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Complexity with Heterogeneous Fundamentalists and a Multiplicative Price Mechanism*

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Abstract

In contrast with the canonical models, Naimzada and Ricchiuti (2008, 2009) show that the interaction of groups of agents who have the same trading rule but present different beliefs about the fundamental value could be a source of instability in financial markets. Differently from Naimzada and Ricchiuti (2008, 2009), we assume the market maker employs a so-called multiplicative price mechanism (Tuinstra, 2002 and Zhu et al., 2009). We show that the occurrence of heterogeneity has an ambiguous role: it may either stabilize or destabilize the market.

JEL: C61, G11, G12, D84
Keywords: mathematical economics, chaos, heterogeneous interacting agents, financial markets.

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1 Introduction

In the last two decades an increasing number of theoretical and empirical works on financial markets have shown how complex dynamics of price fluctuations are related to the interactions between agents with heterogenous beliefs (see Hommes 2006 for a complete survey). Typically, heterogeneity regards the applied trading rule: agents could be in fact fundamentalists, chartists, noise traders and so on. Moreover, as clearly stated by Hommes (2006, p.1175): '*sophisticated traders, such as fundamentalists or rational arbitrageurs typically act as a stabilizing force, pushing prices in the directions of the RE fundamental value. Technical traders, such as feedback traders, trend extrapolators and contrarians typically act as a destabilizing force, pushing prices away from the fundamental*'.

Naimzada and Ricchiuti (2008, 2009, 2012a, 2012b) develop a framework in which the source of instability resides in the interaction of groups that are **homogeneous** in the strategy they use (they are both fundamentalists), but have **heterogeneous beliefs** about the fundamental value of the asset. We strongly believe that heterogeneity in beliefs' structure (not just about rules applied) may imply complex dynamics. The estimations about the future have a subjective dimension: (i) hardly agents reach the true fundamental value and (ii) it is really unlikely that agents have the same expectations. Indeed, in a real stock market, there is a lot of uncertainty on what the "true fundamental" is. As in [Kirman, 1998], our model 'involves agents who may use one of a number of predictors which they might obtain from financial gurus (experts)', the belief is for these agents a focal point.

Brock and Hommes (1998, p. 1258) contains a 3-type asset-pricing model with fundamentalists, upward and downward biased traders. We could interpret this example as a model with three fundamentalists trader types, one with the correct estimate of the fundamental and two other types with a wrong estimate of the fundamental. We consider a two type version of the model; there are two groups of agents which follow a 'guru/expert', both having a wrong/bias estimate of the fundamental value.

Different authors (Brock and Hommes, 1998; De Grauwe and Rovira, 2007 and Rovira, 2010) detect an unknown fundamental price and discuss the presence of biased traders (optimists or pessimists) in financial markets. We assume directly that the expectations are different, without detecting the true fundamental price: we do not know whether agents are or not pessimists (optimists), we just assume that they have different expectations. Moreover, these articles assume that supply and demand are always equal and the former is exogenous, while we assume that the price mechanism is driven by a market maker. Their switching mechanism is based on profitability and have chartists in their analysis and, finally, while they use extensively simulations we have also an analytical approach.

Naimzada and Ricchiuti (2008) employ a simple switching mechanism based on fitness, showing that market instability and periodic, or even, chaotic price fluctuations can be generated; moreover, we show that there are conditions under which an expert can drive another expert out of the market (differently from Brock and Hommes, 1998). On the other hand, Naimzada and Ricchiuti (2009, 2012a, 2012b) use a switching mechanism *à la Brock and Hommes* based on fitness, showing that with heterogeneity there exists at least a fixed point: the average between the two biased fundamental values. Moreover, given the intensity of switching, there exists a positive degree of heterogeneity such that a pitchfork bifurcation occurs. Finally, they show that uniqueness can be achieved even if there is heterogeneity and that a small heterogeneity leads to stability while higher heterogeneity leads to instability of the unique steady state: heterogeneity has an ambiguous role. There is a transition from the an unstable fixed point to a regime in which the fixed point becomes stable and again an unstable period two cycle. Finally, a homoclinic bifurcation transforms a two piece chaotic set into a one piece chaotic set which generates bull and bear markets.

