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Information Disclosure in Procurement Auctions with Horizontally Differentiated Suppliers

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Abstract

This work studies a multidimensional auction in which a buyer needs to procure a given good from either of two potential suppliers with heterogeneous costs and whose qualities are the buyer's private information. Comparing the outcomes of different procurement policies, a trade-off between efficiency and rent-extraction emerges. Buyer's expected utility is maximized via a first score auction and either concealing or privately revealing suppliers' quality - the optimal choice depending on the degree of heterogeneity in costs and qualities. Conversely, efficiency calls for a second score auction or a first score auction with public disclosure of qualities.

1 Introduction

In procurement auctions buyers are frequently interested not only in minimizing the price, but also in obtaining the maximum "value for money". This value may depend on the quality associated to the proposed product, and/or the intrinsic characteristics of each supplier. Multidimensional auction models usually assume that the quality associated with each bid is ex-ante known by the respective bidder. Quality is therefore generally represented as an objective factor that can be either exogenous or endogenously determined as part of the bidding process. In many procurement settings however, a sort of horizontal differentiation exists, in the sense that buyers subjectively assess the quality of different goods according to their individual requirements or tastes. In this context quality can be regarded as a measure of how much the good fits the need of the buyer, and the procurer holds private information about such aspect of the alternative suppliers, while bidders can only guess how their quality will be evaluated.

In this paper we analyze some of the interesting questions associated with procurement auctions in the presence of horizontally differentiated suppliers: should the auctioneer reveal information about the quality of suppliers during the bidding process? If so, how much? How does the auction format influence the outcome of the selection procedure in this context?

These questions are particularly relevant given the increased flexibility in auction formats used in the procurement sector. In many countries the discipline on public procurement has recently enlarged the range of alternative bidding procedures to include reverse electronic auctions, in which differing amounts of information revelation are allowed (see e.g. General Service Administration [15], page 12; European Directives 2004/18/EC, art. 54). Similarly, in the B2B sector private procurers adopt a diverse range of practices with respect to the release of information regarding their qualitative evaluation (see e.g. Teich *et al.* [28] and Kostamis *et al.* [20].).

Following the existing literature (see the next Section) we model suppliers' quality as a random variable that is privately known by the procurers, whose bids are ranked according to a function of both price and quality, the score. In a first score auction the disclosure of this information directly affects suppliers' bidding strategies and consequently it influences the actual outcome of the procedure. We address three different regimes concerning the dissemination of information: concealment, private revelation and public revelation.[1] In the first case, the procurer maintains absolute secrecy regarding her valuation of the qualitative aspects of the bids. In the second case, each supplier learns the value assigned to his quality. In the last case, the procurer publicly reports the scores assigned to the qualitative aspects of all competitors.

The existing literature has studied this issue assuming identical costs for all suppliers. We drop such assumption in the present paper. We also explicitly study the interplay between information regime and auction format. The main conclusions of our model are that i) with a second score auction format the information regime is irrelevant in the sense that it does not affect the auction's outcome and the allocation is surely efficient; ii) the same result can be achieved by means of a first score auction in association with a public revelation regime; iii) the maximization of the procurer's expected utility requires a first score auction format, associated with the appropriate information regime, as follows: when the weight assigned to suppliers' quality is relatively high with respect to the potential asymmetry in their costs, then the optimal information regime is private revelation. In the opposite case the optimal information regime is concealment. A familiar trade-off between efficiency and rent extraction thus emerges as an outcome of our model. In the classic literature on multidimensional auctions the existence of

[1] Apparently, the implementation of a private revelation regime is legally and practically more problematic. However, it can be interpreted as a case in which the buyer publicly announces her preferences about the attributes she cares for in determining the merits of suppliers, therefore overcoming such potential problems.

this trade-off induces the procurer to underweight quality in the optimal scoring function. In our model such trade-off drives the buyer to disclose less information than efficiency would require.

The article is organized as follows. Section 2 provides a concise review of the related literature. Section 3 introduces the general model and the policies available to the buyer. Section 4 analyzes the equilibrium bidding strategies under different procurement policies. Section 5 compares the respective outcomes from two different point of view: their allocative efficiency and their convenience for the buyer. Finally, Section 7 summarizes the main conclusions and identifies some potential extensions of this work. All proofs can be found in the mathematical appendix.

2 Related literature

In view of studying a multidimensional auction procedure in a context in which the quality of each supplier strictly depends on the subjective requirements of the buyer, quality in this paper is assumed to be the buyer's private information. The traditional literature on multidimensional auctions (e.g. Che [8], Branco [6], Naegelen [24], Asker and Cantillon [3].) is not adequate to address this issue, its main focus being on the optimal scoring function given an information setup in which the quality of each bid is either common knowledge or known at least by each specific supplier. Typically the optimal scoring function puts less weight on quality with respect to the true preferences of the buyer.

Rezende [26] extends this optimal mechanism approach by assuming that the buyer is privately informed about the quality of each supplier, and shows that the policy regarding the revelation of information is irrelevant when the buyer is able to commit herself to the adoption of an optimal scoring function. However, from a practical standpoint, the buyer might not be able to implement such an optimal mechanism, due to the buyer's inability to commit to an ex-post inefficient allocation rule[2]. As a consequence, and in accord with commonly observed procurement practices, many authors have assumed that the procurer adopts a "naive" scoring function, equal to her true utility function (see Gal-Or *et al.* [14], Kostamis *et al.* [20], Engelbrecht-Wiggans *et al.* [11] and Katok and Wambach [17]). This approach redefines the goal of the analysis as follows: to come up with the optimal information revelation strategy, given a well defined awarding rule.

[2] Rezende ([26], section 4) also focuses on this case, but he hypothesises that, in absence of commitment power, the buyer is unable to credibly commit herself not to renege the price proposed by the winning bidder. In our analysis we exclude the possibility of a renegotiation phase and for this reason our results are not comparable.

The paper of Gal-Or *et al.* [14] investigates a multidimensional auction according to the above approach. Their model displays the main characteristic of horizontal differentiation, i.e. suppliers cannot be ranked ex-ante objectively according to quality, which in turn is not correlated with production costs. They show that under the hypothesis of risk-neutral suppliers with identical costs private and public revelation are equivalent, while concealment is generally the worst option. Doni and Menicucci [10] extend the analysis of Gal-Or *et al.* to the case of risk-averse bidders; they show that in this case the private revelation always dominates the public revelation, while concealment can be the best option for the buyer when bidders are sufficiently risk averse.

In the present paper we extend the Gal-Or *et al.* model along another direction, namely by allowing for heterogeneity in firms' costs, as usually hypothesized in auction theory. However, we assume that suppliers are perfectly informed about the cost of their competitor. This hypothesis, beside fitting existing real contexts, makes the model more tractable. Heterogeneous costs also appear in the paper by Kostamis *et al.* [20] where they are considered suppliers' private information. The main difference between our model and Kostamis *et al.* [20] is that they assume that the buyer can choose the information regime after having observed suppliers' quality. This hypothesis can be interesting in private procurement while it is not viable in public procurement because usually regulations force the public procurer to announce the information regime before the proposals' submission takes place. However, their analysis is based on a Bayes-Nash equilibrium that is not sequentially rational. Indeed they neglect the effect of buyer's strategy on suppliers' beliefs.

The model considered here can be described as a two-stage game: in the first stage the procurer chooses the procurement policy and in this way selects the auction game that will be played by the suppliers in the second stage. Six alternative policies are possible, involving two auction formats (first score and second score auctions) and three alternative information regimes (concealment, private revelation, and public revelation). Each policy can be associated to some existing model: the Second Score auction resembles the original Vickrey auction. The First Score-Public Revelation policy represents a game with complete information where suppliers are characterized by different valuations and compete à la Bertrand. The First Score-Private Revelation policy makes suppliers compete as in a specific kind of asymmetric auction, already studied at length e.g. by Maskin and Riley [22]. Finally, the First Score-Concealment policy can be analyzed as a horizontally differentiated duopoly where the position of each supplier is exogenously located at either end of the Hotelling line and the buyer's position is distributed symmetrically with respect to the centre of the line. The similarity between a first score auction with concealment and a horizontal differentiated market is also supported by the coincidence between the equilibrium bidding strategies identified by Gal-Or *et al.* [14] and the price equilibrium of the model of horizontal differentiation put forward by Perloff and Salop [25].

The issue of information revelation in an horizontally differentiated set-up is studied by Kaplan [16] where the suppliers' quality depends both on their privately known characteristics and on the buyer's unknown preferences. The paper studies two scenarios which are close to our concealment and private revelation regimes; however the results are not fully comparable with ours as he applies his analysis to a contest, while we consider an auction.

The topic of endogenous information is also analyzed in Ganuza [13]. The author considers an independent private value auction in which the auctioneer holds some private information about the characteristics of the object. Such information can be revealed more or less precisely, and impacts on the bidders' awareness of their own valuation. Without information releasing, bidders' beliefs are completely homogeneous, whereas as more information is revealed bidders become more heterogeneous. The paper's main result emphasizes the existence of a trade off between efficiency and rent extraction: the former would require full revelation while the latter would be obtained without any revelation. So in that model the optimal policy[3] requires that the buyer's private information be partially revealed. Although this finding is very close in spirit to our conclusions, there are relevant differences. In Ganuza [13] bidders must submit a price without knowing with certainty their valuation, so their ex-post payoff in case of winning is uncertain. Conversely, in the present model every bidder knows perfectly his cost; consequently, he automatically defines his ex-post profit by choosing a price level. The uncertainty regarding his quality affects only his probability of winning.

