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Voting in Corporate Boards with Heterogeneous Preferences*

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Abstract

We analyze the voting behavior of a board of directors that has to approve (or reject) an investment proposal with uncertain return. We consider three types of directors: insiders, who are biased toward acceptance of the project, independent outsiders who want to maximize the firm's profit and independent outsiders who care about their reputation. We show that the presence of members with heterogeneous preferences can be beneficial and that the partisan behavior of insiders can be used as a sort of coordinating device by uninformed outsiders. Provided that the size of the board is optimal, there is no gain from increasing the number of outsiders above the strict majority despite the fact that each outsider is informed with positive probability. Substituting profit-maximizing directors with directors concerned about their reputation is not an obstacle to profit maximization provided that an appropriate sequential voting protocol is followed. We also show that a proper board composition makes communication between directors irrelevant in the sense that the same outcome is obtained with and without communication. Finally, as information is costly, our model provides some suggestions on the optimal size of boards.

Key words: Board of directors, Voting, Corporate Governance.

JEL clasification: G30, D71.

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1 Introduction

A peculiar feature of boards is that directors represent different stakes (majority shareholders, minority blockholders, investors, workers, etc.) and perform different tasks. Members of the board range from the CEO and other executives to representatives of mutual funds, members of the family owning the firm, bankers, venture capitalists, university professors, and so on. In general, the diversity of members reflects in their preferences and objectives. As a result, directors may have objectives other than profit maximization even if members are formally chosen by shareholders[1].

On the positive side, a more diverse board is likely to involve a broader range of experiences and perspectives. At the same time, agreeing upon a decision becomes more difficult when members have conflicting interests. Different views are more difficult to reconcile, discussions at the board meetings become more time consuming. Overall diversity of interests makes unanimity more difficult to reach. Despite this, when directors have all the time necessary to discuss and exchange information on the proposal under consideration, it is likely that in the end they will share the same view and vote unanimously. Indeed, in corporate boards many decisions are taken unanimously. When instead communication is prevented or seriously limited, heterogeneity of preferences is a severe obstacle to unanimity. Lack of communication is often reported as one of the main problems of corporate boards. A common complaint of directors is that boards do not meet with the frequency and the time necessary to guarantee proper communication among directors.

In general, unanimity is not necessarily a positive signal. The absence of conflict may indicate apathy rather than harmony as suggested also by the press after the wave of corporate scandals at the beginning of the twenty-first century. "At Enron, for instance, nearly every vote was unanimous. When Fastow and Skilling proposed the strategies that eventually destroyed the company, board members asked a few cursory questions, then signed off. The directors had forgotten that they had the power to say no"[2]. Disagreements and dissent are usually seen as an obstacle to a well functioning board, while consensus is deemed to be a signal of harmonious team work. However, Eisenhardt, Kahwajy and Burgeois [1997] have found that successful companies do encourage disagreements. According to them the real challenge is not to avoid conflict but to help management teams to have a "good fight": "They

[1]For example, it is well documented that the CEO plays an important role in the appointment and confirmation of directors (see Hermalin and Weisbach [1988, 1998]).

[2]Surowiecki, J., Board Stiffs, *The New Yorker,* March 8, 2004

(the managers) know that conflict over issues is natural and even necessary. Reasonable people, making decisions under uncertainty, are likely to have honest disagreements over the best path for their company's future" (page 77).

Our paper formalizes the idea that disagreement may be useful by showing that when communication is constrained the simultaneous presence of directors with heterogenous preferences turns out to be beneficial and an openly split vote reflecting the directors' different preferences may be optimal. In particular, the paper studies the voting behavior of a board that has to decide by simple majority whether to undertake a project whose returns may be positive or negative according to the state of nature. Directors may incur a positive cost to acquire information on project profitability. Once this cost is incurred, they observe the true state of nature with positive probability.

We consider three types of directors where a type is defined by director's preferences. The first type comprises inside directors (the CEO and other executives) and affiliated outsiders (i.e. directors with financial or family ties with the company). These directors (insiders, from now on) are biased in favor of the project. This may happen either because they can extract private benefits from project realization or because the CEO favors the project and insiders do not want to contrast him, since he may be crucial to their careers. For example, the project may refer to a new plant to be built, or a new market to enter. Insiders prefer a larger firm to a smaller one. As a consequence, they disregard any information they may have and, independently of the state of nature, always vote in favor of the project.

The second type includes all independent outside directors whose objective is to maximize expected profit, i.e. directors who care about the board making the right decision. Contrary to insiders, these directors perfectly represent shareholders' interests and want to approve the project only when its return is positive. Therefore they condition their voting strategy on the information on the state of nature. We call this group profit-maximizing outsiders.

The last group of directors is composed of a different type of outsiders that we call reputational. These directors have reputational concerns in the sense that they want to vote 'correctly' but they don't care about the board's final decision. The difference between profit-maximizing and reputational directors depends on the fact that profit maximizing directors want to induce the board to make the right decision while reputation-building directors simply want to cast the right vote. In other words, they want to show to the market that they are right in order to strengthen their reputation but have no interest in the board's final decision being right or wrong. For example, stakeholders representatives may be more concerned about showing that they have been loyal to their "mission" than about the board's final decision. Alternatively, a consultant can use the visibility provided by his position on the board with the final goal to elicit (new) job offers, more directorships or a higher salary.

It is well known that the behavior of economic agents with career concerns may be inefficient and lead to distortions[3]. We study how an optimal composition of the board may minimize the inefficiencies resulting from the diverse objective functions of directors.

We assume that the market can observe individual votes, and more importantly, the realized state of the world so that it can assess which decision was the proper one and reward directors that vote "correctly". This transparency assumption (which is necessary in order to assess the behavior of outsiders with reputational concerns) may look unrealistic but in our opinion it is reasonable in many situations. Even if it is the board decision that is always made public, members with reputational concerns may communicate their votes directly to the market (for example, in a press conference). Moreover, in many cases, records of the votes must acknowledge the existence of minority positions, so the market can verify the information provided by different members. For example, the Italian security and exchange commission, CONSOB, requires listed firms carrying on operations that may involve conflicts of interest to make the opinion of independent directors public, especially when they have a contrarian position.

Our classification resembles the one proposed by Baysinger and Butler [1985]. According to them, the board of directors can be disaggregated into three major components on the basis of the function performed by directors: the executive component that includes corporate officers and other insiders, the monitoring component that includes independent decision makers and other professional directors, and the instrumental component that includes financiers, consultants, legal counsel and stakeholders representatives. The monitoring component's primary function is to ensure that managers stay aligned with shareholders' interests and performs the same function as our profit-maximizing outsiders. The instrumental component's functions are more vaguely defined: these directors help to improve decision-making and provide a link between organizations. It is, then, reasonable to assume that directors belonging to the instrumental component are likely to have reputational concerns.

Our paper shows that a strict majority of outsiders within the board is a necessary and sufficient condition to reach the highest possible level of expected profit. No further gains are obtainable by increasing the number of outside directors, despite the fact that they may be informed on project profitability. This is true both when all outsiders are profit maximizers and when there are also reputational directors. However, in the latter case uniqueness of the optimal equilibrium is only reached under a sequential voting protocol. An interesting implication of this result is that the relationship between board composition and firm performance is highly non linear: all the benefits deriving from the presence of outsiders are secured when their percentage increases from 49% to 51%. This may explain why the

[3] Although Suurmond, Swank and Visser [2004] have shown that reputational concerns may also induce an agent of unknown type to increase his effort with a potential benefit for the principal.

empirical literature has found no clear positive relation between the number of outsiders on the board and firm performance.[4]

We also show that the possibility of pre-voting communication is not crucial. If the board is composed only of insiders and profit-maximizing outsiders, the introduction of pre-voting communication among directors does not affect our previous results provided the initial board composition is optimal. When there are also reputational directors, pre-voting communication ensures optimality even without sequential voting. Finally, as information is costly, our model also provides prescriptions on the optimal size of the board. Given the optimal board composition (which always comprises at least one profit maximizing outsider), the optimal board size is determined by the trade-off between costs and benefits of information acquisition.

We focus our attention on the voting mechanism and we ignore other, undoubtedly relevant, factors that have been studied by other authors. For example, we assume that directors are informed with a costly but exogenously given probability. Thus we assume away any moral hazard problem in becoming informed. Further, since a simple majority rule is commonly used in boards of directors we restrict our attention to this voting rule without investigating if this is indeed the optimal rule.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents the basic model. Section 4 examines the outcome of the voting game in boards with only insiders and profit-maximizing outsiders. In Section 5, we introduce outside directors with reputation concerns and we show how results change when outsiders have different objectives. Finally, Section 6 concludes. All proofs are collected in the Appendix.

2 Related literature

Our paper is related to two strands of literature, the literature on voting and information aggregation and that on board of directors. In the voting literature, preferences are usually assumed to be homogeneous, although few recent papers have addressed the issue of conflict of interest. The main contributions are Feddersen and Pesendorfer [1996] and [1997], who analyze how well simultaneous voting in large elections can aggregate private information and show that the probability of electing the "wrong" candidate asymptotically goes to zero. Conflict of interest is more problematic in small committees where information aggregation may be severely limited by strategic voting induced by divergent interests. In a standard Condorcet Jury Theorem framework, this problem may be partially offset by providing mi-

[4] See for example the survey by Hermalin and Weisbach [2003].

nority members with optimal incentives to participate in voting. Thus their information is not wasted (Chwe [1999]).