In comparison with these previous works, we assume that the market maker employs the so-called multiplicative price mechanism (Tuinstra, 2002 and Zhu, 2009). Instead of the prices first difference, it looks at the rate of change of price (the return) *calculated with the exact formula*, assuming that it is proportional to the excess demand. Two are the main reasons that urge us to analyze this structure. From an empirical point of view the prices first difference specification has a strong lack: the prices are non-stationary. Indeed, even if the returns are just a monotonic transformation of prices, in empirical finance econometricians analyze returns instead of prices because the latter are non-stationary (see Tsay, 2002). On the other hand, an approximation of our specification has been already studied in literature by other authors (Farmer and Joshi, 2002; Westerhoff, 2004; Westerhoff and Reitz, 2005). Specifically, they have used the so-called price impact function, the market maker adjust prices as follows: $p_{t+1}-p_t = ED$ where ED is the excess demand and $p_{t+1} = LogP_{t+1}$. As well known, the logarithmic difference provides a good approximation of the rate of change around zero, and hence its use is rather immaterial as far as this choice refers small variations of prices. However, for example, the structural differences that characterize the distribution of the monthly rate of variations of a turbulent period imply that the method of computation does matter. The relevance of the method used to compute the variation of a price is twofold: first, logarithms provide a poor approximation of the rate of change when this is large; second, and most notably, the logarithmic approximation determines a significantly lower variance of the sample distribution of the rate of variation of a price when there is turbulence. A priori, it is not possible to predict whether the use of logarithms will generate a distribution with a higher or smaller variance, as the logarithm is a contraction mapping on the domain $(1, +\infty)$ and an expansion mapping in the domain $(0, 1)$. But, if the sample values of the rate of change lie disproportionately above or below zero, the use of the logarithmic approximation influences the standard deviation of the transformed values in a predictable way (see Bertoli et al., 2010; for a discussion on the exchange rates). For these reasons we study the rate of change calculated with the exact formula rather than the difference of logarithms.

Even if we introduce a substantial change in the market maker's mechanism, we show that heterogeneity still plays an ambiguous role: first it stabilizes the market and then it destabilizes it. However, this ambiguity depends (differently from Naimzada and Ricchiuti, 2009, 2012a and 2012b) on the condition of the map when expectations are homogenous. If there is monotonic convergence, heterogeneity leads only to a pitchfork bifurcation but the system is still stable. On the other hand, when there is oscillatory convergence heterogeneity affects the sequence of period doubling. Finally, with period cycles and chaos an increasing heterogeneity firstly stabilizes and then destabilizes the system. Moreover, differently from the previous works heterogeneity can lead to the coexistence of stable attractor with a chaotic attractor.

After a presentation of the model, we will discuss the necessary conditions for existence and stability of fixed points, and then we use simulations to study the role of heterogeneity. Finally, the last section of this paper provides brief concluding remarks and suggestions for further research.

2 The Model

In what follows, we have the same structure of the model adopted by Naimzada and Ricchiuti (2009). We have a risk and a risk-free asset. The latter has a perfectly elastic supply at the gross return ($R = (1 + r/k) > 1$), where r is the constant risk-free rate per year and K is the frequency of the trading period per year. The risky asset has a price per share ex-dividend at time t equal to X_t and it presents a (stochastic) dividend process equal to y_t. Defining with $i = 1, 2$ two groups of agents, their wealth at ($t + 1$) is given by:

$$W_{i,t+1} = RW_{i,t} + R_{t+1}q_{i,t} = RW_{i,t} + (X_{t+1} + y_{t+1} - RX_t)\, q_{i,t} \qquad (1)$$

where $R_{t+1} = (X_{t+1} + y_{t+1} - RX_t)$ is the excess capital gain/loss, and $q_{i,t}$ represents the number of shares of the risky asset purchase at time t. Agents have two key beliefs the conditional expectation and the variance of wealth, which can be expressed respectively as follows:

$$E_{i,t}(W_{t+1}) = RW_{i,t} + E_{i,t}\,(X_{t+1} + y_{t+1} - RX_t)\, q_{i,t}, \qquad (2)$$

$$V_{i,t}(W_{t+1}) = q_{i,t}^2 V_{i,t}(R_{t+1}). \qquad (3)$$

We assume that each group of agents has a CARA (constant absolute risk aversion) utility function, $u(W) = -e^{-aW}$, where a is the strictly positive constant risk aversion equal for both groups of agents. Agents maximize the following expected wealth utility function:

$$Max_{q_i,t} = \left[E_{i,t}(W_{i,t+1}) - \frac{a}{2} V_{i,t}(W_{i,t+1})\right]. \qquad (4)$$