3 The general setup

Consider a unique buyer wishing to purchase a single unit of a specific product by means of an auction. There are 2 firms competing to supply the item. Both the buyer and the suppliers are assumed to be risk-neutral.

The two suppliers have different production costs and perfect information both about their own and their opponent's cost[4]. These costs are labelled c_w and c_s

[3]Note that also this issue has been analysed from two different point of view: Ganuza [13] looks for the optimal information policy given a naïve allocation rule, the classic second price auction. Conversely, Esö and Szentes [12] adopt an optimal mechanism approach; they find that if the auctioneer can implement the optimal allocation rule, than she should reveal all the information at her disposal.

[4]The perfect information assumption, with regard to bidders' costs, can be found also in the multidimensional auction model analysed by Burguet and Che [7]. Thank to this similarity, we will take advantage of some of their results.

with $c_w - c_s = \Delta \geq 0$, so we have a "weak" supplier and a "strong" supplier (with subscripts w and s used henceforth in this sense).

The buyer is assumed to value the specific product provided by each seller differently. Let q_i denote the buyer's evaluation of the quality of the product supplied by bidder $i = w, s$. The quality parameters q_i are assumed to be generated by independent and identically distributed random variables whose realizations are privately known by the buyer. The probability distribution governing the uncertainty on qualities is assumed to have a continuously differentiable density function $f(q)$ on the support $\left[\underline{q}, \bar{q}\right]$, where $\underline{q} > 0$, and to satisfy the following:

Assumption 1. *The density function f is log-concave, i.e. $\frac{f'}{f}$ is decreasing.*

This is a rather common assumption in auction theory (see e.g. Branco [6]). Besides, as shown in Bagnoli and Bergstrom [5], this property is satisfied by a wide range of probability distributions and has various interesting consequences.

The utility that the buyer can obtain contracting with a specific supplier depends on the quality of his product and the price asked to provide it:

$$U(q_i, p_i) = q_i - p_i, \quad i = w, s$$

The buyer will only purchase from suppliers providing non-negative utility[5]. The parameter $b = \bar{q} - \underline{q} > 0$ can be interpreted in terms of how much the buyer cares for quality: indeed it represents the maximum the buyer is willing to pay for contracting with the highest-quality type rather than with the lowest possible one.

The score function used to evaluate alternative bids is assumed to coincide with the buyer's utility function, that is:

$$\sigma_i = q_i - p_i, \quad i = w, s$$

Each supplier selects an economic bid p_i in order to maximize his expected profit, equal to his ex-post profit times the probability of being the selected contractor:

$$\max_{p_i} (p_i - c_i) \Pr\{\sigma_i \geq \sigma_j\} \tag{1}$$

The parameter $\underline{q}$ is constrained to be sufficiently high to guarantee a positive score for each bidder in equilibrium. This is a standard assumption in the multi-dimensional auction literature, e.g. Che [8] (Assumption 2, page 671). Implicitly,

[5] This puts an upper bound on the bid any supplier can make, and so rules out practically uninteresting equilibria where suppliers place unboundedly high bids with some positive probability. See, for example, Baye and Morgan [4], who identify equilibria of this type in standard Bertrand games.

this imposes a lower bound on the evaluation of $\underline{q}$ to make sure the score function remains positive in equilibrium. So the analysis targets the cases in which there is enough "gain from trade".

Supplier i's "valuation" of the contract is defined as the social surplus stemming from the exchange between the buyer and himself: $v_i = q_i - c_i, i = w, s$. Remark that this valuation is equal to the maximum score achievable by a supplier under the constraint of a non-negative ex-post profit ($p_i \geq c_i$). In the following analysis $\underline{v}_i$ and $\bar{v}_i$ denote the minimum and maximum possible values for the valuation of supplier i. Notice that the (common) length of the valuations intervals, $\bar{v}_i - \underline{v}_i$, equals b, whereas the shift between the two intervals, $\underline{v}_s - \underline{v}_w$, is equal to Δ. Therefore depending on b and Δ the possible valuations for the two suppliers lie in intervals which may or not overlap.

Analyzing the efficiency and the expected utility of the outcomes achievable by the buyer using different procurement policies is the main goal of the paper. Specifically, a procurement policy consists of an auction format and an information regime. The analysis will compare two possible auction formats and three different information regimes. As a first step, the potential differences arising from the choice of either a first score (FS) or a second score (SS) auction will be compared. The winning bidder will be always the one with the highest score: but in the former case, the price will equal the submitted bid, whereas in the latter the price will be such as to have the winner match the score achieved by the non-winning supplier (in other words if i wins, i.e. $\sigma_i > \sigma_j$, i will be rewarded a price equal to $q_i - \sigma_j$).

We want to compare different information regimes that the buyer can adopt with respect to the revelation of her valuation of the suppliers' qualities. The buyer chooses how much information (if any) to disclose. Practically, she has to choose one of the following information regimes:

a) Concealment (C): no information is revealed, and consequently each supplier must decide the price to submit knowing nothing about the buyer's rating of their product;

b) Private Revelation (Pr): each supplier is informed about the quality rating received by his product, but not that received by his opponent;

c) Public Revelation (Pu): the buyer announces publicly the quality rating of each supplier's product.

To recap, the timing of the model is as follows:

1) the buyer announces a procurement policy, specifying the auction format and the information regime;

2) Nature chooses qualities;

3) the buyer observes the quality of each seller and then she sends a message to everyone according to the chosen information regime;

4) each supplier receives the buyer's message and selects his bid strategy;

5) the auction runs, the winning bidder is selected, and buyer and sellers receive their payoffs.

4 Suppliers' bidding strategies under different procurement policies

This Section derives the equilibrium bidding strategies for suppliers under each of the alternative procurement policies.

4.1 Second score (*SS*) auction

In *SS* auctions suppliers' strategies are determined as in standard second price auctions. Restricting attention to equilibria in weakly dominant strategies[6] implies there is a unique Nash equilibrium and the final allocation is independent of the information regime adopted by the buyer[7]. The standard Vickrey argument allows to characterize the equilibrium strategies and the outcome of the auction as follows:

SS.i) both suppliers bid their marginal cost: $p_i^{SS}(c_i) = c_i$;

SS.ii) bidder i wins if and only if $v_i > v_j$;

SS.iii) in equilibrium, the buyer's utility equals the losing bidder's valuation.

Observe that, with regard to SS.ii), given our assumption on the distribution of qualities, the situation where the bidders' valuations are equal can be ignored as it occurs with zero probability.

4.2 First score (*FS*) auction

This Section shows that the information regime is relevant when the buyer adopts an *FS* rather than an *SS* auction.

[6]Note that in our context the revelation policy can affect the suppliers' knowledge about the procurer's assessment of their quality. This fact could make the suppliers aware of the ranking of their valuation. In such case there is a continuum of weakly-dominated equilibria, admitting both efficient and inefficient outcomes (see Kim and Che [18] for the exact description of such equilibria).

[7]This result may not hold if the procurer adopts a dynamic reverse auction for the submission of the price bid. Katok and Wambach [17] show both theoretically and experimentally that different disclosure policies can significantly affect the outcome of the procurement procedure. However, the second score auction and the multidimensional English auction are outcome-equivalent if every bidder must be always informed of his rank during the dynamic phase, as required by the EU legislation.

4.2.1 Public Revelation (*Pu*).

In this case the suppliers are publicly informed about their quality and their competitor's, prior to submitting their price offers. Given that suppliers also know the production costs, it follows that they are perfectly informed about the valuation of both competitors. As a consequence, the supplier with the highest valuation has a direct bidding advantage over his competitor as in standard Bertrand games with asymmetric players. Thus, in any pure-strategy equilibrium, the winning bidder must be the seller with the highest valuation achieving a score equal to the valuation of his opponent, while the losing bidder must be unable to increase his score any higher and so must be bidding his marginal cost . To avoid problems with regard to the existence of an equilibrium in the presence of perfect information, it is assumed that ties are broken by means of a *SS* auction (which boils down to awarding the contract to the supplier with the highest valuation). This observation provides the intuition to characterize the unique Nash equilibrium in pure strategies of the *FS* auction with Public Revelation (*Pu*)[8]:

Pu.i) bidder i wins if and only if $v_i > v_j$;

Pu.ii) the winning supplier (i) bids and receives price $p_i^{Pu} = q_i - q_j + c_j$ (which implies that $\sigma_i = v_j$);

Pu.iii) the losing supplier (j) bids a price equal to his own marginal cost, $p_j^{Pu} = c_j$.

Notice that the equilibrium outcome of the *Pu* policy corresponds to that of the *SS* policy, so the two are outcome-equivalent. For this reason, henceforth the *Pu* and *SS* auctions will be jointly referred to as the *SS-Pu* policy.