Things become more complicated when the relevant issue is not to find optimal voting rules but rather an optimal way to aggregate "useful (or correct)" information. Li, Rosen and Suen [2001] examine a two-person committee where each member receives a private signal and reports his information. Since members have conflicting interests, strategic considerations induce information misreporting and there is no truth-telling equilibrium. Conflict of interest prevents full information aggregation also in the model by Maug and Yilmaz [2002] where the authors suggest grouping voters with different objectives into separate classes. Indeed, if interest groups are of similar size, a two-class voting mechanism may alleviate the incentive to withhold information and voting decisions become more informative. Wolinsky [2002] suggests solving the problem in a similar way, by partitioning experts in different groups. This may enhance their incentive to reveal information, at least when they think they are pivotal. Although in our model directors are not grouped into different classes, we allow for some separation between types. When directors can communicate, we allow them to exchange messages with other members of the same type, before sending messages across types. An important difference between the above mentioned papers and ours concerns the information structure: in the above papers, committee members observe a noisy signal, in our paper instead we use a simplified setting where directors either observe the true state of the world or they observe nothing.

Cai [2009] develops a model of committee size, where information gathering is costly and preferences are heterogeneous. In his model experts learn their preferences by gathering information. On the contrary, in our model preferences are known from the beginning. Moreover, our main focus is on board composition rather than on board size.

Information aggregation and communication is an important topic also in the literature on board of directors. A common assumption in most of this literature is that insiders are assumed to be better informed on the quality of the project than outsiders. The problem is that insiders' preferences, contrary to those of outside directors, are not aligned with the preferences of the shareholders. It follows that the final decision cannot (always) be delegated to insiders and the question becomes how to induce insiders to share their information with outside directors (see for example Adams and Ferreira [2007], Harris and Raviv [2006], and Raheja [2005])[5]. Our model differs from theirs in that we focus on information aggregation

[5] A related paper by Dessein [2002] suggests that delegating decision power may be better than gathering information, when agents (in our case, insiders) have different preferences from a principal (shareholders). A different approach to study the decision-making process in a board with different preferences is taken by Baranchuk and Dybvig [2008]. They use a new bargaining solution concept called consensus. Directors' preferences are modelled in a spatial model where a director's utility depends on the distance between the decision made and the director's ideal decision. They focus on "grey" directors and do not consider incentive

only among outsiders and we assume that insiders do not communicate any information at all. The positive role of insiders in our model does not rely on their superior information with respect to outsiders, but on the fact that their presence on the board simplifies outside directors' strategies.

Insiders have an information advantage also in the experimental papers by Gillette, Noe and Rebello [2003] and [2008]. These authors examine voting patterns in a board composed of insiders and outsiders (watchdogs) and they show that the inclusion of uninformed outsiders improves board efficiency. Thus, the outsiders' contribution does not derive from their information but from the fact that voting outcomes are more efficient when preferences are heterogeneous. Efficient equilibria however are not coalition-proof (with respect to insiders) even if the evidence of the experiments indicates that they are generally adopted when there is a majority of outsiders.[6] Our model shares with theirs the feature that the inclusion of outsiders may improve board efficiency even if the information available to the board is not improved. In our model all equilibria are coalition proof; however, the relevant information is that of the outsiders rather than that of the insiders and there is no penalty for board dissent. Another major difference is that no communication is allowed in our basic framework.

Another paper related to ours is Warther [1998] who studies the voting mechanism in a board with a manager and two outside directors. The board has to decide whether to retain or dismiss the manager. If an outside director votes to dismiss the manager and the manager is not eventually fired, the dissenting director is ejected from the board. As a result, the board will make the dismissal/retention decision unanimously. In our model there is no penalty for dissenting directors: on the contrary, outside directors find useful the fact that insiders have a different voting strategy.

Finally, unanimity may result also from reputational concerns as shown by Visser and Swank [2007]. They show that members want to vote unanimously since disagreement may signal lack of competence and therefore may decrease the members' reputation. Our result of a split vote stands in sharp contrast with theirs. This follows from the different assumptions made by the two models on what is observable: we assume a transparent voting mechanism where the market can observe the true state of nature while in their model, on the contrary, the state of nature is never observed.

for information sharing, costly effort and delegation.

[6] In the 2008 paper where the performance of the board is related to its structure (one-tier, two-tier), experiments indicate that inclusion of outsiders improves decision-making also when there is a majority of the insiders but having a majority of outsiders (in one of the tiers) greatly improves the outcome.

3 The model

The model analyzes the voting behavior of a board composed of insiders and outsiders. Our aim is to provide insights on how heterogeneous preferences can be exploited to reach the equilibrium outcome desired by the shareholders. To this end, we restrict our attention to a simplified setting where the board has only one task, to approve or reject a project, and we disregard any other task that the board may usually perform. Furthermore we take for granted that insiders are needed on the board although we do not examine the specific tasks performed or the contributions given by insiders.

Project's value and Information

The board, composed of $2n+1$ directors, decides whether to approve a project (voting "yes") or reject it (voting "no") by majority vote. If the proposal is rejected, a value of 0 is realized. If it is accepted, the investment can take one of two values, according to the state of nature: when the state is low (L), the value is $v=-1$; when the state is high (H), the value is $v=1$. In other words, the project can create a profit or a loss for the firm. Each value has the same prior, $\frac{1}{2}$. This implies that when directors have no information on the state of nature there is no one choice that dominates the other.

Information acquisition

Directors can obtain some information on project profitability only by incurring a non-monetary cost, possibly different for different types of director[7]. The cost represents the time and effort a director needs to gather information on the project and its probability of success. Given that, as explained below, insiders always favor the project, they will never incur their information cost. Consequently we focus on the cost for outside directors and we denote it by c. We assume that c is a positive constant equal for all outsiders. By incurring c, an outside director learns the true state with probability $\alpha \in (\frac{1}{2}, 1)$ and with probability $1-\alpha$ he learns nothing. Thus, the choice of whether to acquire information is a binary choice and directors cannot influence the accuracy of this information. As a consequence, the information set of a generic member of the board m is simply $\Omega_m = \{\omega_m\}$, with $\omega_m \in \{H; L\}$, when m is informed. On the contrary, the information set of an uninformed director is $\Omega_m = \{H, L\}$, as he does not know the true state of the world.

Voting and communication

We derive our main results under the assumption that there is no communication between directors. In the basic framework directors first decide whether to get informed, then they

[7]Insiders' information cost might be lower than that of the outsiders. For example inside directors might know the true state of the world at zero cost. Such informational advantage could be a by-product of their managing the firm. However, this is unifluential in our context.

participate in the board meeting and cast their vote. Then we contrast these findings with those obtained in a voting game with pre-voting communication. In this case, we add a communication stage that takes place after information acquisition and before voting. In Section 5, while maintaining the assumption of no pre-voting communication, we consider a sequential voting game. A sequential voting mechanism can be regarded as a case where there is information transmission from predecessors to successors in the voting sequence. Indeed, when directors vote sequentially, the information set Ω_m also contains previous members' voting decisions, d_m.

Each director expresses one vote and abstention is not possible. This is made without loss of generality, as in our context directors would never abstain in equilibrium.

As a consequence of the no-abstention assumption, a director m can only vote "yes", $d_m = y$, to accept the project, or vote "no", $d_m = n$, to reject it. A strategy s_m is a member's voting behavior, conditional on his information set. We concentrate on pure strategy equilibria[8].

Directors' types and utility functions

We assume that directors are risk neutral and that their types are common knowledge. Let I denote insiders, O profit-maximizing outsiders and R reputational outsiders.

A member of type I is an "empire builder" who derives utility from project implementation, for example from the enlargement of the firm[9]. He always supports the investment project, regardless of the value which is *ex post* realized. His utility u_I is therefore a function of the final decision of the board, D, where $D = Y$ when the project is approved ("yes" wins) and $D = N$ when the project is rejected ("no" wins). Accordingly, $u_I(D)$ can take the following two values:

$$u_I(D) : \begin{cases} 1 & if \quad D = Y \\ 0 & if \quad D = N \end{cases} \tag{1}$$

This clearly implies that always voting "yes" is a dominant strategy for an insider. For simplicity, we abstract from additional problems, such as that of a member I's reputation when his proposal creates a loss or is rejected. Furthermore, we abstract from any mechanisms that may mitigate the conflict of interest between insiders' and shareholders, as incentive pay or equity holdings.

A profit-maximizing outsider derives utility from the firm's profit, π, and, in order to obtain the highest level of profit he may want to incur the information cost, c. Let $\Im_c$ be an

[8] In a companion paper (Balduzzi, Graziano and Luporini [2011]) we prove that mixed strategies equilibria are always dominated by pure strategies equilibria.

[9] Alternatively we can interpret the behavior of this type as the consequence of his overconfidence in the success of the project. This however implies that the prior he attaches to the state of the world H ($v = 1$) is higher than $\frac{1}{2}$.

indicator function that takes value 1 if a director decides to acquire information and value 0 if no information is acquired. The utility function of a profit-maximizing outsider, $u_O(\pi, c)$, then takes the form:

$$u_O(\pi, c) = \pi - \Im_c c, \qquad \Im_c \in \{0, 1\}$$

Given this utility function, an O director will choose the strategy that maximizes the expected profit of the firm minus c. To this end, he will condition his strategy on being pivotal, because that is the only case where he can actually influence the outcome of the voting process and therefore his own utility. Since any strategy is optimal when the member is not pivotal, without loss of generality, we concentrate on weakly dominant strategies, that is on strategies which are optimal when the member is pivotal.