Therefore, the demand function for each group is:

$$q_{i,t} = \frac{E_{i,t}(R_{t+1})}{aV_{i,t}(R_{t+1})} = \frac{E_{i,t}(X_{t+1} + y_{t+1} - RX_t)}{aV_{i,t}(R_{t+1})} \qquad (5)$$

We assume that agents have common expectations on dividends ($E_{i,t}(y_{t+1}) = E_t(y_{t+1}) = \bar{y}$)[1]. Moreover, we assume the presence of groups of agents with different expectations on the fundamental price[2]:

[1] The assumption of common expectations on dividends is restrictive. However, the qualitative dynamic behavior of the model is not modified by this assumption.

[2] According to Brock and Hommes [3, p.1245], we could re-write our model assuming the existence of only pure *biased traders* and that all beliefs have the following linear form:

$$E_{i,t}(X_{t+1}) = E_i(X_{t+1}^*) = F^* + b_i = F_i \qquad (6)$$

where F^* is the unknown fundamental value and b_i is the bias of trader type i. The fundamental value is not derived from the model structure and does not have any effect on the dynamics. However, it is focal point to detect optimists (pessimists) agents. $F^* + b_i > 0$, when b_i is positive (negative) the agent is optimist (pessimist). Differently from previous works, our framework allows the analysis of the case in which agents are at the same time both optimists (pessimists).

$$E_{i,t}(X_{t+1}) = E_i(X^*_{t+1}) = F_i \tag{7}$$

F_i is the expectation of trader type i. In the numerical analysis below we will assume that are $F_2 > F_1$.

Equation (5) can be rewritten as follows:

$$q_{i,t} = \alpha(F_i - P_t) \tag{8}$$

where $P_t = RX_t - \bar{y}$ and $\alpha = \frac{1}{a\sigma^2}$ is the positive coefficient of the reaction of investors representing a measure of both risk aversion and reaction to mis-pricing of the biased traders[3].

There is a market maker and two groups who, following a guru, can switch from one expert to another. Differently from Naimzada and Ricchiuti (2008, 2009), we employ a so-called multiplicative price mechanism (Tuinstra, 2002 and Zhu, 2009):

$$P_{t+1} = P_t(1 + \beta ED_t), \tag{9}$$

where β is the market maker's reaction coefficient and ED_t is the following excess of demand:

$$ED_t = w_{t+1}q_{1,t} + (1 - w_{t+1})q_{2,t} \tag{10}$$

where w_{t+1} is the proportion of biased traders of group 1. Agents can switch from one group to the next following an adaptive belief system (Brock and Hommes 1997 and 1998) based on fitness: it depends on the following square errors given by the distance between the *fundamentals* and the price of the last period:

$$SE_{i,t} = (F_i - P_t)^2 \tag{11}$$

the less the sum of the square errors, the higher the quota of agents that emulate the $i's$ guru is. Hence w_{t+1} can be defined as the following frequency:

$$w_{t+1} = \frac{exp[-\gamma(F_1 - P_t)^2]}{\sum_j exp[-\gamma(F_j - P_t)^2]} \tag{12}$$

where γ represents the intensity of choice. Substituting equation (12) into (10) the following dynamic system is achieved:

[3]In terms of the classical Day and Huang' paper (1990), our two groups are $\alpha/investors$.

$$P_{t+1} = P_t \left(1 + \alpha\beta \left[(F_2 - P_t) - \frac{F_2 - F_1}{1 + exp[\gamma(F_2 - F_1)(2P_t - F_1 - F_2)]}\right]\right). \qquad (13)$$

3 Dynamic Analysis

Throughout a mixture of local bifurcation analysis and simulations, we study the existence of steady state(s) and its (their) stability properties.