4.2.2 Private Revelation (*Pr*)

It is shown here that this selection procedure is equivalent to an asymmetric auction. Indeed each supplier chooses his bid by solving $\max_{p_i}(p_i - c_i)\Pr\{\sigma_i \geq \sigma_j\}$ or equivalently

$$\max_{\sigma_i}(v_i - \sigma_i)\Pr\{\sigma_i \geq \sigma_j\}.$$

Now, the *Pr* policy means each bidder is privately informed on his own valuation, while it is common knowledge that suppliers' valuations are drawn from the distributions

$$\begin{aligned} F_s &: \left[\underline{q} - c_s, \bar{q} - c_s\right] \to [0,1]\,, \quad F_s(x) = F(x + c_s) \\ F_w &: \left[\underline{q} - c_w, \bar{q} - c_w\right] \to [0,1]\,, \quad F_w(x) = F(x + c_w) \end{aligned}$$

[8] A formal proof can be found Burguet and Che [7], where the existence of a continuum of payoff-equivalent equilibria in which the winning bidder does as in Pu.ii) and the losing bidder mixes between bidding his cost and bidding his cost plus a positive (sufficiently small) constant is also shown.

given the probability distribution F which determines the qualities. This means we have an asymmetric auction where bidders' distributions are identical except for a shift, whose size is equal to the cost difference between the two suppliers. This setup boils down to bidders choosing a score given a privately known valuation. Applying the results achieved by Maskin and Riley [22] (Proposition 3.5.ii), in a FS auction with Pr the equilibrium bidding strategies can be shown to satisfy the following property:

Pr.i) $v_s(\sigma) > v_w(\sigma)$, $\forall\sigma \in (\sigma_*, \sigma^*)$, where $v_s(\cdot)$ and $v_w(\cdot)$ are the inverse bidding functions of the strong and the weak bidder, while σ_* and σ^* are the lowest and the highest winning bids in equilibrium.

The interpretation is that, in equilibrium, bidders obtain the same score only if the strong has a higher valuation than the weak's. In practice, the two adopt asymmetric bidding functions whereby the weak supplier bids more aggressively than the strong, requiring a smaller mark-up with respect to his cost.

4.2.3 Concealment (C)

When the buyer releases no information about qualities, each supplier chooses his price strategy knowing only the production costs. Therefore, the suppliers' problem reported in (1) can be rewritten as follows:

$$\max_{p_s}(p_s - c_s)\Pr\{z \leq p_w - p_s\} \tag{2}$$

$$\max_{p_w}(p_w - c_w)\Pr\{z \geq p_w - p_s\} \tag{3}$$

where $z = q_w - q_s$. Suppliers share common beliefs with regard to the possible difference between their qualities. The density of the random variable z, $h(z)$, can be obtained by the convolution of $f(q)$, the density generating q, and of its mirror image $f(-q)$. Formally:

$$h(z) = \begin{cases} \int_{\underline{q}}^{\bar{q}+z} f(q-z) f(q)\, dq & if \quad z \in [-b, 0] \\ \int_{\underline{q}+z}^{\bar{q}} f(q-z) f(q)\, dq & if \quad z \in (0, b] \end{cases}$$

This problem is similar to a generalization of the Hotelling model, as shown in Section 5. Some interesting properties of the random variable z can be easily derived as summarized in Lemma 9 in the Appendix. In particular, its density turns out to be log-concave. The first order conditions of problems (2) and (3) and this property (which ensures second order conditions for (2) and (3) are satisfied), yield the following equilibrium bidding strategies:

$$p_s = c_s + \frac{H(z^*)}{h(z^*)}, \qquad p_w = c_w + \frac{1 - H(z^*)}{h(z^*)} \tag{4}$$

where z^* is the solution to the following implicit equation:

$$z^* = \Delta + \frac{1 - 2H(z^*)}{h(z^*)} \tag{5}$$

Notice that z^* corresponds to the difference between qualities such that the auction ends up with an exact tie. We can now state the following Proposition[9].

Proposition 2. *The following properties hold.*
C.i) $z^*(\Delta)$ *is a strictly increasing bijection from* $[0, +\infty)$ *to* $[0, b)$*;*
C.ii) *in equilibrium,* $\Delta > 0 \Rightarrow p_w > p_s$ *and* $0 < p_w - c_w < p_s - c_s$.

Thus under the C policy the strong supplier always asks for a lower price than the weak supplier – but not as much as to win the auction with certainty (because z^* is always less than b). Note that in equilibrium both suppliers bid a price strictly higher than their costs and, as a consequence, both try to obtain an extra-profit i.e. to gain a rent. Such rent is smaller for the weak supplier, who is therefore more aggressive than the strong, which makes this situation similar to a standard asymmetric auction (see Section 4.2.2). For example, under uniformly distributed qualities, the nature of these rents can be clarified by inspecting the suppliers' bidding strategies:

$$\begin{aligned} p_s^C &= c_s + \frac{5(\Delta - b)}{8} + \frac{3\sqrt{(\Delta - b)^2 + 8b}}{8} \\ p_w^C &= c_w - \frac{\Delta - b}{8} + \frac{\sqrt{(\Delta - b)^2 + 8b}}{8} \end{aligned}$$

In general, it is instructive to consider two extreme cases: if $b = 0$, then the weak supplier bids a price equal to his cost and his opponent will win with certainty earning a rent equal to his cost advantage, Δ. This is the classic informative rent for the most efficient bidder in a Bertrand game with cost-asymmetric and quality-homogeneous suppliers. If $\Delta = 0$ then both suppliers bid a price equal to their costs plus a rent equal to $\frac{1}{2h(0)}$ (if qualities are uniformly distributed this equals $\frac{b}{2}$) and each of them will win with a 50% chance. In this case both suppliers earn a positive expected rent in equilibrium, due to the importance the buyer assigns to (quality-induced) product differentiation. For this reason we can define it as Hotelling rent. When both b and Δ are strictly positive, the weak supplier will gain only an expected rent due to the Hotelling effect, while his opponent will get a composition of the two distinct rents.

[9] An equivalent result appears within a different setting in Anderson and Renault [2].

5 A comparative evaluation of different procurement policies

This Section compares the procurement policies from two different points of view. To begin with, the efficiency of each mechanism is evaluated. A mechanism is defined efficient if it always maximizes the social surplus created by the exchange, i.e. if the highest valuation supplier is selected with certainty.

Proposition 3. *For the procurement auction defined in Section 3 the following properties hold:*
1) Under the SS-Pu *policy the mechanism is always efficient, whatever the value of* b *and* Δ*.*
2) Under the Pr *policy the mechanism is efficient if and only if either* $\Delta = 0$ *or* $\Delta - b \geq \frac{1}{f(\overline{q})}$*.*
3) Under the C *policy the mechanism is efficient if and only if* $\Delta = 0$*.*

The above Proposition characterizes the allocative properties of the equilibrium under the various mechanisms. One implication is that the bidders' strategies under heterogeneous costs can induce an inefficient allocation. In particular, if the buyer adopts the FS auction format, the efficiency of the mechanism is achievable only fully revealing the buyer's information. The other regimes, providing only a partial revelation, may generate inefficient outcomes unless suppliers have identical costs (*Pr* and *C* policies) or the cost difference is high enough (*Pr* policy). Notice that with the *Pr* policy efficiency with heterogeneous costs is linked to the so called "Getty Effect" (see Maskin and Riley [22]), after the known art museum icon of the case in which the strong buyer is so strong that his bid beats the weak buyer for sure: remarkably, this never happens under *C*. Overall, Proposition 3 shows that the fact that efficiency holds whatever the information regime, which is true in the Gal-Or *et al.* [14] setup, is not robust to the introduction of a minor heterogeneity in the production costs of suppliers.

It is in principle also possible to see the amount of efficiency loss associated to the *Pr* and *C* policies for the intermediate values of Δ for which both are inefficient. We have done this exercise numerically assuming uniformly distributed qualities and the results can be seen in Figure 1. Remark that the ranking among the three policies according to efficiency is such that *Pu* is better than *Pr* which is better than *C*.

INSERT FIGURE 0 ABOUT HERE

A related but different type of comparison can be done by evaluating each procurement policy from the standpoint of the buyer's expected utility. Since the suppliers' qualities are uncertain ex-ante, the buyer needs choosing the policy resorting to the expected utility generated by each different mechanism on the basis of the probability distribution of qualities.

We start calculating the buyer's expected utility in the case of the C and the *SS-Pu* policies by taking advantage of the similarities between our auction model and a properly defined Hotelling setup. Consider two suppliers exogenously located at the ends of a Hotelling line and a buyer located at a point $z \in [-b, b]$. Let d_s (d_w) denote the distance separating the buyer from the strong (weak) supplier, assumed to be located in $-b$ (b), so $d_s = b+z$ and $d_w = b-z$. Given the price proposed by the suppliers, p_s and p_w, the buyer will purchase from the supplier which maximizes his utility function U^H corresponding to the Hotelling score $\sigma_i^H = k - p_i - \frac{d_i}{2}, \;\; i = s, w$ (where k is a constant).

Lemma 4. *The First Score auction and the Hotelling model are strategically and payoff equivalent provided that:*

i) $z = q_w - q_s$
ii) $k = q^e + \frac{b}{2}$

Note however that there is a difference between the (interpretation of the) two models. Indeed, in our case z does not represent the distribution of consumers' types, but rather the beliefs regarding the type of the unique existing buyer, which can be thought of as a specific value assigned to the difference between the two competitors.

The C regime is equivalent to a horizontally differentiated duopoly in which the suppliers' strategies depend on their beliefs about the buyer's position to the set of feasible proposed prices. On the other hand, the *SS-Pu* regime corresponds to the case in which suppliers strategies are mappings from the set of possible buyer's position to feasible prices. The following Lemma works out the expected utility of the buyer under the two different policies.

Lemma 5. *The buyer's expected utility is:*

$$EU^{SS-Pu} = q^e - c_w + \min\{b, \Delta\} - \int_{-b}^{\min\{b,\Delta\}} H(z)\, dz \tag{6}$$

$$EU^C = q^e - c_w - \frac{1 - H(z^*)}{h(z^*)} + \int_{-b}^{z^*} H(z)\, dz \tag{7}$$

Formula (6) corresponds to the expected value of the second highest valuation: indeed this holds in a *SS* auction context, where the payoff accruing the auctioneer equals the second highest valuation.