Notice that, given the values the investment can take, maximizing $E(\pi)$ is equivalent to maximizing the probability that the board takes the correct decision. Indeed, the latter is given by the sum of the probability that "yes" wins when the actual value of the alternative is 1 and that "no" wins when the actual value of the alternative is -1:

$$\frac{1}{2}\left\{Y(\cdot \mid v = 1) + [1 - Y(\cdot \mid v = -1)]\right\}, \tag{2}$$

where the function $Y(\cdot|\cdot)$ is the conditional probability that the board as a whole votes "yes". The expected profit of the firm is:

$$E(\pi) = \frac{1}{2}[E(\pi|v = 1) + E(\pi|v = -1)] = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] \tag{3}$$

and it is straightforward to notice that expressions (2) and (3) are strategically equivalent.

Finally, as regards reputational outsiders, we assume that their reputation depends only on their own individual votes. In other words, a reputational outsider wants to vote correctly (i.e. vote for approval/rejection of the project if it is profitable/unprofitable) to foster his reputation. Contrary to profit-maximizing outsiders, he is not interested in the final decision being correct and wants to vote correctly even if the final decision of the board is wrong. We do not model explicitly the way in which the market forms and revises its judgements on directors' reputation and we simply capture this process through our assumption on reputational directors' utility.

Recalling that $\Im_c$ takes value 1 only if a director decides to acquire information, an R member has a utility function $u_R(d, c)$ that takes one of the following two values :

$$u_R(d, c) : \begin{cases} u_R(d = y \mid v = 1) = u_R(d = n \mid v = -1) = 1 - \Im_c c \\ u_R(d = y \mid v = -1) = u_R(d = n \mid v = 1) = 0 - \Im_c c \end{cases}, \qquad \Im_c \in \{0, 1\}$$

Given this utility function, an R will do anything to make the correct decision, even if uninformed.

Compensating Strategy

Before analyzing the voting behavior of the board, we introduce the definition of compensating strategy that will be useful in the following sections.

Definition 1 (Compensating strategy) *Two players are compensating for each other when the following conditions are satisfied: i) they are both uninformed; ii) they play "yes" with probabilities whose sum is equal to 1.*

4 Boards with insiders and profit - maximizing outsiders

A board with a majority of insiders would always approve the project, obtaining the best choice for the shareholders half of the time (i.e., when the state of the world is H). The expected profit of the firm would be zero. Our aim is to explore under which conditions a board with both outsiders and insiders can improve this performance. It is easy to see that the presence of outsiders on the board would not affect the result of the voting game as long as they are the minority group. We then focus on a board with a majority of outsiders who can reject the project if they believe it is unprofitable. In the following proposition we determine the equilibria of the game under the assumption that outsiders incur the information cost c. Subsequently, we determine the condition on the size of the board that actually makes it optimal for outsiders to acquire information.

Proposition 1 *In an outsider-controlled board with $2n+1$ members, where outsiders incur the information cost c and insiders always vote in favor of the project, two cases may occur:*
i) if there are n insiders and $n+1$ outsiders there is a unique equilibrium where all uninformed outsiders vote "no".
ii) if there are $n-k$ insiders ($n > k > 0$) and $n+1+k$ outsiders, all the equilibria are such that, when uninformed, $n-k+1$ outsiders vote "no" and the other $2k$ outsiders compensate for each other.

In both cases, outsiders' equilibrium strategies are such that informed directors follow their information, and $E(\pi) = \frac{1}{2}[1-(1-\alpha)^{n+1}] \equiv \widehat{E(\pi)} > 0$.

The behavior of informed outsiders is straightforward: in order to maximize profit they follow their information. The crucial question is how uninformed outsiders vote. Proposition 1 states that in a board with n insiders and $n+1$ outsiders, uninformed outsiders vote "no", contrasting the insiders. In a board with $n-k$ insiders and $n+1+k$ outsiders, $n-k$ outsiders vote "no" contrasting the insiders and the remaining outsiders compensate for each other. To maximize expected profit, uninformed outsiders should not influence the voting outcome and let informed outsiders (if any) be pivotal. This leads to the best outcome for shareholders. Notice that, if there are $n+1$ outsiders and n insiders, an outsider would never find it profitable to abstain even if he were allowed to do so.

As expected, an outsider-dominated board improves shareholders' welfare with respect to an insider-dominated one and reaches positive expected profit $\widehat{E(\pi)} = \frac{1}{2}[1-(1-\alpha)^{n+1}]$. If at least one outsider is informed, the right decision is made. When no outsider is informed, which happens with probability $(1-\alpha)^{n+1}$, the project is rejected and this is the right decision only half of the time. There is however no way to improve this outcome. Even the decision to accept the project would be right only half of the time. What is less obvious is that the level of expected profit is the same irrespective of the proportion of outsiders. In other words, for a given board size, increasing the proportion of outsiders brings no improvement, provided that outsiders have the majority. In particular, this means that even a board entirely composed of outsiders would not be able to improve upon the result of a board with just a strict majority of outsiders[10]. Also such a board would make the wrong decision with the same probability, $\frac{1}{2}(1-\alpha)^{n+1}$, and consequently would reach the same level of expected profit[11]. The following remark highlights this point.

Remark 1 *For a given board size, the expected profit of the firm is not increased by increasing the proportion of outside members above* $\frac{n+1}{2n+1}$.

As long as a majority of outsiders is maintained, having insiders on the board brings no harm to shareholders. On the contrary, insiders have a positive role arising from the fact that their presence simplifies outsiders' strategies: when uninformed, the latter know that they must contrast the biased vote of the former, and there is no need to coordinate with each other. Relying on the result of Proposition 1 and on Remark 1, in the rest of the paper we restrict our attention to boards composed of n insiders and $n+1$ outsiders.

Given the optimal board composition we now have to determine the optimal size of the board. When studying the equilibrium, we have taken for granted that all outsiders pay the information cost c, but each outsider will actually do so only if the advantage of becoming

[10] For a formal analysis of this case, see Balduzzi, Graziano and Luporini [2011].

[11] This result crucially depends on the assumption that directors do not communicate. See Section 4.1 for more details.

informed is at least as large as the cost. Equilibrium expected profit, $\widehat{E(\pi)}$, is increasing in n, because the probability that at least one outsider is informed increases with n. However, $\widehat{E(\pi)}$ increases at a decreasing rate, making the advantage of becoming informed for an individual outsider depend on the size of the board. Notice that the gain from acquiring information is positive only when an outsider is pivotal because it is only in this case that being informed makes a difference. Given the strategies of the insiders, each outsider is more likely to be pivotal when $v = -1$ and therefore votes "no" when uninformed. Information changes his vote (and expected profit) only in the favorable state of the world. Let $E[\pi(n)]$ indicate the expected profit when n outsiders decide to acquire information, and let $\Delta B_O(n+1)$ indicate the marginal benefit of acquiring information for the $n+1^{th}$ outsider. Then

$$\Delta B_O(n+1) \equiv E[\pi(n+1)] - E[\pi(n)] = \frac{\alpha}{2}(1-\alpha)^n \tag{4}$$

is decreasing in n and the next proposition and corollary immediately follow.

Proposition 2 *In a board composed of n insiders and $n+1$ outsiders, the optimal size of the board is $2\overline{n}+1$ where $\overline{n}$ is the smallest integer such that:*

$$\Delta B_O(n+1) = \frac{\alpha}{2}(1-\alpha)^n \geq c. \tag{5}$$

Corollary 1 *$\overline{n}$ is decreasing in c.*

The optimal size of the board is such that all outside directors acquire information. Expanding board size would induce free riding among outsiders who have a lower incentive to become informed. Indeed, from condition (5), it can be seen immediately that $\overline{n}$ is decreasing in c. This happens because the probability that a given director is pivotal decreases as n increases, and this in turn decreases the individual incentive to become informed[12]. Since the cost of acquiring information varies across industries, an empirical implication of Proposition 2 is that the optimal size of the board is smaller in industries (such as innovative ones) where it is relatively costly to get the information or the skills necessary to evaluate investment proposals than in more traditional industries where project evaluation is easier (see also Raheja [2005]).

As to the effects of changes in the probability of getting informed we can establish the following.

Corollary 2 *For a given value of c, the incentive to acquire information is decreasing in α.*

[12] The same endogenous determination of board size as a result of the decreased incentive to become informed arises in Harris and Raviv [2008].

An increase in α has a negative impact on the incentive to acquire information because a higher value of α means a lower probability of being pivotal. Indeed, if one outsider is informed, none of the others is pivotal. On the contrary, when α is low, every single director has a high marginal benefit from acquiring information because he is more likely to be decisive in the voting game. Hence, for a given c, the free-riding problem among outsiders becomes more severe as α increases. This in turn implies that the optimal board size decreases in α.

4.1 Voting outcome with communication

Communication between directors is an important factor that can improve the quality of the decisions made by the board, but cannot be taken for granted even among directors whose objectives do not conflict. In reality, it is quite common that outsiders simply do not have the time to have regular meetings and share information. For this reason, charters and guidelines often prescribe that outsiders should meet periodically in order to share information and discuss their views without insiders who might try and distort both information and its processing[13].

So far we have analyzed a situation where outside directors do not communicate prior to voting. But in our context profit-maximizing outsiders have a clear incentive to communicate their information so as to induce optimal behavior from other outsiders. In this section, we examine the effect of pre-voting communication on the voting outcome.