3.1 Fixed Points

Proposition 1. *Given map (13), at least two fixed points exists, the trivial $P^* = 0$ and $P_M = \frac{(F_1+F_2}{2}$. Two new steady states appear if $(F_2 - F_1) > \sqrt{\frac{2}{\gamma}}$. Finally, with the exception of $P^* = 0$, the set of steady states belong to the interval (F_1, F_2).*

Proof. Let $P_{t+1} = P_t = P^*$ be the condition to have a steady state. It is straightforward that $P^* = 0$ is a fixed point of the map (13). Moreover, The other possible steady states have to satisfy the following equation:

$$\frac{F_2 - P^*}{(F_2 - F_1)} = \frac{1}{1 + exp[\gamma(F_2 - F_1)(2P^* - F_1 - F_2)]}. \qquad (14)$$

It is easy to show that $P_M = \frac{(F_1+F_2)}{2}$ satisfies this relation. Let us assume that $F_2 > F_1$. The LHS (the blue line, in the Figures 1(a)-(b)) is a decreasing linear function in P^*, it crosses the x-axis in F_2 and the y-axis in $\frac{F_2}{F_2 - F_1}$, that is greater than one. The RHS (the red line) is a decreasing function and it has a limit for $P^* \to +\infty = 0$ and $P^* \to -\infty = 1$; therefore, the RHS must intersect the LHS at least in one point. Moreover, the second derivative of the RHS is zero in $P^* = P_M$. It can be easily shown that it is the unique inflection point, therefore P_M is a steady state always existing.

The derivatives of eq. 14 are the following:

$$-\frac{1}{(F_2 - F_1)} = -\frac{2\gamma(F_2 - F_1)exp[\gamma(F_2 - F_1)(2P^* - F_1 - F_2)]}{[1 + exp[\gamma(F_2 - F_1)(2P^* - F_1 - F_2)]]^2}. \qquad (15)$$

There are three intersections (steady states) when the derivative of the RHS of eq 14, calculated in the inflection point P_M is higher than that of the LHS, that is when:

$$(F_2 - F_1) > \sqrt{\frac{2}{\gamma}}. \tag{16}$$

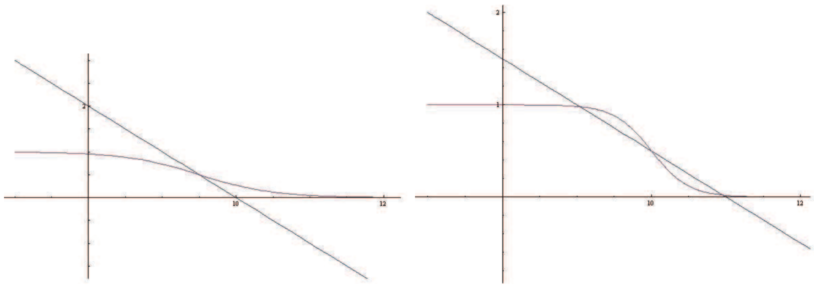

Figure 1: Steady states conditions

Therefore, given the intensity of choice, a large enough distance between the two biases leads to the appearance of two further steady states.

Finally, even if we do not know an analytical expression for the steady states, we can state that with the exception of $P^* = 0$, the set of steady states belong to the interval (F_1, F_2). Equation (14) can be rewritten as follows:

$$-\frac{F_1 - P^*}{F_2 - P^*} = exp[\gamma(F_2 - F_1)(2P^* - F_1 - F_2)], \tag{17}$$

The LHS has an asymptote for (F_2) and crosses the x-axis at (F_1). Given that the RHS is an exponential function, that is always positive, the possible steady states belong to the interval (F_1, F_2). This means that, each group cannot conquered the entire market. □

To study stability properties of both $P^* = 0$ and P_M we use both an analytical and a simulation approach.

3.2 Stability

To study the stability of the steady states we work out the following first derivative of map (13):

$$\frac{\partial P_{t+1}}{\partial P_t} = 1 - \frac{\alpha\beta(b_2 - b_1)}{1 + exp[\gamma(b_2 - b_1)(2P_t - 2F^* - b_1 - b_2)]} + \alpha\beta(b_2 + F^* - 2P_t) + \\ + \frac{2\gamma\alpha\beta P_t(b_2 - b_1)^2 exp[\gamma(b_2 - b_1)(2P_t - 2F^* - b_1 - b_2)]}{[1 + exp[\gamma(b_2 - b_1)(2P_t - 2F^* - b_1 - b_2)]]^2} \tag{18}$$

