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COMMODITY PRICING VOLATILITY SHIFTS IN A HIGHLY TURBULENT TIME PERIOD. A TIME-VARYING TRANSITION PROBABILITY MARKOV SWITCHING ANALYSIS

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The pricing of six highly liquid futures commodity contracts is investigated using a Markov switching procedure. The data set spans an exceptionally turbulent time period, characterized by a complex interplay of economic/financial and political shocks. Markov switching analysis exploits time series nonlinearity in order to identify the nature and the timing of the implicit changes of regime. Building on a HAM framework, we use the time varying parameterization of the transition probability estimates in order to link these shifts to exogenous variables. We provide in this way additional information on the co-movement of the time series and on their eventual regime shifts. The WTI oil futures price and DJIA stock index turn out to be the main common drivers of the changes in regime of most futures commodity prices.*

Keywords: HAM Commodity pricing, Markov Switching, Time-Varying Transition Probabilities

JEL Codes G11, G12, G18, Q40

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The data set spans an exceptional time period, characterized by financial crises in the US (the subprime crisis and its Lehmann collapse appendix) and in the Euro area (sovereign debt and banking crises). Its turbulence is further compounded by the worldwide deflationary impact brought about by the COVID pandemic and the outbreak of the Russo-Ukrainian war. The effects on commodity markets were correspondingly huge as their smooth operational activity was seriously disrupted. Regime shifts are therefore to be expected, as crises impinge on the standard pricing mechanisms. The interconnection between commodity price rates of returns is investigated in a booming literature related to issues on portfolio management efficiency. The large hedging effectiveness and safe haven literatures are related to them.

In this paper, taking advantage of some improvements on Markov switching modelling, we analyse these phenomena from a different perspective. The shifts in the pricing impact of trend following and of fundamentalist speculation due to commodity market turmoil are analyzed at first.

Having identified crisis and non crisis periods, we focus on the impact of predetermined variables on the probabilities of shifting from one regime to the other. The significance and the sign of the latter are liable to provide some novel insights on the interaction between the commodity markets investigated below.

The empirical analysis identifies two major drivers of the time-varying transition probabilities, the rate of change of the WTI contract 1 futures price and the rate of change of the Dow Jones Industrial Average (DJIA) index.[1] WTI oil and US stock market prices do maintain a central role for economic policy making, in spite of the recent unprecedented events that have affected the World economy. Their shifts are, at times, exogenously determined. WTI pricing is strongly influenced by supply shifts due to

[1] The Nasdaq index has a smaller but qualitatively analogous impact.

political considerations.[2] Dow Jones stock market behaviour, in turn, is highly responsive to the monetary policy stance of the US Federal Reserve and to its interaction with US macroeconomic forecasts.

One of the main properties of the Markov switching procedure, which justifies its use in the investigation of business cycles, is the endogenous dating of the regime shifts. Its commodity markets implementation allows to assess the timing of the shifts of the commodity futures prices rates of change. The implications of this analysis for portfolio management are relevant, since an appropriate monitoring of the shifts of the time series might shed light on the co-movement of major asset prices and explain phenomena of contagion.

The main contributions of the paper can be described as follows.

It provides a careful assessment of the Heterogeneous Agents Model (HAM) over periods of stress. Changes in the behaviour of chartists and fundamentaist speculators over the pricing cycles are clearly detected and are associated with changes in the sign and significance of the relevant heterogeneous agents model coefficients;

it provides an accurate assessment of the timing of the commodity price volatility regime shifts and of their interaction;

it introduces a simple two-step approach for the data driven selection of the variables affecting the time-varying transition probabilities, alternative to Bazzi et al. (2017);

it identifies, at first, with the help of a standard Principal Component Analysis, the rates of growth of the NYMEX WTI futures 1 oil contract price and of the Dow Jones Industrial Average index as main drivers of the co-movements between the one-step ahead

[2] On the oil supply and price stabilization policy of Saudi Arabia see Nakov and Nuño (2011). See Santabárbara (2017) for details on the November 2014 and December 2015 innovative OPEC oil supply policies. The Great Recession affecting the World economy, important technological innovations (shale oil in particular) and geopolitical turmoil (Middle-East conflicts, Saudi Arabia energy policy shifts) interact in the second half of the sample. A recent investigation of oil supply manipulation by producing countries and of the corresponding financial consequences is found in Omar et al. (2017).