Communication is introduced as a pre-voting stage where outsiders send costless messages about their information sets. We consider situations where the decision to become informed is made before the message stage. In other words, we refer to situations where the process of collecting information takes time, so that an outsider cannot strategically postpone such decision after the message stage in order to free ride on other, possibly informed, outsiders. Messages are cheap talk. Recall that the information set of a generic board member m is $\Omega_m = \{\omega_m\}$, with $\omega_m \in \{H; L\}$, when m is informed, and $\Omega_m = \{H, L\}$ when m is uninformed. Consequently, member m can send a message $\sigma_m \in \{\omega_m; 0\}$, where $\sigma_m = 0$ means that m sends no information. Messages are exchanged simultaneously among outside directors and enter the information set of all other outsiders[14]. Finally, we assume that the communication stage cannot be observed by the public.

[13] Gilette, Noe and Rebello [2003], for example, report that since 1994 General Motors requires that outsiders meet without insiders at least two or three times a year.

[14] This is not crucial in our model. Alternatively we can assume that messages are exchanged among all directors and enter everybody's information set. Notice however that insiders cannot commit to send truthful messages because of their strong bias. Thus, they would never be believed. This is equivalent to assuming that insiders do not send any message, i.e. $\sigma_I = 0$. On the other hand, given their preferences, insiders would not change their strategies even if they received a message revealing that the state of nature is L. For these reasons we focus on the message strategies of outside directors.

As in the case of no communication, we first determine the equilibrium of the voting game under the assumption that all outsiders incur the information cost and then we determine which size and composition of the board induces them to incur such a cost. Assuming that, when indifferent, an informed outsider always sends a truthful message, the equilibrium strategies are given in the following proposition.

Proposition 3 *When outsiders pay c and can communicate in a board with n insiders and $n+1$ outsiders there are multiple equilibria all of which yield $E(\pi) = \widehat{E(\pi)}$. Each informed outsider sends a truthful message and then votes according to his information; each uninformed outsider votes according to the message(s) received, if he receives at least one $\sigma_m = L$, and otherwise votes indifferently either "yes" or "no".*

When at least one outsider is informed, his information is revealed to all outsiders at the message stage, and the board surely makes the correct decision. When no information is revealed, outsiders now know that nobody is informed and have no reason to contrast insiders and make other outsiders pivotal. Then multiple equilibria arise. The effect of communication on the expected profit of the firm, keeping constant board composition, is null because the right decision is still made with probability 1 when at least one outsider is informed and with probability $\frac{1}{2}$ when all outsiders are uninformed. To put it differently, the wrong decision is made with probability $\frac{1}{2}$ only when all outsiders are uninformed. This is precisely what happens in the case without communication. Indeed, the voting strategies of outsiders who cannot communicate, by compensating and leaving the decision to possibly informed directors, minimize the information required to reach the best possible outcome and the corresponding expected profit, $\widehat{E(\pi)}$.

The only effect of allowing for communication is the expansion of the set of equilibria; there now also exist equilibria where some outsiders vote "no" after observing $\sigma_m = H$. As outsiders receiving such a message know that they are not pivotal, they can cast any vote. These additional equilibria appear highly unrealistic, nonetheless they all yield $E(\pi) = \widehat{E(\pi)}$[15].

The optimal number of outsiders is still determined by the incentive to acquire information. Only incurring the information cost, each outsider may contribute to aggregate information and consequently may positively affect the outcome. Provided that all outsiders incur c, the probability of making the wrong decision decreases, and consequently expected profit increases, as the number of outsiders grows up to the point in which the marginal cost of information acquisition equals the marginal benefit. Given that equilibrium expected profit is still equal to $\widehat{E(\pi)}$, the marginal benefit from incurring c for the $n + 1^{th}$ outsider

[15] Notice that, contrary to what happens in the case examined at point ii) of proposition 1, this multiplicity does not entail a coordination problem: whatever the choice of the outsiders receiving message $\sigma_m = H$, an equilibrium with expected profit $\widehat{E(\pi)}$ is reached.

is the same as in the case of no communication, i.e. $\Delta B_O(n+1) = \frac{\alpha}{2}(1-\alpha)^n$. Then the following analog of Proposition 2 immediately follows.

Proposition 4 *When directors can communicate, the optimal number of outsiders does not change with respect to the case with no communication. The optimal number of insiders is undetermined: any composition with $\overline{n}+1$ outsiders and n insiders (with $1 \leq n \leq \overline{n}$) is optimal where $\overline{n}$ is the smallest integer that satisfies $\Delta B_O(n+1) = \frac{\alpha}{2}(1-\alpha)^n \geq c$.*

The striking result is that there is no difference in the performance of a board where directors do or do not communicate if the number of outsiders is $\overline{n}+1$. Propositions 1 and 3 imply that the expected profit reaches the same level $\widehat{E(\pi)} = \frac{1}{2}[1-(1-\alpha)^{n+1}]$ independently of communication. Consequently, the optimal number of outsiders remains unchanged with respect to the case without communication; the difference lies in the fact that now the number of insiders is undetermined: insiders can be in any number as long as they are the minority group. The reason is that in the absence of communication, the role of insiders is to be a sort of coordination device for uninformed outsiders and to this end they need to be exactly one less than the outsiders. With communication, instead, there is no need of a coordination device. Outsiders are uninformed only when none of them observes the state of nature (and are aware of that): then voting "yes" or "no" yields the same expected profit.

5 Boards with insiders and heterogeneous outsiders

As we noted in the introduction, outside directors are not necessarily a homogeneous group. To capture this we extend our basic model by also considering outsiders who are concerned more about their reputation than about the firm's profit. Thus, we allow outsiders to be either pure representatives of shareholders' interests who maximize the firm's profit (O) or independent directors who care about their own reputation (R). Let $k \leq n$ be the number of reputational outsiders where n is the number of insiders.

Note that, given their different preferences, reputational and profit-maximizing outsiders also differ in their willingness to acquire information. For a reputational outsider the benefit from becoming informed is measured by the increase in reputation stemming from voting correctly. Recalling that the gross utility from voting correctly is equal to 1, the expected value of the benefit for an uninformed reputational outsider is equal to $\frac{1}{2}$ (independently of the strategy adopted when uninformed). If an R decides to become informed, her expected benefit becomes $\alpha + \frac{(1-\alpha)}{2} = \frac{\alpha+1}{2}$. Thus, the gain from acquiring information for reputational outsiders is

$$\Delta B_R = \frac{\alpha}{2}$$

which is independent of board size.

A profit-maximizing outsider instead, continues to evaluate the increase in expected profit in order to make his decision. In particular, if an O anticipates that an equilibrium where all uninformed outsiders vote "no" and $E(\pi) = \widehat{E(\pi)}$ will occur, the value of $\Delta B_O(n+1)$ does not change with respect to (4). Notice that $\Delta B_R > \Delta B_O(n+1)$. Intuitively, the marginal gain from information is higher for a reputational than for a profit-maximizing outsider because reputational members fully enjoy the benefit from information, whereas profit maximizing members can benefit from information only if they are pivotal. Thus the condition for all outsiders to incur c is still given by (5).

In other words, c must be small enough to induce profit maximizing outsiders to acquire information: if profit-maximizing outsiders find it profitable to acquire information, *a fortiori* information acquisition is profitable for reputational outsiders.

To analyze the voting behavior of a board with both profit-maximizing and reputational outsiders we start again by assuming that directors do not communicate and then we discuss the role of communication. The following proposition characterizes the equilibrium strategies in the absence of communication.

Proposition 5 *In a board with n insiders, $n+1-k$ profit-maximizing outsiders and k reputational outsiders, if condition (5) is satisfied there are multiple equilibria, only one of which ensures that all outsiders pay c and expected profit equals $\widehat{E(\pi)}$. In this equilibrium, any informed outsider votes according to his information and any uninformed outsider votes "no". In all other equilibria we have that: i) profit maximizing outsiders do not pay c and vote "no"; ii) reputational outsiders pay c only if $c \leq \frac{\alpha}{2}$ and are indifferent as to whether they vote "yes" or "no" when they are uninformed; iii) $E(\pi) < \widehat{E(\pi)}$.*

As before, profit-maximizing outsiders know that they can influence the firm's profit only when they are pivotal; for this reason, they act as if they actually decided the voting outcome. The presence of reputational outsiders does not change their behavior: given that insiders always vote "yes", the best reply of an uniformed O is to contrast them, i.e. to vote "no". On the contrary, reputational outsiders do not condition their strategy on being pivotal. Since their payoffs are independent of the board's final decision, uninformed reputational outsiders are indifferent between approving or rejecting the project. As a consequence, the equilibrium with expected profit $\widehat{E(\pi)}$ obtains only when all uninformed Rs vote "no".

It can be immediately verified that the equilibrium where all uninformed Rs vote "no" is the only one that induces profit maximizing outsiders to incur the information cost. Provided that reputational outsiders acquire information (i.e. provided that $c \leq \frac{\alpha}{2}$)[16], any O who does

[16] As shown in the proof of Proposition 5, for $c > \frac{\alpha}{2}$, neither reputational nor profit-maximizing outsiders

not incur c will always vote "no" in order to contrast the insiders and leave the decision to a possibly informed reputational director. Again, if an O acquires information, expected profit changes only in the favorable state of the world. But in any equilibrium where at least one R votes "yes", an O is never pivotal when $v = 1$. Hence he has no incentive to incur c. As a consequence of the low acquisition of information in these equilibria expected profit is lower than $\widehat{E(\pi)}$. In the next subsection we identify a voting protocol which ensures that the equilibrium with $E(\pi) = \widehat{E(\pi)}$ is the unique solution of the game.