Proposition 2. *The steady state $P^* = 0$ is always unstable.*

Proof. Eq. (18) evaluated in $P^* = 0$ is:

$$\frac{\partial P_{t+1}}{\partial P_t}_{(P^*=0)} = 1 - \alpha\beta \left[\frac{(b_2 - b_1)}{1 + exp[\gamma(b_2 - b_1)(-2F^* - b_1 - b_2)]} - (F^* + b_2) \right] \tag{19}$$

$P^* = 0$ is stable if:

$$-2 < -\alpha\beta \left[\frac{(b_2 - b_1)}{1 + exp[\gamma(b_2 - b_1)(-2F^* - b_1 - b_2)]} - (F^* + b_2) \right] < 0 \tag{20}$$

The expression in the squared parenthesis is negative, therefore

$$-\alpha\beta \left[\frac{(b_2 - b_1)}{1 + exp[\gamma(b_2 - b_1)(-2F^* - b_1 - b_2)]} - (F^* + b_2) \right]$$

is positive, the eigenvalue is greater than 1 and, hence, $P^* = 0$ is always unstable. □

Proposition 3. *When $b_1 = b_2 = b$, P_M can lose stability through a period doubling bifurcation. When $b_1 \neq b_2$, P_M loses stability through a pitchfork bifurcation and also via a period doubling bifurcation.*

Proof. To evaluate the stability properties of P_M, we work out Eq. (18) for $P^* = P_M$:

$$\frac{\partial P_{t+1}}{\partial P_t}_{(P^*=P_M)} = 1 - \frac{\alpha\beta(b_2 - b_1)}{2} + \alpha\beta(-F^* - b_1) + \frac{1}{2}\alpha\beta\gamma(b_2 - b_1)^2(\frac{1}{2}b_1 + F^* + \frac{1}{2}b_2) \tag{21}$$

When $b_1 = b_2 = b$, Eq. (21) becomes simple as follows:

$$\frac{\partial P_{t+1}}{\partial P_t}_{(P^*=P_M)} = 1 - \alpha\beta(F^* + b) \tag{22}$$

In this case, P_M can lose stability only through a period doubling bifurcation if $\alpha\beta(F^* + b) = 2$, that is either when the reaction'coefficients or the expectation of future price are high enough. On the other hand, when $b_1 \neq b_2$, P_M is stable if:

$$-2 < g < 0 \tag{23}$$

where

$$g = -\frac{\alpha\beta(b_2 - b_1)}{2} + \alpha\beta(-F^* - b_1) + \frac{1}{2}\alpha\beta\gamma(b_2 - b_1)^2(\frac{1}{2}b_1 + F^* + \frac{1}{2}b_2).$$

Specifically, P_M loses its stability through a pitchfork bifurcation if:

$$(b_2 - b_1) = \sqrt{\frac{2}{\gamma}} \tag{24}$$

In this case, g=0 and the eigenvalue is equal to one. On the other hand, there are sets of parameters for which g=-2 and a flip bifurcation may occur. □

4 Numerical Analysis

When $b_1 = b_2$, the homogeneous case, the map (13) is topologically conjugated to the logistic and $P_M = F^* + b$ is the unique positive steady state. We use numerical simulations, to study the effects of an increasing heterogeneity, that is an increasing distance between b_1 and b_2. As already stated, differently from Brock and Hommes (1998) and De Grauwe and Rovira (2007), we focus on a situation in which the groups of agents are both optimists, that is $b_i > 0$. Specifically, we start from the homogeneous case ($b_1 = b_2$) setting up the following parameters $\alpha = 0.5, b_1 = b_2 = 1, F^* = 10, \gamma = 0.5$. Then, we increase β, the market maker's reaction coefficient, to analyze how heterogeneity affects the initial status. Given the propositions discussed above, the pitchfork bifurcation arises when $b_2 = 3$ holding $b_1 = 1$. In Fig. 2, as we already know from above, we can see that an increase in β leads to chaos through a cascade of period doubling bifurcations. Starting from different values of β we analyze how the occurrence of heterogeneity affects the dynamics.