regime 1 predicted probabilities of the HAM commodity pricing estimates, obtained with a constant transition probabilities procedure;

it measures subsequently the sensitivity of time-varying transition probabilities to these proxies of macroeconomic activity, providing a tentative explanation of the origin of regime shifts, variously analyzed in the literature;

it improves significantly the accuracy and the quality of fit of each commodity contract estimate with respect to the corredsponding constant transition probability version.

1. A Short Survey of the Literature

Frankel and Froot (1986) focused on the importance of the interaction between standard financial market operators, such as chartists and fundamentalist speculators, as a driver of an endogenous non-linear law of motion in foreign exchange rate dynamics. In the same vein, numerous analyses on commodity pricing, building on heterogeneous agent models (HAM) by Brock and Hommes (1997, 1998) and Westerhoff (2004), among many others, posited that agents reactions to differing information sets are reflectet in market prices, which are weighted averages of their heterogeneous reactions. Following Westerhoff and Reitz (2005) and Reitz and Westerhoff (2007), a model is developed in which two categories of agents interact: noise traders, and fundamentalist speculators. Noise traders react to past price changes only. They can either stabilize the market, behaving as contrarians, i.e. as negative feedback traders, or destabilize it as trend followers, i.e. as positive feedback traders.[3] Fundamentalist speculators, among whom we include institutional investors, respond to deviations of market returns from equilibrium. In this case, a destabilizing behavior would be due to lack of confidence in the mean-reverting nature of market prices.[4]

[3] We build here on Kao et al. (2016), who add contrarians to positive feedback traders and fundamentalists.
[4] For a comprehensive survey of this ample literature, see, among others, Hommes (2005).

In a path-breaking article, Hamilton (1989) developed a highly successful technique for analyzing the nonlinearity of macroeconomic time series over the business cycle, which was subsequently used to investigate the dynamics of multiple real and financial phenomena. The transition probabilities were initially assumed to be invariant. Following the studies of Filardo (1994) and Diebold et al. (1994) time-varying transition probability models (i.e. models where the parameters shift according to a finite-state Markov chain with time-varying transition probabilities) were extensively analyzed. As pointed out by Psaradakis and Sola (2017), the implementation of time-varying transition probabilities is to be found in investigations dealing with business cycles (Filardo and Gordon, 1998), exchange rates (Engel and Hakkio, 1996), interest rates (Bekaert and Harvey, 1995) and, more recently, financial crises (Alvarez Plata and Schrooten, 2006, and Brunetti et al., 2008), among others.

2. A Markov Switching Heterogeneous Agents Model of Commodity Futures Pricing

2.1 A simplified Heterogenous Agents Model

Prices are set in an order-driven market. Every period traders revise their long/short positions; price changes from t to $t+1$ are a function of their excess demands and can be parameterized by the following log-linear function

$$f_{t+1} = f_t + \alpha(D_t^C + D_t^F) + e_{t+1} \tag{1}$$

f_t is the logarithm of the futures price, α is a positive market reaction coefficient and D_t^C, D_t^F denote the demand of chartists (feedback traders) and fundamentalists. The residual e_{t+1} accounts for additional factors that may affect prices. The demand of feedback traders at time t is

$$D_t^C = a_1(f_t - f_{t-1}) \tag{2}$$

Coefficient a_1 is positive as feedback traders expect the existing price trend to persist in the subsequent time-period. They will buy the contract if Δf_t is positive and sell it if Δf_t is negative.