5.1 Sequential voting

Under a sequential protocol, the observed behavior of a director can convey some information to successors in the voting sequence. This changes reputational directors' behavior as shown in the following proposition where we assume that all outsiders pay the information cost. The condition on the size of the board ensuring that c is actually paid is established in the sequel.

Proposition 6 *In a board with n insiders, $n + 1 - k$ profit-maximizing and k reputational outsiders, where all the outsiders incur cost c and at least one O votes before any R, there is a unique equilibrium which yields expected profit $\widehat{E(\pi)}$. The insiders' position in the voting sequence is irrelevant. Outsiders vote according to their information whenever informed. Profit maximizing outsiders vote "no" when uninformed; reputational outsiders vote "no" when uninformed, unless any previous outsider (O or R) voted "yes". When the first outsider to vote is an R, additional equilibria emerge, where $E(\pi) \leq \widehat{E(\pi)}$.*

The insiders' behavior is uninformative because of their bias. Consequently their position in the voting sequence is irrelevant. On the contrary, the relative position of outsiders is important: to ensure $E(\pi) = \widehat{E(\pi)}$, profit maximizing outsiders must vote before reputational ones. An uninformed profit maximizer cannot elicit any useful information from observing previous directors' voting behavior. Since each profit-maximizer votes "no" when uninformed, his behavior does not change when state L is revealed. But even if a predecessor's vote revealed that the state of the world is H, a profit maximizer has no need to change his strategy, as he is no longer pivotal. On the contrary, a reputational director does not care about being pivotal: he only cares about appearing informed. Therefore, whenever uninformed, reputational members imitate preceding outsiders. Consider the first R to vote. If he is uninformed and observes at least one O voting like an insider, then he follows him, as that profit-maximizing outsider is certainly informed. But even if no profit-maximizing director

have an incentive to acquire information.

deviates and plays like the insiders, the uninformed reputational member follows Os' behavior, as there is a positive probability that the latter are informed. Subsequent uninformed Rs simply follow their predecessors in the voting sequence. In fact, if there are other Rs among predecessors, those Rs are either uninformed and follow preceding Os, or are informed and follow their own information[17].

When the first outsider to cast his vote is an uninformed reputational one, he can indifferently vote either "no" or "yes". If he votes "no", his vote corresponds to what a profit-maximizing outsider would have done. Subsequent uninformed reputational directors imitate him, and the equilibrium profile corresponds to that where the first voter is an O and $E(\pi) = \widehat{E(\pi)}$. If instead the first R votes "yes", he (an uninformed R) determines the final decision of the board; but then the correct decision is made only half of the time and the corresponding expected profit is equal to 0. In this equilibrium, where subsequent reputational directors imitate predecessors and profit-maximizers always vote "no", profit is positive only when $v = 1$ and the first R to vote is informed. Hence the expected profit is $E(\pi) = \frac{1}{2}\alpha$ which is lower than $\widehat{E(\pi)}$.

When the first outsider to vote is a profit maximizer, the sequential structure works as an "implementation mechanism" for the strategy profile desired by the shareholders. Given the appropriate order of vote, such a board performs just like a board composed only of insiders and profit-maximizing outsiders because reputational members imitate profit maximizing ones. This leads to the following remark.

Remark 2 *When all outsiders pay c and voting is sequential, the proportion of reputational to profit-maximizing outsiders is irrelevant, provided that the first outsider to vote is an O.*

This implies that with sequential voting it is not necessary to know the type of all the outsiders, to reach the $\widehat{E(\pi)}$ it is sufficient to know the type of a single O and to let him vote first. This is important because while it is relatively easy to distinguish between insiders and outsiders, knowing whether an outsider is profit-maximizing or reputational may be much more difficult.

We now turn to the condition ensuring that all outsiders actually pay c when the appropriate protocol is adopted. Again, such condition is related to the size and composition of the board. With sequential voting, the gain from information acquisition for a reputational outsider depends on his position in the voting sequence as well as on how many profit-maximizing outsiders vote before the reputational ones. Let us consider the case where there is only one

[17] There may arise a case where profit maximizers are all uninformed and vote "no", while an informed reputational votes "yes". In this case, in equilibrium subsequent reputational directors follow him.

profit-maximizing outsider who acquires information and votes first. A reputational director, if deciding not to become informed, can imitate the behavior of the profit maximizer. Given that the profit-maximizing outsider has incurred the cost of becoming informed, the reputational director, by imitating him, votes correctly with probability

$$\alpha + \frac{(1-\alpha)}{2} = 1 - \frac{(1-\alpha)}{2}$$

where $\frac{(1-\alpha)}{2}$ in the right-hand side expression represents the probability of casting the "wrong" vote. Consider however that for reputational uninformed directors there is a positive probability (equal to $\alpha + \frac{(1-\alpha)}{2}$) to obtain information from any predecessor, independently of the latter's type. The benefit for a reputational outsider who does not incur c then is:

$$B_R(z, no \text{ info}) = 1 - \frac{(1-\alpha)^z}{2}$$

where $z = 1, 2, ..., k = n$ represents the order in the reputational voting sequence (i.e. $z = 1$ means that this is the first R_j to cast his vote). The gross benefit obtained by a reputational outsider who pays c is:

$$B_R(z, info) = \alpha + (1-\alpha)\left[1 - \frac{(1-\alpha)^z}{2}\right] = 1 - \frac{(1-\alpha)^{z+1}}{2}.$$

Hence the gain from acquiring information is:

$$\Delta B_R^z = \frac{\alpha}{2}(1-\alpha)^z$$

which is decreasing in z. This implies that the gain for the n^{th} reputational outsider in the sequence, coinciding with the $n+1^{th}$ outsider to cast his vote, is

$$\Delta B_R^n = \frac{\alpha}{2}(1-\alpha)^n$$

which is equal to the value of the marginal benefit of acquiring information for the $n+1^{th}$ outsider in the case where all outsiders are profit-maximizers, $\Delta B_O(n+1)$. Notice that the values of $B_R(.)$ would not change if there were more than one profit-maximizing outsiders voting before the reputational one. In that case however $z = 1....n$ would indicate the order in the sequence of outsiders following the first one.[18] Moreover for a profit maximizer, the benefit from information acquisition (given that the reputational members pay c) is still $\Delta B_O(n+1)$. Then the next proposition immediately follows.

[18]For instance, if there are 3 Os, $z = 3$ indicates the first reputational outsider to cast his vote.

Proposition 7 *In a board with n insiders, $n+1-k$ profit-maximizing outsiders and k reputational outsiders, when voting is sequential, the optimal size and the expected profit are the same as those of a board where outsiders are all profit-maximizers, provided that the first voter in the sequence is an O.*

Summarizing, boards with heterogeneous outsiders can perform efficiently provided the order vote is properly chosen. We can then conclude that the same level of expected profit is obtained with a majority of outsiders independently of their types, provided that at least one outsider is a profit maximizer and that reputational directors (if present) follow profit maximizers in the voting sequence.

5.2 Voting outcome with communication

When there are both types of outside directors on the board, some differences emerge at the communication stage because profit-maximizing outsiders have an incentive to coordinate on the messages they send to reputational outsiders.

Reputational outsiders have no incentive to reveal their informed status, as their utility only depends on their vote[19]. Then, for simplicity, we assume that they do not send any message so that only profit-maximizing outsiders send messages to other directors. Furthermore, we assume that, when indifferent between telling the truth or lying, they always send a truthful message.

Messages are sent simultaneously, but we now allow for selective communication, in which a director sends a message to a subset of board members. We now consider a three-stage game where at the first stage messages (denoted by σ^O) are exchanged between profit maximizers, at the second stage messages (denoted by σ^R) are sent from profit maximizers to reputational outsiders, and finally voting takes place. We denote by the superscript the receivers of the message, and we omit the senders because they are always (all) the profit-maximizing outsiders.

Communication among outsiders modifies the outcome of the simultaneous voting game as shown in the next proposition where we assume that all outsiders pay the information cost.

Proposition 8 *When directors can communicate and all outsiders pay c, in a board with n insiders, $n+1-k$ profit-maximizing outsiders and k reputational outsiders, there are*

[19] If reputation depended on relative performances, reputational members may even have an incentive to send wrong messages. As we do not impose that members vote consistently with their messages, reputational members may in fact send wrong messages. By doing so, R directors could destroy the reputation of uninformed members of their same type. But this would be anticipated and messages would not be believed.

multiple equilibria yielding expected profit equal to $\widehat{E(\pi)}$. At the first stage a profit-maximizing outsider always sends a truthful message to other profit maximizing outsiders. If informed at the second stage, he also sends a truthful message to reputational directors and finally votes according to his information. If still uninformed at the second stage, a profit maximizing outsider sends a false message $\sigma^R = L$ to reputational directors and then votes "no". Each uninformed profit-maximizing outsider votes according to the message(s) received, if he has received at least one $\sigma^O = L$, and otherwise votes indifferently either "yes" or "no". A reputational outsider votes according to his information, if informed, and votes according to the received messages, if uninformed.

Profit-maximizing outsiders find it profitable to transmit their information on the state of nature to reputational directors. If all profit maximizers are uninformed, they now coordinate on the message they send to reputational members to induce them to vote "no". Reputational directors, when observing message L, are aware that the message may be false but they follow the message because the conditional probability that the true state is L is higher than $\frac{1}{2}$. Expected profit $\widehat{E(\pi)}$ is then guaranteed: as in the sequential case without communication, such equilibrium is reached because all reputational members vote like profit-maximizing ones[20].

