra, J., 2002. Endogenous fluctuations under evolutionary pressure in Cournot competition. *Games and Economic Behavior* 40 (2), 232–269.
- Fukuda, K., Ouchida, Y., 2020. Corporate social responsibility (CSR) and the environment: Does csr increase emissions? *Energy Economics* 92, 104933.
- Gioffré, A., Tampieri, A., Villanacci, A., 2021. Private versus public companies with strategic csr. *Journal of Economics* 133, 129–166.
- Hirose, K., Matsumura, T., Lee, S., 2017. Environmental corporate social responsibility: a note on the first-mover advantage under price competition. *Economics Bulletin* 37, 214–221.
- Hommes, C. H., Ochea, M. I., Tuinstra, J., 2018. Evolutionary competition between adjustment processes in cournot oligopoly: instability and complex dynamics. *Dynamic Games and Applications* 8 (4), 822–843.
- Kopel, M., Lamantia, F., 2018. The persistence of social strategies under increasing competitive pressure. *Journal of Economic Dynamics and Control* 91, 71–83.
- Kopel, M., Lamantia, F., Szidarovszky, F., 2014. Evolutionary competition in a mixed market with socially concerned firms. *Journal of Economic Dynamics and Control* 48, 394–409.
- KPMG, 2020. The KPMG survey of sustainability reporting 2020. Tech. rep., KPMG.
- Lamantia, F., Negriu, A., Tuinstra, J., 2018. Technology choice in an evolutionary oligopoly game. *Decisions in Economics and Finance* 41 (2), 335–356.
- Lambertini, L., Palestini, A., Tampieri, A., 2016. CSR in an asymmetric duopoly with environmental externality. *Southern Economic Journal* 83, 236–252.
- Lambertini, L., Pignataro, G., Tampieri, A., 2020. Competition among coalitions in a Cournot industry: a validation of the porter hypothesis. *Japanese Economic Review*, 1–35.
- Lambertini, L., Tampieri, A., 2015. Incentives, performance and desirability of socially responsible firms in a Cournot oligopoly. *Economic Modelling* 50, 40–48.
- Leal, M., García, A., Lee, S., 2018. Corporate social responsibility and environmental taxation with endogenous entry. *Hitotsubashi Journal of Economics* 59, 25–43.

- Li, D., Wang, L., 2021. Does environmental corporate social responsibility (ECSR) promote green product and process innovation? *Managerial and Decision Economics* 59, 1–9.
- Liu, C., Wang, L., Lee, S., 2015. Strategic environmental corporate social responsibility in a differentiated duopoly market. *Economic Letters* 129, 108–111.
- Nie, P., Wang, C., Meng, Y., 2018. An analysis of environmental corporate social responsibility. *Managerial and Decision Economics* 40, 384–393.
- Porter, M., 1991. America's green strategy. *Scientific American* 264, 168.
- Tichý, T., Radi, D., Lamantia, F., 2020. Hybrid evolutionary oligopolies and the dynamics of corporate social responsibility. *Journal of Economic Interaction and Coordination* 17, 87–114.
- Weibull, J., 1995. *Evolutionary Game Theory*. MIT Press, Cambridge, MA, USA.
- Xu, L., Chen, Y., Lee, S., 2022. Emission tax and strategic environmental corporate social responsibility in a Cournot–Bertrand comparison. *Energy Economics* 107, 105846.
- Xu, L., Lee, S., 2018. Corporate social responsibility and environmental taxation with endogenous entry. *Hitotsubashi Journal of Economics* 59, 61–82.
- Xu, L., Lee, S., 2022. Non-cooperative and cooperative environmental corporate social responsibility with emission taxes. *Managerial and Decision Economics* , 1–14.