For what concerns the *Pr* policy, the calculation of the buyer's expected utility is more complex. However, we can take advantage of the coincidence between the Pr regime and an asymmetric auction in order to exploit some general results shown by Maskin and Riley [22], and consider the limiting cases of $\Delta = 0$ and large Δ (with respect to b) to state the following Proposition.

Proposition 6. *The ranking among policies, from the point of view of the auctioneer, satisfies the following properties.*
1) If $\Delta = 0$ *then* $Pr \sim SS - Pu \succ C$ *provided that* f *is monotone.*
2) For large enough Δ, $C \succ Pr \succ SS - Pu$.
3) For all $\Delta > 0$, $Pr \succ SS - Pu$ *provided that* F *is convex.*

This Proposition suggests that, when heterogeneous costs are introduced there is not a dominant procurement policy for any value of the cost difference. To get an intuition as to why C becomes the most attractive regime for the buyer when the cost difference is high enough, consider the rationale of releasing information to the suppliers: it is a way of inducing more competition by "suggesting" the weak supplier can win as well. But this is precisely what becomes (almost) not viable past a threshold for Δ (because the strong wins almost surely): in that case passing information to the suppliers (as it happens in Pr and *SS-Pu*) can only harm the buyer.

Moreover, as it can be shown, the buyer's expected utility is linearly homogeneous in Δ and b: in other word doubling both the cost difference and the size of the quality support doubles the expected utility of the procurer in each information regime, without altering the ranking among such different regimes. So, what really plays a key role in determining the respective positions of the information regimes is the relative importance of the two sources of heterogeneity of our model, namely cost and quality differences. Another implication is that, for intermediate values of Δ, the global ranking depends on the specific probability distribution of the qualities: while Pr always dominates *SS-Pu*, Proposition 6 leaves their relative position with respect to C indeterminate.

We present some numerical results (for a number of different density functions) to illustrate some regularities in the ranking of the information regimes from the buyer's expected utility perspective.

INSERT FIGURES 2 AND 3 ABOUT HERE

Figures 2 and 3 show how under the different regimes[10] the buyer's expected utility (gross of the weak supplier's cost[11]) varies as a function of the cost difference, assuming the normalization $b = 1$. What emerges quite clearly is that, for

[10] For what concerns the regimes C and *SS-Pu*, we applied (6) and (7). With regard to the Pr regime, we computed the buyer's expected utility by means of the BIDCOMP^2 software (see Li and Riley [21]).

[11] The rationale for this is that a given value for Δ may result from different combinations of c_s and c_w. This potential source of ambiguity is ruled out by adding c_w to buyer's expected utility.

positive but not too large values of Δ, which information regime (between C and Pr) dominates depends on the specific probability distribution of the qualities. In all the numerical estimates we carried out for various choices of distributions (only part of which are shown in Figures 1 and 2) we observe that the expected utilities as functions of Δ cross once and only once. Furthermore, comparing the buyers's expected utility plots of two monotone and symmetric distributions, the crossing point in the decreasing case lies to the right with respect to that of the increasing case. Our simulations suggest that such properties may have some kind of generality, and perhaps proving it calls for future research endeavour. Finally, the distance between Pr and C stems from two different aspects of the density distribution: for low values of Δ the distance between Pr and C increases with the variance. For higher values of Δ such distance increases with the mean of the distribution (both these facts are a by-product of the analytical characterization about the pairwise differences in the buyer's expected utility shown in the proof of Proposition 6).

6 More than two sellers

Let us now consider the case of procurement auctions where more than two sellers are competing to supply the item to the buyer. In what follows we will refer to the previous setup of Section 3 adapted with the following changes/integrations:

- there are n sellers, with costs equal to $c_1 \leq c_2 \leq \cdots \leq c_n$;
- $\Delta_j = c_{j+1} - c_j$;
- $f_{k,n}(q)$ is the probability density function of the k^{th} largest quality among n sellers.

We first address the problem of evaluating the allocative efficiency of different procurement mechanisms.

Proposition 7. *For the procurement auction with n suppliers the following properties hold:*
1) Under the SS-Pu *policy the mechanism is always efficient.*
2) Under the Pr *policy the mechanism is efficient if and only if either (i) $\Delta_i = 0, i = 1, \ldots, k-1$ and $\Delta_k > b$ for some $k \in \{2, \ldots, n-1\}$ or (ii) $\Delta_1 - b \geq \frac{1}{f_{1,k}(\bar{q})}$ where $k = \#\{j : c_j = c_2\}$.*
3) Under the C *policy the mechanism is efficient if either (i) $\Delta_i = 0, \forall i$ or (ii) $\Delta_i = 0, i = 1, \ldots, k-1$ and $\Delta_k > b\left(1 + \frac{1}{k\int_{\underline{q}}^{\bar{q}} f_{2,k}(q)f(q)dq}\right)$ for some $k \in \{2, \ldots, n-1\}$*

From the standpoint of the buyer's expected utility the following holds.

Proposition 8. *The ranking among policies, from the point of view of the auctioneer, satisfies the following properties.*
1) If $\Delta_i = 0, \forall i$ then $Pr \sim SS - Pu \succ C$ provided that f is increasing.
2) For large enough Δ_1, $C \succ Pr \succ SS - Pu$.
3) If $\Delta_1 > 0$ and $\Delta_i = 0, \forall i > 1$ then $Pr \succ SS - Pu$ provided that f is increasing.[12]

Summing up, *SS-Pu* retains its efficiency property, while the other regimes guarantee efficient outcomes only for quite special configurations of cost differences. Concerning the buyer's expected utility discussed in Proposition 6 the ranking $Pr \sim SS - Pu \succ C$ is robust to the introduction of multiple homogeneous suppliers provided that f is increasing, while the ranking $C \succ Pr \succ SS - Pu$ continues to hold if there is a large enough difference between the strong supplier and its competitors.

7 Conclusion

In many procurement settings, a supplier's perceived quality can depend on (or be defined by) the buyer's subjective requirements or tastes. In these cases buyers have to choose what kind of information to provide the suppliers before their submission of the economic bid. In this paper we have adopted a multidimensional auction model with horizontally differentiated suppliers in order to analyze the outcomes generated by different information regimes: concealment, private and public revelation. Gal-Or *et al.* [14] show that, in the case of suppliers with identical costs, all three procurement policies guarantee an efficient allocation of the contract. Besides, private and public revelation are equivalent from the buyer's expected utility standpoint, whereas concealment is generally inferior.

In our setting there are in effect two independent sources of suppliers' heterogeneity: cost structure and quality of the output, as perceived by the procurer. Under the assumption that production costs are perfectly known to the suppliers (whose number we limit to two), but unknown to the buyer, our main findings are as follows:

i) an efficient mechanism requires public revelation of the suppliers' quality; concealing or privately revealing such information can lead to inefficient allocations when the cost difference between suppliers is strictly positive;

ii) private and public revelation fail to be equivalent when suppliers' costs are heterogeneous; private revelation induces an asymmetric auction and increases the buyer's expected utility with respect to public revelation;

[12] Kirkegaard [19] also shows that 3) can be extended to slightly more general situations.

iii) private and public revelation are preferable to concealment for sufficiently low values of the ratio between the suppliers' cost difference and the weight assigned to quality considerations. However, for sufficiently large values of this ratio, concealment becomes the best strategy.

The interplay between different dimensions of heterogeneity is fundamental in shaping the relative desirability of the three information regimes, from the viewpoint of the procurer's expected utility from the auction. Our results support the choice of a concealment regime when one of the firms is known to have large cost advantages with respect to the competitors. Observe that, if the buyer is uncertain about the cost difference, the optimal information regime depends on buyer's beliefs regarding such costs. Whenever such cost differences are (or are believed to be) comparatively less important our model suggests the private revelation policy. Publicly revealing the information about the qualities is socially efficient but it is never in the best interest of the buyer.

The paper also shows that the outcome associated to the public revelation is coincident with the one achievable with a second score auction, whatever the information regime. Therefore, a corollary of our conclusions is that in this framework first and second score mechanisms are not always equivalent.

Some of the assumptions upon which this work is built may be generalized further. For this reason we think that future research should extend our analysis along various directions. A first extension would be to allow for an endogenous number of bidders: in such case, the degree of competition in the auction could be influenced by the information regime and so the resulting ranking of alternative policies could change. A further extension could consider vertically differentiated suppliers. In many situations the quality of two different goods/suppliers can in fact be objectively ranked and in such case the uncertainty solely regards the buyer's willingness to pay for purchasing the higher quality product instead of the lower quality one. Future research should then investigate how different procurement policies affect not only the buyer's expected utility, but also the incentives to invest in product quality when suppliers can choose to vertically differentiate their products.

Finally, procurement policies evaluation should keep into account their vulnerability to phenomena of corruption and collusion. Therefore, analyzing the pros and cons of these different procurement policies when either qualitative evaluations are expressed by agents of public administration who can pursue their private interest, or when bidders can create a cartel, should also be part of the future research agenda.