On the one hand, communication simplifies the outcome of the simultaneous game by eliminating equilibria where expected profit is not maximized. On the other hand, as in the case with no reputational outsiders (see section 4.1),[21] communication introduces some unrealistic equilibria, where uninformed profit-maximizers vote "no" after receiving $\sigma^R = H$. Nonetheless, in all these new equilibria $E(\pi) = \widehat{E(\pi)}$.

We know that, in the absence of communication, boards with heterogenous outsiders reach the same outcome only if directors follow the optimal order of vote with at least a profit maximizer voting before reputational members. Such protocol serves as a means to transmit information from predecessors to successors in the sequence. When direct communication is possible, sequentiality is obviously not needed.

We now check the condition ensuring that all outsiders incur the information cost $c > 0$. Consider that following the message obtained from profit maximizers implies that uninformed reputational outsiders make the wrong decision with probability $\frac{(1-\alpha)^{n-k+1}}{2}$. Then the gross

[20] The same outcome can be obtained if we exclude selective communication and allow directors to simultaneouly send messages to all other board members. In this case an informed O_i always sends a truthful message $\sigma_i^{OR} = \omega_i$ and votes according to his information; if uninformed he sends a false message $\sigma_i^{OR} = L$ and always votes "no"; an informed reputational outsider votes according to his information, an uninformed reputational director votes according to his received messages if they are consistent, and votes "yes" if messages report conflicting information.

[21] Also the same comment on the fact that multiple equilibria does not create any coordination problem applies here.

benefit obtained by a reputational outsider who does not incur c is now:

$$B_R(no\ info) = 1 - \frac{(1-\alpha)^{n-k+1}}{2}$$

while the gross benefit if he pays c is:

$$B_R(info) = \alpha + (1-\alpha)\left[1 - \frac{(1-\alpha)^{n-k+1}}{2}\right] = 1 - \frac{(1-\alpha)^{n-k+2}}{2}$$

resulting in the following gain from information acquisition:

$$\Delta B_R = \frac{\alpha}{2}(1-\alpha) > \Delta B_O(n+1)$$

However, provided that all other outsiders pay c, the benefit from information acquisition for a profit maximizer is still $\Delta B_O(n+1)$ as defined in (4). Then (5) is still the condition that constrains board size and the following analog of Propositions 2 and 7 holds.

Proposition 9 *In a board with n insiders, $n+1-k$ profit-maximizing outsiders and k reputational outsiders, when directors can communicate the optimal size and the expected profit are the same as those of a board where outsiders are all profit-maximizers.*

6 Conclusions

The model has analyzed the voting behavior of directors faced with the acceptance/rejection decision on an investment project with uncertain prospects. In order to have a positive probability of becoming informed on project's profitability, directors have to incur an information cost. We have considered three types of directors: insiders, who are biased toward acceptance of the project, independent outsiders who want to maximize the firm's profit, and independent outsiders who care about their own reputation.

The main results can be summarized as follows. First, a strict majority of outsiders over insiders is a necessary and sufficient condition to maximize the profit of the firm. Outsiders are decisive for the voting outcome: if informed, they vote according to their information so as to induce the "right" decision; if uninformed, they optimally contrast the insiders' bias. Provided that the size of the board is optimally chosen, increasing the number of outsiders above a strict majority does not produce any gain despite the fact that each outsider is informed with positive probability. Thus, our result suggests that corporate governance reforms requiring a proportion of independent outsiders above strict majority would impose an unnecessary cost on companies because finding independent directors with a good knowledge of the business sector where the company operates can be a challenging task.

Moreover our model suggests that the gains obtained by the presence of outsiders on the board are discontinuous: all gains are obtained when the number of outsiders increases from 49% to 51%. This may explain why the empirical literature has found no clear positive relation between the number of outsiders on the board and firm performance.

Second, mixing up the identity of outsiders (i.e., substituting part of the profit-maximizing outsiders with reputational ones) is not an obstacle to profit maximization provided that at least one director is of the profit-maximizing type and that an appropriate sequential voting protocol is adopted. Third, a proper board composition makes communication among directors irrelevant in the sense that the same outcome is obtained with and without communication. This last result is particularly important because communication in the real world is difficult and quite often directors lack the time needed to exchange information and communicate.

Fourth, the optimal dimension of the board is determined by the trade-off between costs and benefits of expanding board membership. A large (outsider dominated) board is expected to collect more information but marginal returns from information acquisition decrease in board size for profit maximizing outsiders. Thus expanding the board may have the drawback of inducing free riding. Consequently, the optimal dimension of the board is given by the largest membership that avoids the free riding of profit-maximizing outsiders. The optimal size and composition of the board prescribed by our model does not take into consideration other tasks that directors (especially insiders) may perform. This implies that the actual board size may be different from the one that is optimal for project selection. If the board size is larger than optimal, we should observe under-investment in the sense that the project is less likely to be approved and the expected profit is smaller than it would be with the optimal size. This follows from the fact that a larger than optimal board induces the "additional outsiders" not to acquire information. Thus, a testable implication of our model is that we should have under-investment in companies with large boards but high cost of acquiring information, for example in innovative industries.

Despite being built on board-of-directors experiences, the model can be extended to represent different committees. For instance, some juries (e.g.: the Italian Constitutional Court) have members appointed by different constituencies or individuals. The same heterogeneity may be found in technical committees, where politicians, bureaucrats and experts meet to provide advice.

The model could be enriched along several directions. The identity of members (their preferences) could be private information; in addition, outsiders could have different probabilities of becoming informed. The model also appears to be particularly suitable for a laboratory experiment. We will develop our future research along these lines.

7 Appendix

7.1 Proof of Proposition 1

In equilibrium, each informed outsider plays according to his information, as this maximizes the probability of making the correct decision. Thus, in what follows we only focus on the voting strategies of uninformed members. Recall that outsiders choose their strategies as if they were pivotal, as what they do when they are not pivotal is irrelevant for the voting outcome. Thus, we concentrate on equilibria in weakly dominant strategies.

Let o_i be the probability that an uninformed member O_i votes "yes", $o_i \in \{0,1\}$, with $i = 1, 2, ..., n+1$, when there are $n+1$ outsiders, and $i = 1, 2, ..., n+1+k$ when there are $n+1+k$ outsiders. We prove that:

i) if the board is composed of $n+1$ outsiders and n insiders, there exists a unique equilibrium where each outsider chooses $o_i = 0$ when uninformed;

ii) if the board is composed of $n+1+k$ outsiders and $n-k$ insiders, all the equilibria are such that $n-k$ outsider chooses $o_i = 0$ when uninformed and the remaining outsiders compensate for each other.

iii) any of the above equilibria yields expected profit $\widehat{E(\pi)} = \frac{1}{2}[1-(1-\alpha)^{n+1}]$.

i) Board with $n+1$ outsiders and n insiders.

To prove that the unique equilibrium is the one where all outsiders vote "no" whenever uninformed ($o_i = 0; i = 1, 2, ..., n+1$), consider outsider O_{n+1}.

When $v = 1$, O_{n+1} is pivotal only if all other outsiders are uninformed and vote "no", which happens with probability:

$$(1-\alpha)^n \prod_{j=1}^{n}(1-o_j).$$

When $v = -1$, O_{n+1} is pivotal if:

a) all other outsiders are uninformed and vote "no", which happens with probability

$$(1-\alpha)^n \prod_{j=1}^{n}(1-o_j),$$

b) all the other outsiders are informed, which happens with probability

$$\alpha^n,$$

c) at least one (but not all) of the other outsiders is informed and the others vote "no" when uninformed, which happens with probability

$$\sum_{h=1}^{n} \frac{n!}{h!(n-h)!} \alpha^{n-h} (1-\alpha)^h \prod_{j=1}^{h} (1-o_j).$$

where $\frac{n!}{h!(n-h)!}$ represents the number of combination with h uninformed outsiders and $n-h$ informed outsiders. It is straightforward that O_{n+1} is pivotal with a higher probability in the bad state. Hence O_{n+1} chooses $o_{n+1} = 0$. As the same reasoning holds for any other outsider $j \neq n+1$, it follows that every outsider will vote "no" when uninformed.

Finally, note that we have not restricted o_j, $j \neq n+1$, to any particular value, so the result also proves that this equilibrium is unique.

ii) Board with $n+1+k$ outsiders and $n-k$ insiders.

In the case of $n-k$ insiders ($n > k > 0$) and $n+1+k$ outsiders, there exist multiple equilibria with $n-k+1$ outsiders voting against the project and $2k$ outsiders compensating for each other.

We prove the existence of this equilibrium in three steps. In the first step, we prove that when $n-k$ outsiders vote against the project and $2k$ outsiders compensate each other, the remaining outsider has still an incentive to vote against the project; in the second step, we prove that when $n-k$ outsiders vote against the project to contrast the $n-k$ insiders, and a majority of the other outsiders also vote against the project, the remaining outsider has an incentive to compensate, voting "yes". Finally, we show that there are no other equilibria.

1. If n outsiders choose $o_j = 0$, and k outsiders choose $o_j = 1$, the best response of O_i, $i \neq j$, is to choose $o_i = 0$.