Alongside feedback traders, we posit the existence of professional (institutional) investors, labelled here fundamentalists, who exploit their commodity market expertise for portfolio diversification and/or informed speculation purposes. Their demand of futures contracts at time t is

$$D_t^F = a_2\left(\overline{\overline{f}}_t - f_t\right) \tag{3}$$

Fundamentalists react to deviations of the futures prices from their equilibrium value $\overline{\overline{f}}_t$, proxied here by an N period log-price moving average ($\overline{\overline{f}}_t = \frac{\sum_{d=1}^{N} f_{t-d}}{N-1}$).[5] The coefficient a_2 indicates how fundamentalists' beliefs about market prices affect their behaviour. A positive value of coefficient a_2 implies that the majority of fundamentalists believes that the price will revert to its equilibrium value. This will lead them to buy if $\overline{\overline{f}}_t > f_t$ and to sell in the opposite case. If the coefficient a_2 takes on a negative value, fundamentalists, disbelieving in the mean-reverting nature of the price, will sell if $\overline{\overline{f}}_t > f_t$ and buy in the

[5] It is assumed in this paper that N = 4, $\overline{\overline{f}}_t$ being a monthly (four weeks) log-futures price moving average.

opposite case.[6] In all cases, empirical findings suggest that fundamentalists enter or exit the market depending on their perception of market price misalignment.

Replacing D_t^C and D_t^F by their determinants in equation (1), we obtain the following reduced form

$$r_{ft+1} = \theta_1 r_{ft} + \theta_2(\overline{\overline{f_t}} - f_t) + e_{ft+1} \tag{4}$$

where $\theta_1 = \alpha a_1$, $\theta_2 = \alpha a_2$.

In our investigation we use, at first, a Markov-switching model with constant transition probabilities. Equation (3) is accordingly adapted to a two-state Markov switching framework, in which the drivers of futures returns are assumed to switch between two different processes determined by the state of the market. It reads as follows

$$r_{ft} = \theta_{0s_t} + \theta_{1s_t} r_{ft-1} + \theta_{2s_t}(\overline{\overline{f_{t-1}}} - f_{t-1}) + e_{rfs_tt} \tag{4'}$$

where $e_{rfs_tt} \equiv \epsilon\sigma_{s_t} \sim N(0, \sigma_{s_t}^2)$ and the unobserved random variable s_t indicates the state in which is the market.

According to the Markov hypothesis, the value of the current regime s_t is assumed to depend on the state of the previous period, s_{t-1}, and the transition probability $P\{s_t = j | s_{t-1} = i,\} = P_{ij}$ gives the probability that state *i* will be followed by state *j*.

In the two-state case $P_{11} + P_{12} = 1$ and $P_{22} + P_{21} = 1$, and the corresponding transition matrix reads as

$$\begin{bmatrix} P_{11} & 1 - P_{22} \\ 1 - P_{11} & P_{22} \end{bmatrix} \tag{5}$$

[6] Fundamentalists, wary of the mean-reverting nature of futures prices, believe that the present prices of futures contracts will last for some time and persist in their long/short trades. This is a symptom of the failure of the price signaling process during periods of turbulence and is consistent with fundamentalists destabilizing the market, their traditional stabilizing behaviour being associated with a positive value of a_2 (see Shleifer and Vishny, 1997 and Chia et al., 2014, among others, for further details).

The parameters of equation (4′) and the transition probabilities parameters of matrix (5) are jointly estimated.

The joint probability of r_{ft} and s_t is given by the product

$$P\big(r_{ft}, s_t = j|Y_{t-1}, \omega\big) = h\big(r_{ft}|s_t = j; Y_{t-1}, \omega\big). P(s_t = j|Y_{t-1}, \omega) \qquad j = 1, 2 \qquad (6)$$

where Y_{t-1} is the information set that includes all past information on the population parameters and $\omega = (\theta_{0s_t}, \theta_{1s_t}, \theta_{2s_t}, \log(\sigma^2_{s_t}))$ is the vector of parameters to be estimated, h is the density of r_{ft}, conditional on the random variable s_t and $P(.)$ is the conditional probability that s_t will take the value j.