Appendix

Lemma 9. *Let z be the difference between two independent random variables whose common density is a log-concave function $f(q)$. Then the following properties hold:*

- *Its density function $h(z)$ is log-concave.*
- *z has monotone increasing hazard rate and monotone decreasing reverse hazard rate, i.e.:*

$$\frac{d}{dz}\left(\frac{h(z)}{1-H(z)}\right) \geq 0 \tag{8}$$

$$\frac{d}{dz}\left(\frac{h(z)}{H(z)}\right) \leq 0 \tag{9}$$

- *$h(z)$ is symmetric and therefore $H(0) = \frac{1}{2}$.*

Proof. Notice first that from the log-concavity assumption on $f(q)$ trivially follows that $f(-q)$ is also log-concave. Hence $h(z)$ is log-concave, due to the fact that this property preserves under convolution, and so is the distribution $H(z)$, because log-concavity of a density function implies log-concavity of its cumulative distribution function (see An [1] and Bagnoli and Bergstrom [5]). This property implies z has monotone increasing hazard rate and monotone decreasing reverse hazard rate (see An [1]). Finally, it is straightforward that $h(z)$ is symmetric and therefore $H(0) = \frac{1}{2}$. ☐

Proposition (2, page 11)**.** *The following properties hold.*
C.i) *$z^*(\Delta)$ is a strictly increasing bijection from $[0, +\infty)$ to $[0, b)$;*
C.ii) *in equilibrium, $\Delta > 0 \Rightarrow p_w > p_s$ and $0 < p_w - c_w < p_s - c_s$.*

Proof. C.i) Notice that, applying the Implicit Function theorem to equation (5), we get $\frac{dz^*(\Delta)}{d\Delta} = \left(\frac{\partial}{\partial z^*}\left(z^* - \frac{1-2H(z^*)}{h(z^*)}\right)\right)^{-1} = \left(1 - \frac{\partial}{\partial z^*}\left(\frac{1-H(z^*)}{h(z^*)}\right) + \frac{\partial}{\partial z^*}\left(\frac{H(z^*)}{h(z^*)}\right)\right)^{-1} > 0$ due to (8) and (9), which shows that $z^*(\Delta)$ is strictly increasing. This and the fact that $H(0) = \frac{1}{2}$ imply that equation (5) is satisfied for $\Delta = 0$ if and only if $z^* = 0$. Further, given that the inverse function $\Delta(z^*) = z^* - \frac{1-2H(z^*)}{h(z^*)}$ is also monotone increasing and that $\lim_{z^* \to b} \Delta(z^*) = +\infty$, we conclude that $z^*(\Delta)$ is a bijection from $[0, +\infty)$ to $[0, b)$.
C.ii) If $\Delta > 0$ then $p_w - p_s = \Delta + \frac{1-2H(z^*)}{h(z^*)} = z^* > 0$ in equilibrium (due to C.i). Further $p_w - c_w - p_s + c_s = \frac{1-2H(z^*)}{h(z^*)} = \frac{1-H(z^*)}{h(z^*)} - \frac{H(z^*)}{h(z^*)} \leq 0$ due to (9), (8) and the fact that $z^* > 0$ in equilibrium. ☐

Proposition (3, page 12). *For the procurement auction defined in Section 3 the following properties hold:*
1) Under the SS-Pu *policy the mechanism is always efficient, whatever the value of* b *and* Δ*.*
2) Under the Pr *policy the mechanism is efficient if and only if either* $\Delta = 0$ *or* $\Delta - b \geq \frac{1}{f(\bar{q})}$*.*
3) Under the C *policy the mechanism is efficient if and only if* $\Delta = 0$*.*

Proof. 1) The result for the *SS-Pu* case immediately follows from the fact that, as we remark in Section 4, both these procurement policies imply the selection of the supplier with the highest contract valuation.
2) The setup with a *FS* payment rule and a *Pr* regime when $\Delta = 0$ reduces to a symmetric Independent Private Values auction and therefore is efficient. When $\Delta > 0$ the auction is asymmetric. To analyze this case, assume first that the weak supplier bids his own valuation. Then the strong supplier's expected utility is $\pi_s^E(s_s) = (v_s - s_s) F_w(s_s)$ to be maximized with respect to the score s_s, where v_s is his valuation and $F_w(v) = F(v + c_w) = F(q)$ is the distribution function of the weak bidder valuation. The objective function has derivative

$$\frac{d}{ds_s}\pi_s^E(s_s) = (v_s - s_s) f_w(s_s) - F_w(s_s)$$

Under the assumption $\Delta - b \geq \frac{1}{f(\bar{q})}$ the optimal score turns out to be $\bar{v}_w$. Indeed observe that if $s_s > \bar{v}_w = \bar{q} - c_w$ then $\frac{d}{ds_s}\pi_s^E(s_s) = -1$ which means that the strong supplier has no incentive to bid higher than his competitor's maximum possible bid. Also,

$$\begin{aligned}
&\underline{v}_s - \bar{v}_w = \Delta - b \geq \frac{1}{f(\bar{q})} = \frac{F_w(\bar{v}_w)}{f_w(\bar{v}_w)} \Rightarrow \\
&v_s \geq \underline{v}_s \geq \bar{v}_w + \frac{F_w(\bar{v}_w)}{f_w(\bar{v}_w)} \geq x + \frac{F_w(x)}{f_w(x)} \Rightarrow \\
&(v_s - x) f_w(x) - F_w(x) \geq 0
\end{aligned}$$

whenever $x < \bar{v}_w$ thanks to the logconcavity of F.[13] So $s_s^* = \bar{v}_w$. In turn, as the strong bidder wins with certainty, bidding his own valuation is a best response strategy for the weak bidder. Finally Maskin and Riley ([23], Proposition 1) prove that this equilibrium is, in essence, unique.

[13] Indeed log-concavity of the density f (which we assume in Assumption 1) implies the log-concavity of the distribution function F (see Bagnoli and Bergstrom [5], Theorem 1). This implies that $x + \frac{F_w(x)}{f_w(x)}$ is increasing.

Conversely, when $0 < \Delta < b + \frac{1}{f(\bar{q})}$,

$$\begin{aligned} \underline{v}_s &< \bar{v}_w + \frac{F_w(\bar{v}_w)}{f_w(\bar{v}_w)} \Rightarrow \exists v_s \geq \underline{v}_s : \\ & (v_s - \bar{v}_w) f_w(\bar{v}_w) - F_w(\bar{v}_w) < 0 \end{aligned}$$

so that the strong bidder optimal score can be less than $\bar{v}_w$, thus leaving the weak bidder with a positive winning probability. Consequently, there is a non-empty interval of winning bids (s_*, s^*) (see Pr.i) in Section 4.2.2) over which suppliers' inverse bidding function are both defined and such that $v_s(s) > v_w(s)$, $\forall s \in (s_*, s^*)$. Then, by continuity of the inverse bidding functions, there exists $\varepsilon > 0$ such that $v_s(s) > v_w(s+\varepsilon)$, or, equivalently, given a couple of valuations $v_s > v_w$ for which the bidders' scores are the same, there is $\delta > 0$ such that $v_s > v_w + \delta$ and $s_s(v_s) < s_w(v_w + \delta)$ so the weak bidder wins the auction despite having the lowest valuation.
3) In Section 4.2.3 we have seen that, under the C regime, the solution of the equation

$$z^* = c_w - c_s + \frac{1 - 2H(z^*)}{h(z^*)}$$

where H is the distribution of $q_w - q_s$, is such that, in equilibrium, $z^* = p_w - p_s$. Therefore, when $q_w - q_s > (<) z^*$ supplier $w(s)$ wins the auction. Now notice that efficiency requires that $q_w - q_s > c_w - c_s$ when w wins and vice-versa; hence equilibrium and efficiency conditions coincide if and only if $\frac{1-2H(z^*)}{h(z^*)} = 0$. Due to property C.i), this equality holds when $z^* = 0$ and in turn this is true if and only if $c_w - c_s = \Delta = 0$. □

Lemma (4, page 13)**.** *The First Score auction and the Hotelling model are strategically and payoff equivalent provided that:*

i) $z = q_w - q_s$
ii) $k = q^e + \frac{b}{2}$

Proof. We first show that the outcome does not depend on the chosen setup. That is $\sigma_s^H < \sigma_w^H$ if and only if $\sigma_s < \sigma_w$. We have:

$$\begin{aligned} \sigma_s^H &< \sigma_w^H \Leftrightarrow k - p_s - \frac{d_s}{2} < k - p_w - \frac{d_w}{2} \\ &\Leftrightarrow q_s - p_s < q_w - p_w \Leftrightarrow \sigma_s < \sigma_w \end{aligned}$$

As a consequence, the suppliers' expected profit at any set of prices is identical in the two models. Indeed, for the strong supplier we have:

$$E\pi_s^H(p_s, p_w) = (p_s - c_s)\mathcal{P}\left\{\sigma_s^H \geq \sigma_w^H\right\} = (p_s - c_s)\mathcal{P}\left\{\sigma_s \geq \sigma_w\right\} = E\pi_s(p_s, p_w)$$

and analogously for the weak supplier. Strategic equivalence follows (as well as suppliers' payoff equivalence). We now show that the buyer's expected utility is the same in the two setups. Indeed, let z^* be such that the strong seller wins if and only if $z \leq z^*$. Using i), ii) and the definitions of d_s and d_w (see page 13), we can write

$$\begin{aligned}
EU^H &= E\left(\sigma_s^H \mid z \leq z^*\right) + E\left(\sigma_w^H \mid z > z^*\right) \\
&= E\left(k - p_s - \frac{d_s}{2} \mid z \leq z^*\right) + E\left(k - p_w - \frac{d_w}{2} \mid z > z^*\right) \\
&= E\left(q^e + \frac{b}{2} - p_s - \frac{b + q_w - q_s}{2} \mid q_w - q_s \leq z^*\right) + \\
&\quad E\left(q^e + \frac{b}{2} - p_w - \frac{b - q_w + q_s}{2} \mid q_w - q_s > z^*\right) \\
&= E\left(q^e - \frac{q_w + q_s}{2}\right) + E\left(q_s - p_s \mid q_w - q_s \leq z^*\right) + E\left(q_w - p_w \mid q_w - q_s > z^*\right) \\
&= E\left(q_s - p_s \mid q_w - q_s \leq z^*\right) + E\left(q_w - p_w \mid q_w - q_s > z^*\right) = EU
\end{aligned}$$

which concludes our proof. $\square$

Lemma (5, page 13)**.** *The buyer's expected utility, as a function of Δ, is:*

$$\begin{aligned}
EU^{SS-Pu}(\Delta) &= q^e - c_w + \min\{b, \Delta\} - \int_{-b}^{\min\{b,\Delta\}} H(z)\, dz \\
EU^C(\Delta) &= q^e - c_w - \frac{1 - H(z^*)}{h(z^*)} + \int_{-b}^{z^*} H(z)\, dz
\end{aligned}$$