The first case is showed in Fig 3. It is worth noting that with homogeneity (Fig. 3a) the max of the map is on the right of the fixed point, that is there is a monotonic convergence. When the biases are different enough a pitchfork bifurcation arises (see Fig.3b): P_M becomes unstable and two new steady states, P_L and P_H arise,

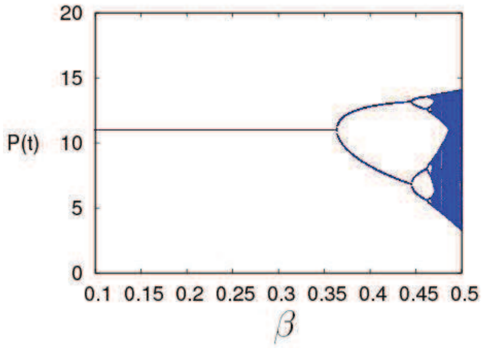

Figure 2: From stability to chaos in the homogeneous case, $\beta\epsilon[0.1, 0.5]$, $\alpha = 0.5, b_1 = b_2 = 1, F^* = 10, \gamma = 0.5$

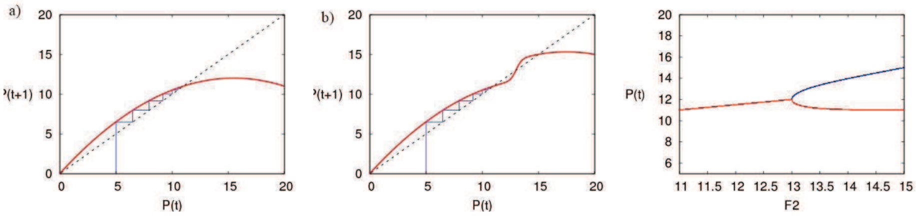

Figure 3: From Stability to Pitchfork. $\alpha = 0.5, \beta = 0.1, b_1 = 1, F^* = 10, \gamma = 0.5, b_2 = 1 \quad in \quad a), b_2 = 4.5 \quad in \quad b)$

with $P_L < P_M < P_H$. Moreover, as showed in Fig. 3c, we can see that the new steady states are stable.

In Fig. 4a, for a higher β there is an oscillatory convergence. As in the first case, when $b_2 = 3$ a pitchfork bifurcation occurs and two stable steady states arise. However, as shown in Fig. 4b and 4c, an increasing b_2 leads to two coexistent attractors. Indeed, there is the coexistence of a stable attractor around the lower steady state P_L and a chaotic attractor around the higher steady state, P_H. This is different from Naimzada and Ricchiuti (2009), where the dynamic behavior is symmetric around the two new steady states.

In the next figures we analyze the case in which P_M with homogenous biases is not stable. Specifically, we study the effect of increasing heterogeneity when the map shows either a period four cycle, chaos or divergence to infinity. In the

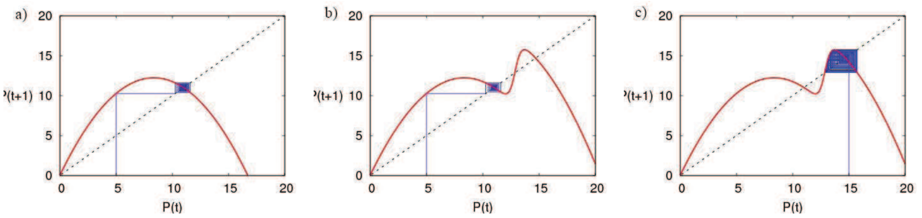

Figure 4: From oscillatory convergence to coexistent attractors. $\alpha = 0.5, \beta = 0.35, b_1 = 1, F^* = 10, \gamma = 0.5, b_2 = 1 \quad in \quad a), b_2 = 4.5 \quad in \quad b) \quad and \quad c)$

Fig. 5 and Fig. 6, β is set such that, with homogeneity, there is a cycle 4 or chaos respectively. In both cases, an increasing heterogeneity leads to a period halving bifurcation: the system switches to a new behavior with half the period of the map with homogeneity, a series of period-halving bifurcations leads the system from chaos to order. For $b_2 = 3$ the pitchfork bifurcation arises, moreover an increasing distance between the two biases leads two coexistent attractors (Fig. 5b-c and Fig. 6b-c), in both cases after the pitchfork there is a cascade of period-doubling bifurcations leading to chaos.