2.2 The Time-Varying Transition Probabilities Markov Switching Estimation Procedure

The Markov switching models of the previous paragraph have constant transition probabilities. Following an approach originally set forth by Diebold et al. (1994), and Filardo (1994), the present model allows for time-varying logistic parameterization probabilities. It follows that $P\{s_t = j|s_{t-1} = i,\, Q_{t-1}, \varphi\} = P_{ij}(Q_{t-1}, \varphi)$ gives the probability that state *i* shall be followed by state j, where $Q_t = (1, q_{1t}, \dots, q_{n-1t})'$ is the $(n\, x\, 1)$ vector of exogenous observable variables that may affect the transition probabilities and φ is the $(n\, x\, 1)$ vector of coefficients obtained from a standard multinomial logit specification

$$P(s_t = j|s_{t-1} = i, Q_{t-1}, \varphi) = \frac{\exp(Q'_{t-1}\varphi_j)}{1+\exp(Q'_{t-1}\varphi_j)} = P_s(Q_{t-1}, \varphi) \quad s = i, j, \;\; i \neq j \qquad (7)$$

[7]

In the two-state case $P_{11}(Q'_{t-1}\,\varphi_{11}) + P_{12}(Q'_{t-1}\,\varphi_{12}) = 1$ and $P_{22}(Q'_{t-1}\,\varphi_{22}) + P_{21}(Q'_{t-1}\,\varphi_{21}) = 1$, and the transition matrix is adjusted accordingly.

[7] As pointed out by Filardo (1994, page 302), the logistic functional form for the transition probabilities maps the explanatory variables into the interval (0,1) guaranteeing in this way a well defined log-likelihood function.

It reads as $$\begin{bmatrix} P_{11}(Q'_{t-1}\,\varphi_{11}) & 1-P_{22}(Q'_{t-1}\,\varphi_{22}) \\ 1-P_{11}(Q'_{t-1}\,\varphi_{11}) & P_{22}(Q'_{t-1}\,\varphi_{22}) \end{bmatrix} \quad (8)$$

The full log-likelihood is a normal mixture

$$l(\omega,\varphi) = \sum_{t=1}^{T} log\left[\sum_{s=1}^{2}\frac{1}{\sigma_s}h\left(\frac{e_{rfs_t t}}{\sigma_s}\right).P_s(Q_{t-1},\varphi)\right] \quad (9)$$

We assume, following Filardo (1994), that the exogenous variables, that are likely to impinge on the transition probabilities and enter the information vector Q_t, do not differ from commodity to commodity. Their selection, however, is necessarily arbitrary and cumbersome, as stressed by Bazzi et al. (2017). An innovative approach is followed here, based on the Principal Components Analysis of the robust correlation matrix of the low volatility one-step ahead predicted probabilities produced by the Markov regime switching estimation procedure. This technique determines the pattern of the co-movement of the elements of a data set with minimal loss of information. The data are projected onto fewer dimensions, so that their variability (i.e. their information measure) is retained in the smaller number of dimensions. In this way, a vector U_t of z correlated variables is transformed in a smaller vector V_t of w ≤ z uncorrelated ones. We can thus select the common variables that impact on the transition probabilities of a set of commodity futures time series from the principal components of the correlation matrix of the one-step ahead predicted regime probabilities of a preliminary constant transition probabilities version of the estimates.

3. Markov Switching Model Estimation of the Heterogeneous Agents Pricing Model

3.1 Preliminary Statistical Analysis

Our weekly data span the 28 April 2004 – 17 August 2022 time period, with the exception of the Brent futures oil price, which terminates on 7 August 2020. Futures prices correspond to the highly liquid 1 month (nearest to delivery) futures contract.[8] Returns are computed as first differences of the logarithms of the price levels. The model is tested for six commodities belonging to different commodity sectors: cotton (industrial materials), copper (industrial metals), crude oil, natural gas (energy), gold (precious metals), and corn (grains). All the futures contracts are taken from Bloomberg and are expressed in US dollars. The gold futures price is the COMEX Gold Composite Commodity Future Continuation 1 price. The copper futures price is the COMEX Copper Composite Commodity Future Continuation 3 price. The corn futures price is the CBoT Corn Composite Commodity Future price. The cotton price is the ICE-US Cotton No. 2 Futures Electronic Commodity Future Continuation 2 price. The Brent futures price is the ICE Europe Brent Crude Electronic Energy Future price. The natural gas futures price is the NYMEX Henry Hub Natural Gas Electronic Energy Future Continuation 1 price. The