Proof. In equilibrium the prices proposed by the winning seller (who will be the strong if $\Delta > z$ and the weak otherwise) under the $SS{-}Pu$ regime is $p_i = c_j + q_i - q_j$

$$
\begin{aligned}
EU^{SS-Pu} &= \int\limits_{-b}^{\min\{b,\Delta\}} \left(k - c_w + z - \frac{z}{2} - \frac{b}{2}\right) h(z)\, dz + \int\limits_{\min\{b,\Delta\}}^{b} \left(k - c_s - z + \frac{z}{2} - \frac{b}{2}\right) h(z)\, dz \\
&= \int\limits_{-b}^{\min\{b,\Delta\}} \left(k - c_w + \frac{z}{2} - \frac{b}{2}\right) h(z)\, dz + \int\limits_{\min\{b,\Delta\}}^{b} \left(k - c_s - \frac{z}{2} - \frac{b}{2}\right) h(z)\, dz \\
&= \int_{-b}^{b} \left(k - c_w - \frac{b}{2}\right) h(z)\, dz + \int\limits_{-b}^{\min\{b,\Delta\}} \frac{z}{2} h(z)\, dz + \int\limits_{\min\{b,\Delta\}}^{b} \left(\Delta - \frac{z}{2}\right) h(z)\, dz \\
&= k - c_w - \frac{b}{2} + \Delta \int\limits_{\min\{b,\Delta\}}^{b} h(z)\, dz + \int_{-b}^{0} \frac{z}{2} h(z)\, dz + \\
&\quad \int\limits_{0}^{\min\{b,\Delta\}} \frac{z}{2} h(z)\, dz - \left(\int_{0}^{b} \frac{z}{2} h(z)\, dz - \int\limits_{0}^{\min\{b,\Delta\}} \frac{z}{2} h(z)\, dz \right) \\
&= k - c_w - \frac{b}{2} + \Delta \int\limits_{\min\{b,\Delta\}}^{b} h(z)\, dz + \int\limits_{0}^{\min\{b,\Delta\}} z h(z)\, dz + \\
&\quad \int_{-b}^{0} \frac{z}{2} h(z)\, dz - \int_{0}^{b} \frac{z}{2} h(z)\, dz \\
&= k - c_w - \frac{b}{2} + \Delta \int_{\min\{b,\Delta\}}^{b} h(z)\, dz + \int\limits_{-b}^{\min\{b,\Delta\}} z h(z)\, dz \\
&= k - c_w - \frac{b}{2} + \Delta \left(H(b) - H(\min\{b,\Delta\})\right) + \min\{b,\Delta\} H(\min\{b,\Delta\}) + \\
&\quad bH(-b) - \int\limits_{-b}^{\min\{b,\Delta\}} H(z)\, dz \\
&= q^e - c_w + \min\{b,\Delta\} - \int\limits_{-b}^{\min\{b,\Delta\}} H(z)\, dz
\end{aligned}
$$

Let us now turn to $EU^C(\Delta)$. In this case we have $p_s = c_s + \frac{H(z^*)}{h(z^*)}$, $p_w = c_w + \frac{1-H(z^*)}{h(z^*)}$ and the strong (weak) will win if $z < z^*$ ($z > z^*$), where z^* is such that $z^* = \Delta + \frac{1-2H(z^*)}{h(z^*)}$ as obtained in Section 4.2.3.

Therefore:

$$\begin{aligned}
EU^C(\Delta) &= \int_{-b}^{z^*} \left(k - c_s - \frac{H(z^*)}{h(z^*)} - \frac{z}{2} - \frac{b}{2}\right) h(z)\,dz + \\
&\quad \int_{z^*}^{b} \left(k - c_w - \frac{1-H(z^*)}{h(z^*)} + \frac{z}{2} - \frac{b}{2}\right) h(z)\,dz \\
&= \int_{-b}^{b} \left(k - c_w - \frac{b}{2}\right) h(z)\,dz + \int_{-b}^{z^*} \left(\Delta - \frac{H(z^*)}{h(z^*)} - \frac{z}{2}\right) h(z)\,dz + \\
&\quad \int_{z^*}^{b} \left(-\frac{1-H(z^*)}{h(z^*)} + \frac{z}{2}\right) h(z)\,dz \\
&= k - c_w - \frac{b}{2} + \left(\Delta - \frac{H(z^*)}{h(z^*)}\right) \int_{-b}^{z^*} h(z)\,dz - \frac{1-H(z^*)}{h(z^*)} \int_{z^*}^{b} h(z)\,dz + \\
&\quad \int_{-b}^{z^*} \left(-\frac{z}{2}\right) h(z)\,dz + \int_{z^*}^{b} \frac{z}{2} h(z)\,dz \\
&= k - c_w - \frac{b}{2} + \left(\Delta - \frac{H(z^*)}{h(z^*)}\right) H(z^*) - \frac{1-H(z^*)}{h(z^*)} (1 - H(z^*)) - \\
&\quad \int_{-b}^{0} \frac{z}{2} h(z)\,dz - \int_{0}^{z^*} \frac{z}{2} h(z)\,dz + \int_{0}^{b} \frac{z}{2} h(z)\,dz - \int_{0}^{z^*} \frac{z}{2} h(z)\,dz \\
&= k - c_w - \frac{b}{2} + H(z^*) \left(\Delta - \frac{H(z^*)}{h(z^*)} + \frac{1-H(z^*)}{h(z^*)}\right) - \frac{1-H(z^*)}{h(z^*)} + \int_{z^*}^{b} zh(z)\,dz \\
&= k - c_w - \frac{b}{2} + H(z^*)\,z^* - \frac{1-H(z^*)}{h(z^*)} + b - z^* H(z^*) - \int_{z^*}^{b} H(z)\,dz \\
&= q^e - c_w - \frac{1-H(z^*)}{h(z^*)} + b - \int_{z^*}^{b} H(z)\,dz \\
&= q^e - c_w - \frac{1-H(z^*)}{h(z^*)} + \int_{-b}^{b} H(z)\,dz - \int_{z^*}^{b} H(z)\,dz \\
&= q^e - c_w - \frac{1-H(z^*)}{h(z^*)} + \int_{-b}^{z^*} H(z)\,dz
\end{aligned}$$

□

Proposition (6, page 14)**.** *The ranking among policies, from the point of view of the auctioneer, satisfies the following properties.*
1) If $\Delta = 0$ *then* $Pr \sim SS - Pu \succ C$ *provided that* f *is monotone.*
2) For large enough Δ, $C \succ Pr \succ SS - Pu$.
3) For all $\Delta > 0$, $Pr \succ SS - Pu$ *provided that* F *is convex.*

Proof. 1) The above Lemma 5 and the Revenue Equivalence Theorem show that:

$$\begin{aligned} EU^{SS-Pu}(\Delta=0) &= q^e - c_w - \int_{-b}^{0} H(z)\,dz \\ EU^{C}(\Delta=0) &= q^e - c_w - \frac{1}{2h(0)} + \int_{-b}^{0} H(z)\,dz \\ EU^{Pr}(\Delta=0) &= EU^{SS-Pu}(\Delta=0) = q^e - c_w - \int_{-b}^{0} H(z)\,dz \end{aligned}$$

We now show that the above formula are indeed equivalent to those found in Gal-Or *et al.* [14]. To this purpose suppose $c_w = 0$. Using the fact that[14]

$$\int_{-b}^{0} H(z)\,dz = \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{\underline{q}}^{\bar{q}} F^2(z)\,dz$$