When $v = 1$, O_i is pivotal if all the outsiders are uninformed or if at least one of those k outsiders who choose $o_j = 1$ when uninformed, is in fact informed. Thus, O_i is pivotal with probability

$$(1-\alpha)^n \left[\sum_{j=0}^{k} \frac{k!}{j!(k-j)!} \alpha^{k-j} (1-\alpha)^j \right] = (1-\alpha)^n$$

where $\frac{k!}{j!(k-j)!}$ represents the number of combination with j uninformed outsiders, $k-j$ informed outsiders and the term in bracket is equal to 1 from the binomial theorem. When

$v=-1$, O_i is pivotal if all the outsiders are uninformed or if at least one of those n outsiders who chooses $o_j = 0$ when uninformed, is in fact informed. Then O_i is pivotal with probability

$$(1-\alpha)^k \left[\sum_{j=0}^{n} \frac{n!}{j!(n-j)!} \alpha^{n-j} (1-\alpha)^j \right] = (1-\alpha)^k$$

Given that $(1-\alpha)^k > (1-\alpha)^n$, the probability that O_i is pivotal is higher when $v=-1$ than when $v=1$. Hence O_i chooses $o_i = 0$.

2. If $n+1$ outsiders choose $o_j = 0$ and $k-1$ outsiders choose $o_j = 1$, the best response of O_i, $i \neq j$ is to choose $o_i = 1$

When $v=1$, O_i is pivotal if only one of the $n+1$ outsiders choosing $o_j = 0$ is informed and votes “yes”. This happens with probability

$$(n+1)(1-\alpha)^n \alpha.$$

On the contrary, O_i is never pivotal when $v=-1$. Hence, he chooses $o_i = 1$.

Finally, note that any outsider can be in the position of O_i or of an O_j voting "yes", or also of an O_j voting "no". Thus, there is a multiplicity of equilibria such as the one we are considering.

3. There cannot exist other equilibria than those characterized at points 1 and 2.

We must now consider what happens if either a) more than n outsiders vote “no” and the others vote “yes”, or b) more than k outsiders vote “yes” and the rest vote “no”.

a) *If $n-h$ outsiders choose $o_j = 0$, and $k+h$ outsiders choose $o_j = 1$, $n \geq h > 0$, the best response of O_i, $i \neq j$, is to choose $o_i = 0$* because O_i is never pivotal when $v=1$ while he may be pivotal when $v=-1$. This happens in the case where h of those $n+h$ outsiders who choose $o_j = 1$ if uninformed, are in fact informed. As this is true for any $h > 0$, we are back to the case examined at point 1 above.

b) If $n+h$ outsiders choose $o_j = 0$, and $k-h$ outsiders choose $o_j = 1$, $k \geq h > 1$, the best response of O_i, $i \neq j$, is to choose $o_i = 1$ because O_i is never pivotal when $v=-1$ while he may be pivotal when $v=1$. This happens in the case where h of those $n+h$ outsiders who choose $o_j = 0$ if uninformed, are in fact informed. As this is true for any $h > 1$, we are back to the case examined at point 2 above.

iii) Expected profit

Recall that expected profit is equal to

$$E(\pi) = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)].$$

Both in the case with n insiders and $n + 1$ outsiders and in the case of $n - k$ insiders ($n > k > 0$) and $n + 1 + k$ outsiders, the unique outcome is such that $Y(\cdot|v = -1) = 0$ and that $Y(\cdot|v = 1)$ is equal to the probability that at least one of the $n+1$ outsiders who choose $o_j = 0$ is informed. Then the expected profit is equal to

$$\frac{1}{2}\sum_{j=0}^{n} \frac{(n+1)!}{j!(n+1-j)!}\alpha^{n+1-j}(1-\alpha)^j = \frac{1}{2}\left[1 - (1-\alpha)^{n+1}\right].$$

7.2 Proof of Corollary 2

The Corollary follows immediately by differentiating $\Delta B_O(n + 1)$ with respect to α :

$$\frac{\partial[\frac{\alpha}{2}(1-\alpha)^n]}{\partial\alpha} = \frac{1}{2}(1-\alpha)^{n-1}[1 - \alpha(1+n)] < 0.$$

7.3 Proof of Proposition 3

With probability $(1-\alpha)^{n+1}$ no O_i is informed, $i = 1, 2, ..., n+1$, whereas with probability $1-(1-\alpha)^{n+1}$, at least one outsider, say O_j, obtains information and consequently sends a truthful message $\sigma_{O_j} = \omega_{O_j}$. In the latter case, subsequent voting strategies are straightforward: O_i votes according to his information and all $O_{i\neq j}$ vote according to the received information. More precisely, if the revealed information is $\sigma_{O_j} = L$, each outsider is pivotal and votes “no”. On the contrary, if the revealed information is $\sigma_{O_j} = H$, then those outsiders who receive the message are no longer pivotal (as the correct decision has already been made) and, given that the sender votes "yes", can cast any vote. The argument immediately generalizes to the case where more than one outsider obtains information.

When instead no outsider is informed, any O_i chooses $o_i \in \{0, 1\}$. Given equal priors about the states of the world, utility is independent of o_i, so any probability $o_i \in \{0, 1\}$ is utility maximizing for O_i.

It follows that the equilibrium (weakly dominant) strategy (in terms of probability of voting "yes") for O_i is:

$$s^*_{O_i}:\left\{\begin{array}{c} 1 \mid \Omega_{O_i}=\{\omega_{O_i}=H\}; \\ 0 \mid \Omega_{O_i}=\{\omega_{O_i}=L\}; \Omega_{O_i}=\{\sigma_{O_j}=L\}; \\ \forall o_i \in \{0,1\} \mid \Omega_{O_i}=\{\sigma_{O_j}=H\}; \Omega_{O_i}=\{H,L;0\} \end{array}\right\}$$

where $j \neq i$, $i,j = 1,2,.....n+1$. When at least one outsider is informed (which happens with probability $1-(1-\alpha)^{n+1}$), the correct decision is made with probability equal to 1 and the expected profit of the firm is:

$$\frac{1}{2}[Y(\cdot|v=1)-Y(\cdot|v=-1)=[\frac{1}{2}1-\frac{1}{2}0]=\frac{1}{2}.$$

When no outsider is informed, the expected profit of the firm is:

$$\frac{1}{2}[Y(\cdot|v=1)-Y(\cdot|v=-1)=0,$$

as $Y(\cdot|v=1)=Y(\cdot|v=-1)$ when no information is available.

Thus, the expected profit of the firm is

$$E(\pi)=\frac{1}{2}\left[1-(1-\alpha)^{n+1}\right]=\widehat{E(\pi)}$$

7.4 Proof of Proposition 5

Informed outsiders, independently of their type, vote according to their information, as this is always correct.

As regards uninformed outsiders, first of all let o_i be the probability that an uninformed member O_i votes "yes", $o_i \in \{0,1\}$, $i=1,2,...,n-k+1$, and let r_i be the probability that an uninformed member R_i votes "yes", $r_i \in \{0,1\}$, $i=1,2,...,k$.

Let us first consider reputational outsiders. If uninformed ($\Omega_{R_i}=\{H,L\}$), R_i chooses the probability $r_i \in \{0,1\}$, which maximizes his expected utility. It is straightforward to work out that R_i's expected utility is:

$$E(u_R)=\frac{1+\alpha}{2},$$

which is independent of the probability r_i. This means that any probability $r_i \in \{0,1\}$is utility maximizing for R_i. It follows that the dominant strategy for any R_i (in terms of probability of voting "yes") is the following:

$$s^*_{\forall R_i \in R} : \left\{ \begin{array}{c} 1 \mid \Omega_{r_i} = \{\omega_{R_i} = H\}; \\ 0 \mid \Omega_{r_i} = \{\omega_{R_i} = L\}; \\ r_i \in \{0,1\},\ \Omega_{R_i} = \{H, L\} \end{array} \right\}.$$

As regards profit-maximizing outsiders, using the same argument as Proposition 1, the probability of one of them being pivotal when $v = -1$ is *a fortiori* higher than the probability of being pivotal when $v = 1$. Thus, the dominant strategy for any O_i (in terms of probability of voting "yes") is the following:

$$s^*_{\forall O_i \in O} : \left\{ \begin{array}{c} 1 \mid \Omega_{O_i} = \{\omega_{O_i} = H\}; \\ 0 \mid \Omega_{O_i} = \{\omega_{O_i} = L\}; \\ 0 \mid \Omega_{O_i} = \{H, L\} \end{array} \right\}$$

This leads to multiple equilibria: one equilibrium in which every outsider votes "no" and other equilibria in which at least one reputational outsider votes "yes" while all profit maximizers vote "no".

We know from Proposition 1 that expected profit is $\widehat{E(\pi)}$ in the equilibrium where every outsider pays c and votes "no" when uninformed. As a consequence condition (5) ensures that all outsiders pay c.

To analyze the other equilibria where at least one R votes "yes", let us denote with δ the number of reputational outsiders who choose $r_j = 0$. Then $k - \delta$ is the number of reputational outsiders who choose $r_j = 1$ where $k > \delta \geq 0$.

In order to determine the expected profit, let us first determine the probability that the board makes the wrong decision. Given that we are considering equilibria where at least one reputational outsider chooses $r_j = 1$, such probability is equal to 0 when $v = 1$. When $v = -1$, instead the probability of a wrong decision is equal to the probability that at least one of those reputational outsiders choosing $r_j = 1$ is uninformed:

$$\sum_{j=1}^{k-\delta} \frac{(k-\delta)!}{j!(k-\delta-j)!} \alpha^{k-\delta-j}(1-\alpha)^j = 1 - \alpha^{k-\delta}.$$

Consequently the expected profit is

$$E[\pi(k-\delta)] = \frac{1}{2}\left[1 - (1-\alpha^{k-\delta})\right] = \frac{1}{2}\alpha^{k-\delta} < \frac{1}{2}\left[1 - (1-\alpha)^{n+1}\right] = \widehat{E[\pi]}$$

where the inequality holds because $1 > \alpha^{k-\delta} + (1-\alpha)^{k-\delta}$ and $n \geq k$.