Table 1. Preliminary Statistical Analysis
Weekly Futures Rates of Change

02/01/2004 – 17/07/2022

	Copper	Cotton	Corn	Gold	Brent	Nat.Gas
Mean	0.001	0.000	0.001	0.001	0.000	0.000
St.Dev.	0.030	0.031	0.034	0.020	0.043	0.056
Sk..	-0.494	-0.162	-0.363	-0.702	-0.607	0.115
Kurt.	5.707	4.743	5.238	6.334	9.950	4.214
BDS(2)	7.646 [0.00]	5.131 [0.00]	4.506 [0.00]	4.113 [0.00]	7.426 [0.00]	6.220 [0.00]
AR(1)	53.849 [0.00]	73.107 [0.00]	64.465 [0.00]	36.352 [0.00]	57.062 [0.00]	36.253 [0.00]
AR(2)	63.745 [0.00]	75.115 [0.00]	66.175 [0.00]	37.960 [0.00]	57.114 [0.00]	36.605 [0.00]
ARCH(1)	67.656 [0.00]	33.127 [0.00]	56.162 [0.00]	21.366 [0.00]	202.150 [0.00]	20.123 [0.00]
ARCH(2)	161.69 [0.00]	87.785 [0.00]	60.326 [0.00]	26.945 [0.00]	311.26 [0.00]	41.560 [0.00]
ADF(n, c)	-24.07 [0.00] (0, no c)	-23.00 [0.00] (0, no c)	-23.40 [0.00] (0, no c)	-25.15 [0.00] (0, no c)	-22.27 [0.00] (0, no c)	-25.22 [0.00] (0, no c)
JB	328.851 [0.00]	124.489 [0.00]	219.180 [0.00]	441.399 [0.00]	1752.716 [0.00]	60.527 [0.00]

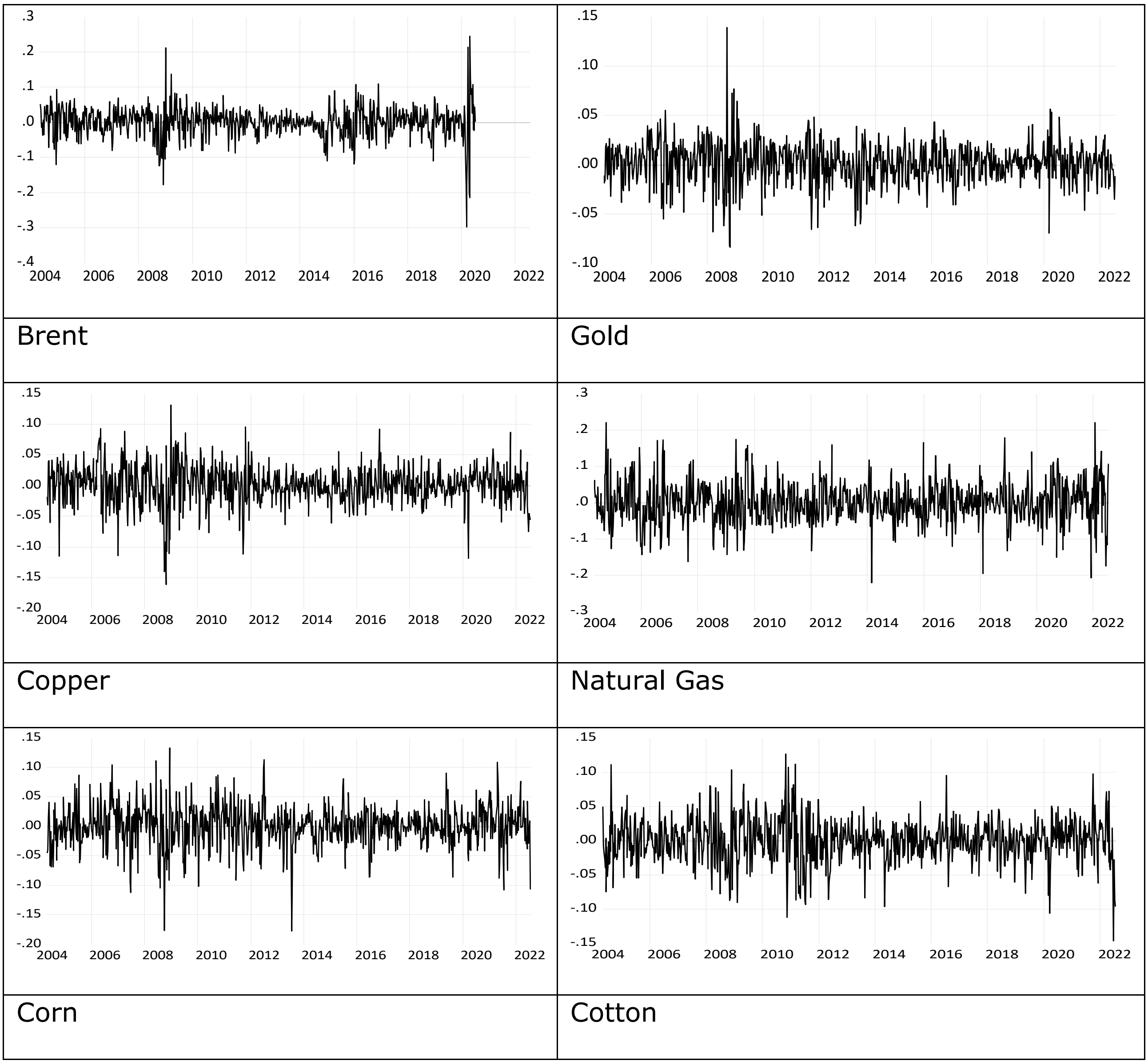
Notes. Sk.: skewness; Kurt.: kurtosis; Probability values in square brackets; AR(k): Ljung-Box test statistic for k-th order serial correlation of the time series; ARCH(k): Ljung-Box test statistic for k-th order serial correlation of the squared time series; ADF(n, c): Augmented Dickey Fuller unit root test statistic, with a constant term and n^{th} order autoregressive component; BDS(k): test statistic, with embedding dimension k, of the null that the time series, filtered for a first order autoregressive structure, is independently and identically distributed; JB: Jarque Bera test statistic for normality of the data distribution.