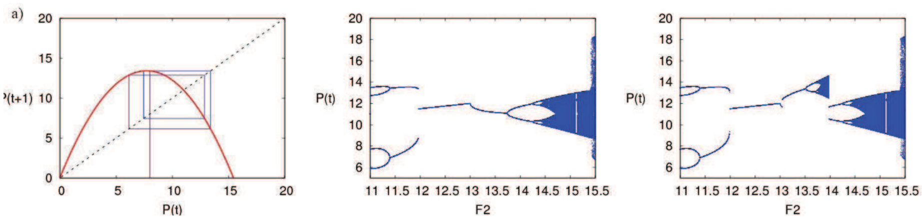

Figure 5: Period 4 cycle. $\alpha = 0.5, \beta = 0.454, b_1 = 1, F^* = 10, \gamma = 0.5, b_2 = 1 \quad in \quad a), P_0 = 11 \quad in \quad b) and P_0 = 7 \quad in \quad c)$

Finally setting $\beta = 0.6$, the map with homogeneity diverges (see Fig 7), clearly because the first iterate of the maximum lies out of the admissible interval. As expected heterogeneity acts such that the map becomes a self-map and so any initial condition in the admissible interval does not diverge.

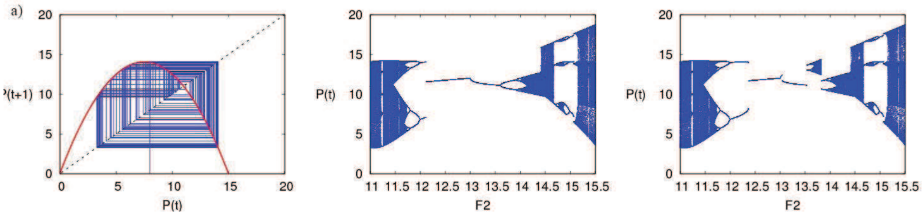

Figure 6: Chaos. $\alpha = 0.5, \beta = 0.5, b_1 = 1, F^* = 10, \gamma = 0.5,\ b_2 = 1 \quad in \quad a), P_0 = 11 \quad in \quad b) and P_0 = 7 \quad in \quad c)$

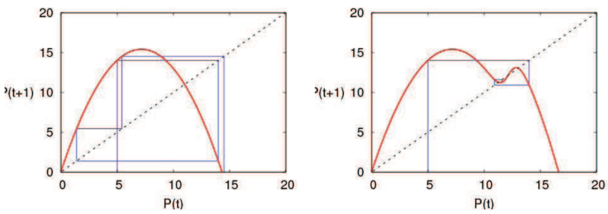

Figure 7: From Divergence to Self map. $\alpha = 0.5, \beta = 0.6, b_1 = 1, F^* = 10, \gamma = 0.5$

5 Conclusions

Heterogeneity has been developed in various models which have aided in explaining the financial market dynamics. Differently from the canonical models, we focus on agents that employ the same trading rule (i.e., fundamentalists) but heterogeneous beliefs, that is, they predict different *biased* expected prices. Moreover, agents can move from one expert to another following an adaptive rational mechanism.

As in Naimzada and Ricchiuti (2008,2009), we show that even interactions between agents that use the same trading rule can lead to complex price fluctuations and, in contrast to common knowledge, fundamentalists can **destabilize** the market. However, there are interesting differences. On one hand, heterogeneity can lead to the coexistence of stable attractor with a chaotic attractor. Therefore, the price variations on the market crucially depend on the initial conditions. On the other hand, heterogeneity plays an ambiguous role: first it stabilizes the market and then it destabilizes it. The idea is that when agents (or better gurus) disagrees but their distance is small, the aggregate effect is an agreement around an 'average' price (that for other economists is the representative agent). On the other hand, when agents/gurus opinions diverge consistently, the price variations are strong. This ambiguity depends (differently from Naimzada and Ricchiuti, 2009) on the condition of the map when the biases are homogenous. If there is monotonic convergence, heterogeneity leads only to a pitchfork bifurcation but the system is stable. On the other hand, when there is oscillatory convergence heterogeneity affects the sequence of period doubling. Finally, with period cycles and chaos an increasing heterogeneity firstly stabilizes and then destabilizes the system.

We would like to indicate some additions to our simple behavioral financial model. First of all, it would be interesting to analyze a case when there are more than two gurus and agents who have long memory. In such a case, it would be possible to analyze the interactions between many views within the economic system and their survival in an evolutionary environment based on historical data. A higher complexity could be reached by either studying the dynamics when agents follow the guru that guarantees higher profits rather than the better forecast or adding, more realistically, a group of chartists.

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