To check that profit-maximizing outsiders never pay the information cost c, let us first consider the case where $c \leq \frac{\alpha}{2}$ and reputational outsiders pay c. In these equilibria a profit-maximizing outsider is never pivotal when $v = 1$. The only case in which he may be pivotal is when $v = -1$, but then his dominant strategy is to vote "no" with no need to acquire any information. When $c > \frac{\alpha}{2}$, we know that reputational outsiders do not acquire information and these equilibria always result in approval of the project. As profit maximizing outsiders are never pivotal, they will never pay c.

7.5 Proof of Proposition 6

For simplicity and without loss of generality, let the cardinality of a member also indicate his ordinality in the voting process among voters of his own type. That is, R_i is the i^{th} voter among the reputational outsiders: $R_i \in R = \{R_1, R_2, ..., R_k\}$; and O_i is the i^{th} voter among the reputational outsiders: $O_i \in O = \{O_1, O_2, ..., O_{n+1-k}\}$. Let again r_i be the probability that member R_i votes "yes" when uninformed.

Informed outsiders always vote according to their information, as this is always correct.

As regards uninformed outsiders, let's start by assuming now that the entire set R votes first. R_1 has no additional information but the information he may have acquired by paying c (if any) on which to base his action; so he behaves as in the simultaneous voting game. His utility maximizing strategy (in terms of probability of voting "yes") is:

$$s^*_{R_1} : \left\{ \begin{array}{c} 1 \mid \Omega_{r_1} = \{\omega_{R_1} = H\}; \\ 0 \mid \Omega_{r_1} = \{\omega_{R_1} = L\}; \\ r_1 \in \{0,1\}\,,\ \Omega_{R_1} = \{H, L\} \end{array} \right\}.$$

On the contrary, any subsequent $R_i \in R \backslash R_1$, if uninformed, has an incentive to follow R_{i-1}. From Bayes updating, the probability of being right by following R_{i-1} is higher than that of being right by choosing the alternative vote. Thus R_i follows R_{i-1}. For subsequent $R_i \in R \backslash R_1$, the optimal equilibrium strategy (in terms of probability of voting "yes") is:

$$s^*_{R_i \in R\backslash R_1} : \left\{ \begin{array}{c} 1 \mid \Omega_{R_i \in R\backslash R_1} = \{\omega_{R_i \in R\backslash R_1} = H; d_{R_1}, d_{R_2}, ..., d_{R_{i-1}}\}; \\ 0 \mid \Omega_{R_i \in R\backslash R_1} = \{\omega_{R_i \in R\backslash R_1} = L; d_{R_1}, d_{R_2}, ..., d_{R_{i-1}}\}; \\ d_{R_{i-1}} \mid \Omega_{R_i \in R\backslash R_1} = \{H, L; d_{R_1}, d_{R_2}, ..., d_{R_{i-1}}\} \end{array} \right\},$$

where d_{R_i} is the decision made by director R_i (and $d_R : \{d_{R_1}, d_{R_2}, ..., d_{R_k}\}$).

As regards profit maximizing outsiders, nothing changes with respect to simultaneous voting. Given that every O_i conditions his strategy on being pivotal, with sequential voting he elicits no useful information from observing other members' votes, in the same spirit of Dekel and Piccione [2000]. If every previous outsider votes "no ", it is optimal for him to vote "no " and leave the decision to other, possibly informed, outsiders. If at least one $O_{j\neq i}$ votes "yes", then O_i is no longer pivotal and can keep on voting "no " without changing the outcome of the voting process. Thus, for any $O_i \in O$, the optimal equilibrium strategy (in terms of probability of voting "yes") is:

$$s^*_{\forall O_i \in O} : \left\{ \begin{array}{c} 1 \mid \Omega_{O_i} = \{\omega_O = H; d_R\}; \\ 0 \mid \Omega_{O_i} = \{\omega_O = L; d_R\}; \\ 0 \mid \Omega_{O_i} = \{H, L; d_R\} \end{array} \right\},$$

We know from Proposition 1 and 5 that $E(\pi)$ is maximized when $r_i = 0, \forall R_i \in R$. It follows that, if the Rs vote first, there are multiple equilibria, no one of which is necessarily optimal. Hence in any equilibria where an R votes first, $E(\pi) \leq \widehat{E(\pi)}$.

Consider now the case where all the Os vote first. Although nothing changes for them, R_1 can elicit some information from their behavior. By the same reasoning as above, R_1, when uninformed, has an incentive to follow previous members. So R_1 has the following equilibrium strategy (in terms of probability of voting "yes"):

$$s^*_{R_1} : \left\{ \begin{array}{c} 1 \mid \Omega_{R_1} = \{\omega_{R_1} = H; d_O\}; \\ 0 \mid \Omega_{R_1} = \{\omega_{R_1} = L; d_O\}; \\ d_{O_{n+1-k}} \text{ if } \Omega_{R_1} = \{H, L; d_O\} \wedge d_{O_i} = \text{“no”}, \forall O_i \\ d_{O_j} \text{ if } \Omega_{R_1} = \{H, L; d_O\} \wedge d_{O_j} = \text{“yes”} \end{array} \right\}$$

The last line follows from the fact that R_1 has an incentive to follow any O_j deviating from playing "no". Given profit maximizers' optimal strategy, this means that such an O_j is surely informed. For the remaining Rs, again, nothing changes:

$$s^*_{R_i \in R \setminus R_1} : \left\{ \begin{array}{c} 1 \mid \Omega_{R_i \in R \setminus R_1} = \{\omega_{R_i \in R \setminus R_1} = H; d_O; d_{R_1, R_2, \ldots, R_{i-1}}\}; \\ 0 \mid \Omega_{R_i \in R \setminus R_1} = \{\omega_{R_i \in R \setminus R_1} = L; d_O; d_{R_1, R_2, \ldots, R_{i-1}}\}; \\ d_{R_{i-1}} \mid \Omega_{R_i \in R \setminus R_1} = \{H, L; d_O; d_{R_1, R_2, \ldots, R_{i-1}}\} \end{array} \right\}$$

Notice that this order of vote implies an optimal equilibrium. Every R_i behaves like a profit maximizing outsider, unless he is informed or follows someone who is informed for sure. This implies $E(\pi) = \widehat{E(\pi)}$.

By using similar arguments, we can state that it is sufficient to have O_1 voting before R_1

to ensure $E(\pi) = \widehat{E(\pi)}$.

7.6 Proof of Proposition 8

Profit-maximizing outsiders have an incentive to coordinate on the messages they send to reputational outsiders. This implies that they all send truthful messages to other profit-maximizers. With probability $(1-\alpha)^{n+1-k}$ no profit maximizer is informed, whereas with probability $1-(1-\alpha)^{n+1-k}$, at least one of them, say O_j, is informed and sends signal $\sigma_j^O = \omega_j$, $\omega_j \in \{H, L\}$. As to the message from the Os to the Rs, this is $\sigma_i^R = \omega_j$, $i = 1, 2...n-k+1$, if at least one O_j is informed, $j \in \{1, 2, ...n-k+1\}$. If all Os are uninformed instead, they send the false message $\sigma_i^R = L$ in order to induce the Rs to vote "no".

The voting strategies of the Os are straightforward: if at least one O is informed, every O votes accordingly. As in Proposition 3, if the revealed information is $\sigma_j^O = L$, each outsider is pivotal. On the contrary, if the revealed information is $\sigma_j^O = H$, only one of the outsiders is pivotal. There may then also arise different equilibria where all outsiders but one can cast any vote. The equilibrium weakly dominant strategy (in terms of probability of voting "yes") of a generic O_i is

$$s_{O_i}^* : \left\{ \begin{array}{c} 1 \mid \Omega_{O_i} = \{\omega_{O_i} = H\}; \\ 0 \mid \Omega_{O_i} = \{\omega_{O_i} = L\}; \Omega_{O_i} = \{\sigma_j^O = L\};\ \Omega_{O_i} = \{H, L; 0\} \\ \forall o_i \in \{0, 1\} \mid \Omega_{O_i} = \{\sigma_j^O = H\} \end{array} \right\}.$$

Reputational outsiders follow their information when informed, and follow the message from the Os when uninformed. They do so because, from Bayes updating, they know that it is $v = 1$ when $\sigma^R = H$, and that the probability that $v = -1$ when $\sigma^R = L$ is higher than $\frac{1}{2}$. The strategy of a generic R_i is

$$s_{R_i}^* : \left\{ \begin{array}{c} 1 \mid \Omega_{R_i} = \{\omega_{R_i} = H\}; \Omega_{R_i} = \{\sigma^R = H\}; \\ 0 \mid \Omega_{R_i} = \{\omega_{R_i} = L\}; \Omega_{R_i} = \{\sigma^R = L\}; \end{array} \right\}.$$

When $v = 1$, the correct decision is made if at least one outsider is informed. When $v = -1$, it is made if at least one profit-maximizing outsider is informed. Finally, when nobody is informed the project is rejected and zero profit is realized. Then, expected profit is again given by:

$$E(\pi) = \frac{1}{2}\left[1 - (1-\alpha)^{n+1}\right] = \widehat{E(\pi)}$$

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