[14]Indeed

$$\begin{aligned} \int_{-b}^{0} H(z)\,dz &= \int_{-b}^{0}\left(\int_{\underline{q}}^{\bar{q}} f(x)\,F(x+z)\,dx\right)dz \\ &= \int_{-b}^{0} F(\bar{q}+z)\,dz - \int_{\underline{q}}^{\bar{q}}\left(\int_{-b}^{0} F(x)\,f(x+z)\,dz\right)dx \\ &= \int_{-b}^{0} F(\bar{q}+z)\,dz - \int_{\underline{q}}^{\bar{q}} F(x)\,F(x+z)|_{-b}^{0}\,dx = \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{\underline{q}}^{\bar{q}} F^2(z)\,dz \end{aligned}$$

we get

$$\begin{aligned}
EU^{SS-Pu}(\Delta=0) &= q^e - \int_{-b}^{0} H(z)\,dz \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{-b}^{0} H(z)\,dz \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{-b}^{0}\left(\int_{\underline{q}}^{\bar{q}} f(x)\,F(x+z)\,dx\right)dz \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{-b}^{0}\left(F(x)\,F(x+z)|_{\underline{q}}^{\bar{q}} - \int_{\underline{q}}^{\bar{q}} F(x)\,f(x+z)\,dx\right)dz \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{-b}^{0} F(\bar{q}+z)\,dz + \int_{\underline{q}}^{\bar{q}}\left(\int_{-b}^{0} F(x)\,f(x+z)\,dz\right)dx \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{-b}^{0} F(\bar{q}+z)\,dz + \int_{\underline{q}}^{\bar{q}} F(x)\,F(x+z)|_{-b}^{0}\,dx \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{\underline{q}}^{\bar{q}} F(z)\,dz + \int_{\underline{q}}^{\bar{q}} F^2(z)\,dz \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,(2-F(z))\,dz = E\left(q_{(2)}\right)
\end{aligned}$$

where the last equality stems from the fact that the cumulative distribution function of the second-order statistic for q is given by $F_{(2)} = F(2-F)$ and $E\left(q_{(2)}\right) = \int_{\underline{q}}^{\bar{q}} z f_{(2)}(z)\,dz = \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,(2-F(z))\,dz$. As for the expected revenue under concealment we have:

$$\begin{aligned}
EU^{C}(\Delta=0) &= q^e - \frac{1}{2h(0)} + \int_{-b}^{0} H(z)\,dz \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \frac{1}{2h(0)} + \int_{-b}^{0} H(z)\,dz \\
&= \bar{q} - \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \frac{1}{2\int_{\underline{q}}^{\bar{q}} f^2(x)\,dx} + \int_{\underline{q}}^{\bar{q}} F(z)\,dz - \int_{\underline{q}}^{\bar{q}} F^2(z)\,dz \\
&= \bar{q} - \frac{1}{2\int_{\underline{q}}^{\bar{q}} f^2(x)\,dx} - \int_{\underline{q}}^{\bar{q}} F^2(z)\,dz \\
&= E\left(q_{(1)}\right) - \frac{1}{2\int_{\underline{q}}^{\bar{q}} f^2(x)\,dx}
\end{aligned}$$

where the last equality uses the fact that the cumulative distribution function of the first-order statistic for q is given by $F_{(1)} = F^2$ and $E\left(q_{(1)}\right) = \int_{\underline{q}}^{\bar{q}} z f_{(1)}(z)\, dz = \bar{q} - \int_{\underline{q}}^{\bar{q}} F^2(z)\, dz$. The reader can check that these are indeed the formulas of Proposition 3 in Gal-Or *et al.* [14]. In light of the assumption that $f(x)$ is monotone the very same Proposition shows that when $\Delta = 0$ the *SS-Pu* information regime (or equivalently the *Pr*) dominates C in terms of the procurer's expected revenue. Indeed Gal-Or *et al.* [14] actually uses the assumption $f'(x) \geq 0$. However, if $f'(x) \leq 0$ then its mirror image on the support $\left[\underline{q}, \bar{q}\right]$, $f\left(\bar{q} + \underline{q} - x\right)$ is also log-concave and increasing and, as shown in Doni and Menicucci [10] (Proposition 1), it leads to sort the expected revenues in the same order as does $f(x)$.

2) Let us now turn to the case of large Δ. In the Pr format, condition $\Delta - b \geq \frac{1}{f(\bar{q})}$ ensures efficiency, hence that the strong bidder outbids his competitor offering $\bar{q} - c_w$ regardless of his quality. This fact, together with formulas in Lemma 5, allows to determine the ranking for large Δ. Indeed when Δ is large enough the difference between EU^{Pr} and EU^{SS-Pu} equals $\bar{q} - q^e$ and in turn the difference between EU^C and EU^{Pr} approaches $q^e + b - \bar{q} = q^e - \underline{q}$. Therefore for large Δ the ranking from the point of view of the auctioneer is $C \succ Pr \succ SS - Pu$.

3) This directly follows from to Proposition 4.3 of Maskin and Riley [22]: indeed the *Pr* policy corresponds to the high-bid auction and the *SS-Pu* to the open auction. It can be easily shown that the extra technical requirements of such Proposition are redundant given that we assume a log-concave density without probability masses. ☐

Proposition 10 (7, page 7)**.** *For the procurement auction with n suppliers the following properties hold:*
1) Under the SS-Pu *policy the mechanism is always efficient.*
2) Under the Pr *policy the mechanism is efficient if and only if either (i) $\Delta_i = 0, i = 1, \ldots, k-1$ and $\Delta_k > b$ for some $k \in \{2, \ldots, n-1\}$ or (ii) $\Delta_1 - b \geq \frac{1}{f_{1,k}(\bar{q})}$ where $k = \#\{j : c_j = c_2\}$.*
3) Under the C *policy the mechanism is efficient if either (i) $\Delta_i = 0, \forall i$ or (ii) $\Delta_i = 0, i = 1, \ldots, k-1$ and $\Delta_k > b\left(1 + \frac{1}{k \int_{\underline{q}}^{\bar{q}} f_{2,k}(q) f(q) dq}\right)$ for some $k \in \{2, \ldots, n-1\}$*

Proof. 1) This immediately stems from the fact that under complete information the seller with the highest valuation always wins.

2) The proof is, in its essence, the same as in the $n = 2$ case with some minimal caveats as follows.
(i) When more than one seller shares the lowest cost and all the other competitors have lower valuations with certainty then:
- a strong seller with the lowest possible quality cannot have a positive profit in equilibrium (because a positive profit is not compatible with his winning probability being zero in equilibrium);

- as a consequence of $\Delta_k > b$ all the non-strong sellers cannot win;
- therefore the competition reduces to the subset of strong players and the outcome is efficient due to symmetry considerations.

To show that such conditions are necessary we need to study two separate situations. The case $\Delta_1 > 0$ is analyzed in (ii). Suppose now that $\Delta_i = 0, i = 1, \ldots, k-1$ and $\Delta_k \leq b$. In this case (some) non-strong bidders can win in equilibrium and they will bid more than the strong sellers for any given valuation (see e.g. Li and Riley [21], Prop. 6). Continuity implies that a weak bidder can win the auction despite there being a strong seller with an higher valuation.
(ii) When there is only one strong seller, condition $\Delta_1 - b \geq \frac{1}{f_{1,k}(\bar{q})}$ is necessary and sufficient to ensure that the strong supplier optimal bid equals his competitors' maximum possible bid (details are the same as in the case $n = 2$).

3) (i) In any symmetric equilibrium the n homogeneous sellers will bid equally and therefore the most efficient seller wins the auction with certainty. (ii) Assume instead that $\Delta_i = 0, i = 1, \ldots, k-1$ and $\Delta_k > b\left(1 + \frac{1}{k\int_{\underline{q}}^{\bar{q}} f_{2,k}(q)f(q)dq}\right)$ for some $k \in \{2, \ldots, n-1\}$. In this case if the strong suppliers bid as if the non-strong are not in the game, each of them bids $p = \frac{b}{k\int_{\underline{q}}^{\bar{q}} f_{2,k}(q)f(q)dq}$ (see Gal-Or et al. [14], Prop. 1). But doing so, given that $\Delta_k > b\left(1 + \frac{1}{k\int_{\underline{q}}^{\bar{q}} f_{2,k}(q)f(q)dq}\right)$, will effectively rule out any chance of winning for the non-strong suppliers. So this is an equilibrium and it is efficient. Such symmetric equilibrium (i.e. in which the strong play symmetrically) is also unique: suppose instead that another equilibrium exists in which the strong play symmetrically; it would be such that some non-strong sellers have a positive probability of winning (otherwise contradicting uniqueness in Gal-Or et al. [14], Prop. 1). $\square$

Proposition 11 (8, page 16). *The ranking among policies, from the point of view of the auctioneer, satisfies the following properties.*
1) If $\Delta_i = 0, \forall i$ then $Pr \sim SS - Pu \succ C$ provided that f is increasing.
2) For large enough Δ_1, $C \succ Pr \succ SS - Pu$.
3) If $\Delta_1 > 0$ and $\Delta_i = 0, \forall i > 1$ then $Pr \succ SS - Pu$ provided that f is increasing.[15]

Proof. 1) This comes again from the Revenue Equivalence Theorem and the correspondence of our model under homogeneous costs with that of Gal-Or et al. [14].
2) The proof of the fact that $Pr \succ SS - Pu$ is the same as for $n = 2$ (see part 2 of Proposition 6), with the obvious caveat that Proposition 7 needs to be invoked instead of Proposition 3. Notice that If $\Delta_i = 0, i = 2, \ldots, k$ the size of Δ_1 which ensures the result decreases as a consequence of the fact that $f_{1,k}(\bar{q})$ increases with k. We know $C \succ Pr$ holds for $n = 2$. In the C regime, adding more non-strong

[15] Kirkegaard [19] also shows that 3) can be extended to slightly more general situations.

suppliers will make those with cost equal c_2 more aggressive, which in turn will make the strong supplier more aggressive as well. As a consequence the buyer's expected utility will (weakly) increase, whereas it is unchanged for the Pr regime (which comes from the above argument whereby in Pr a large enough Δ_1 ensures that the strong will certainly outbid the other suppliers).
3) See Kirkegaard [19], Prop. 12. $\square$