[8] The futures contract expires on the 3rd business day prior to the 25th calendar day of the month preceding the delivery month. If the 25th calendar day of the month is a nonbusiness day, trading ceases on the third business day prior to the business day preceding the 25th calendar day.

WTI futures price is the EIA Cushing, OK Crude Oil Future Contract 1 price. The VIX/VXO index is provided by the Cboe. Summary statistics are presented in Table 1.

As expected the log price first differences are serially correlated and conditionally heteroskedastic, volatility clustering being extremely large between 2008 and 2009 and again at the end of the sample period. The distributions of the returns are always significantly skewed and leptokurtic, the departure from normality being confirmed by

Figure 1. Commodity Futures Prices Weekly Rates of Change 02/01/2004 – 17/07/2022



the size of the corresponding Jarque Bera (JB) test statistics. The presence of nonlinearities is detected by the significance of the BDS test statistics of Brock et al. 1987).[9]

3.2 Constant Transition Probabilites Estimation

In Table 2 are set forth the Markov switching estimates of equation (4’), obtained with the constant transition probabilities procedure of equations (5) and (6).

Table 2. Markov Switching Constant Transition Probabilities Estimates

02/01/2004 – 17/07/2022

$$r_{ft} = \theta_{0s_t} + \theta_{1s_t} r_{ft-1} + \theta_{2s_t}(\overline{\overline{f_{t-1}}} - f_{t-1}) + e_{rfs_t t} \qquad (4') \qquad \begin{bmatrix} P_{11} & 1-P_{22} \\ 1-P_{11} & P_{22} \end{bmatrix} \qquad (5)$$