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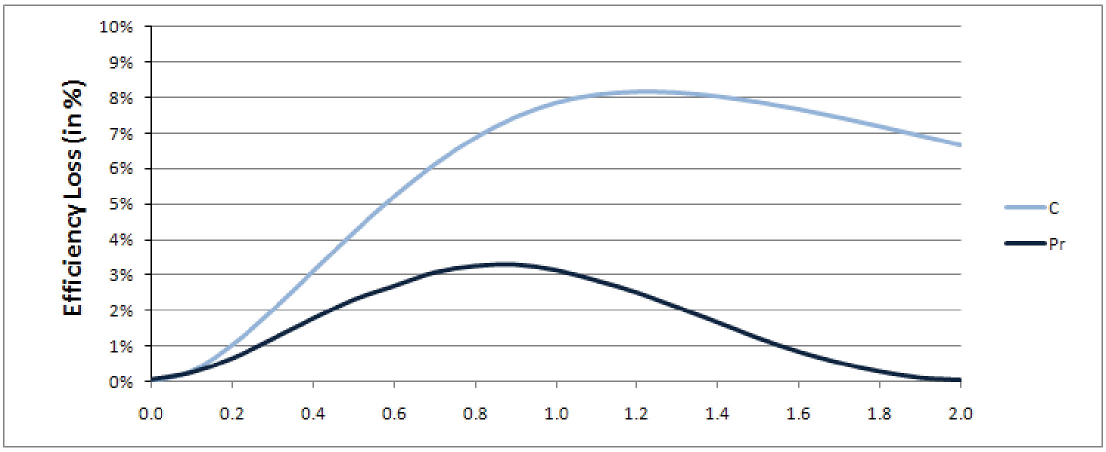

Figure 1: Loss in social surplus for intermediate values of Δ.

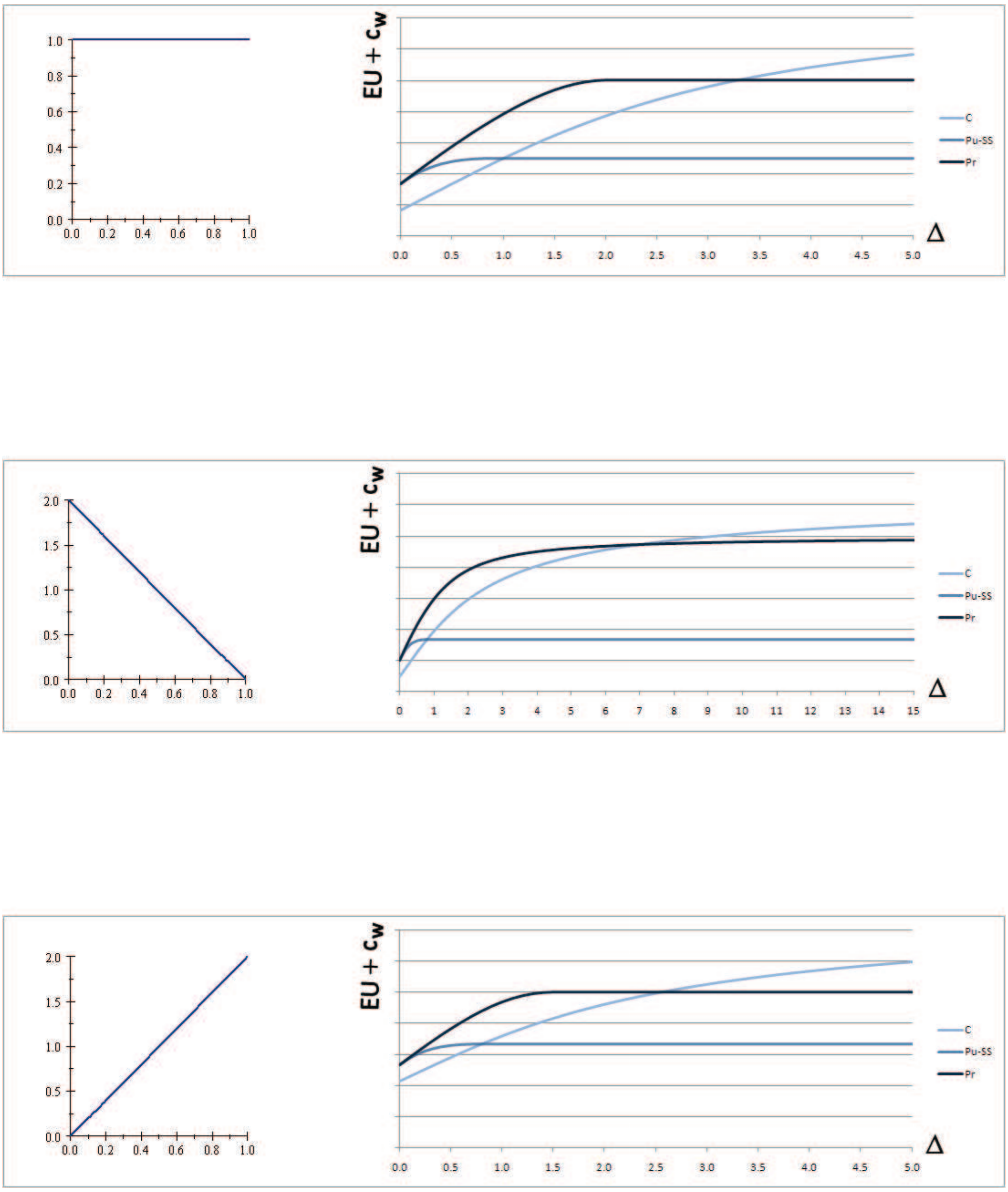

Figure 2: Top: uniform - Middle: $f(x) = 2 - 2x$ - Bottom: $f(x) = 2x$. Notice the different scales on the horizontal axis.

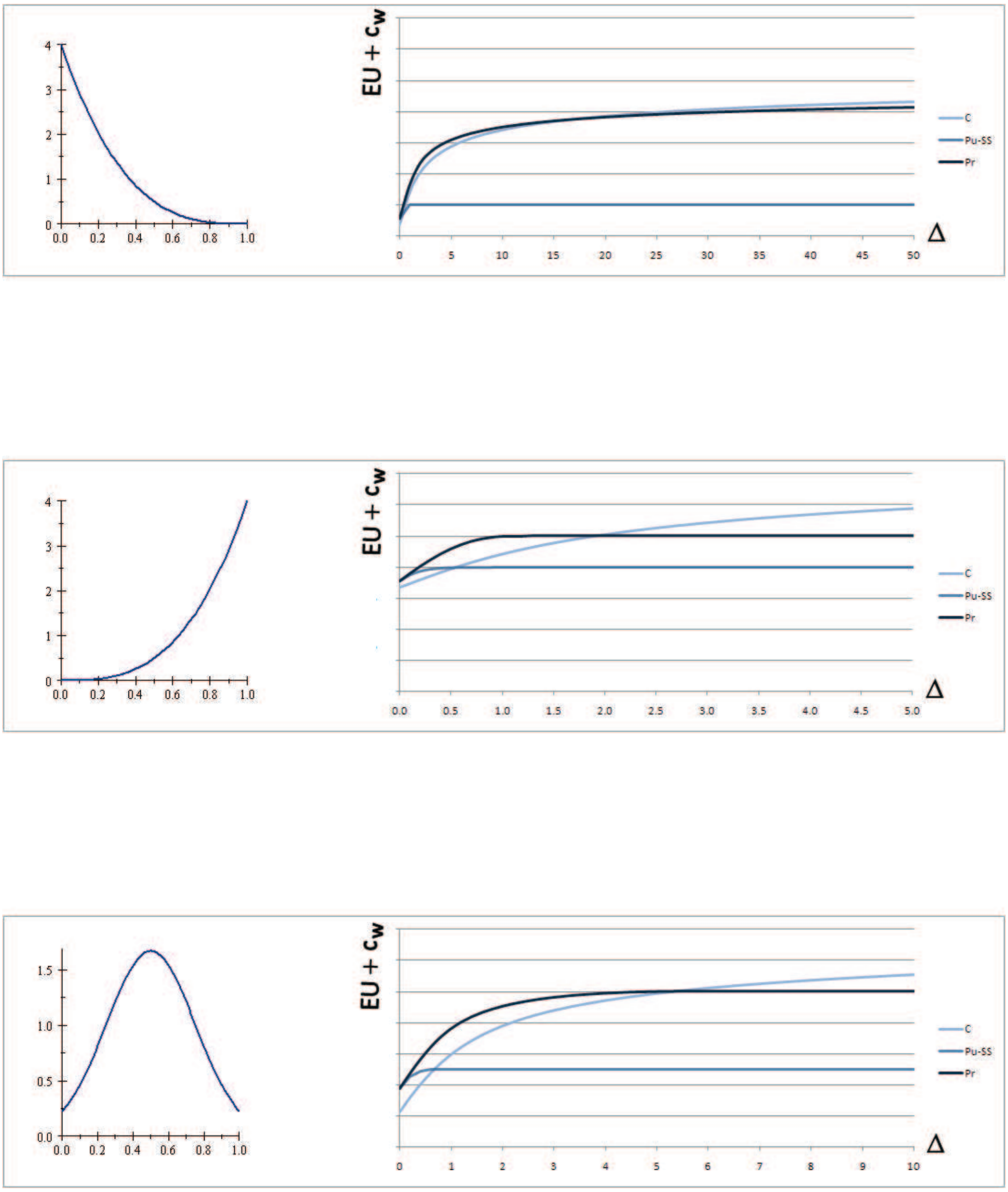

Figure 3: Top: $f(x) = 4(1-x)^3$ - Middle: $f(x) = 4x^3$ - Bottom: truncated normal ($\mu = 0.5,\ \sigma = 0.25$) in $[0, 1]$. Notice the different scales on the horizontal axis.