	Copper		Cotton		Corn		Gold		Brent		Nat.Gas	
	L	H	L	H	L	H	L	H	L	H	L	H
s_t	1	2	1	2	1	2	1	2	1	2	1	2
$P_{s_t,\, not\, s_t}$	0.02	0.06	0.02	0.05	0.05	0.09	0.01	0.03	0.01	0.07	0.05	0.07
Exp-Dur	45.29	15.42	50.03	20.95	19.34	10.56	116.7	28.68	118.10	13.91	18.56	13.42
θ_{0s_t}	0.00 (81.76)	-0.00 (-0.24)	0.00 (1.86)	-0.00 (-0.96)	0.00 (0.51)	0.00 (0.40)	0.00 (2.21)	0.00 (0.62)	0.00 (1.56)	-0.02 (-1.65)	-0.00 (-0.76)	0.00 (0.65)
θ_{1s_t}	0.30 (4.11)	0.04 (0.29)	0.37 (5.32)	0.26 (2.23)	0.18 (1.12)	0.28 (3.75)	0.33 (5.00)	0.24 (1.93)	0.28 (2.92)	0.61 (2.15)	0.26 (4.34)	0.22 (3.27)
θ_{2s_t}	0.13 (1.92)	-0.25 (-2.11)	0.14 (2.16)	-0.05 (-0.45)	-0.05 (-0.35)	0.19 (0.26)	0.11 (1.86)	0.09 (0.76)	0.05 (0.44)	0.51 (1.42)	0.01 (0.38)	0.05 (1.08)
$log\sigma_t^2$	-3.84 (-90.6)	-3.09 (-42.5)	-3.84 (-91.9)	-3.11 (-54.3)	-4.18 (-42.4)	-3.23 (-84.1)	-4.19 (-14.0)	-3.34 (-55.5)	-3.47 (-97.0)	-2.42 (-21.1)	-3.35 (-86.9)	-2.60 (-49.5)
Function value	2071.422		2055.197		1957.140		2463.277		1614.741		1472.438	
$LR_{\sigma_{1t}^2=\sigma_{2t}^2}$	65.372 [0.00]		93.474 [0.00]		90.240 [0.00]		124.054 [0.00]		109.214 [0.00]		92.07 [0.00]	
AR(1)	0.56 [0.45]		0.41 [0.52[		0.02 [0.89]		0.05 [0.82]		0.71 [0.40]		0.00 [0.98]	
AR(2)	1.88 [0.39]		0.64 [0.73]		1.00 [0.61]		0.43 [0.80]		0.81 [0.66]		0.39 [0.82]	
ARCH(1)	1.80 [0.18]		0.16 [0.69[		0.60 [0.44]		0.75 [0.39]		0.91 [0.34]		1.40 [0.24]	
ARCH(2)	1.80 [0.41]		0.16 [0.92]		5.02 [0.08]		0.75 [0.69]		9.80^{**} [0.01]		1.56 [0.46]	

Note. $LR_{\sigma_{1t}^2=\sigma_{2t}^2}$: Likelihood Ratio test of the null hypothesis that $\sigma_{1t}^2 = \sigma_{2t}^2$; *Exp-Dur*: Expected Duration.

The quality of fit is satisfactorty, serial correlation and conditional heteroskedasticity are accounted for by the Markov switching parameterization.[10] The hypothesis of equal

[9] Analogous results are obtained for unfiltered returns, with embedding dimensions varying from 2 to 6.
[10] It should be noticed that closed forms of the limit distributions of the AR and ARCH tests are not yet available, which reduces their reliability.

residual variances over the regimes is strongly rejected by LR tests performed for each commodity model. Low and high volatility regimes of a different expected duration are clearly detected, regimes characterized by changes in the behaviour of chartists and fundamentalists.

The probability of switching from a low variance to a high variance state P_{12} is always lower than the probability of switching from a high variance to a low variance state P_{21} . In the same way the average expected duration of being in state 1 is systematically larger than the high volatility one. The number of weeks of high volatility is on the whole rather small.[11]

The equation (4′) estimates differ significantly across regimes. In the low volatility regime, in the case of copper, cotton and gold, chartists and fundamentalist have significant positive coefficients. This certifies that the former tend to destablize and the latter to stabilize the corresponding futures prices.

Regime shifts seem to affect fundamentalists more than chartists. Indeed chartists do not seem to be strongly affected by regime shifts. In both regimes they do exert a positive destabilizing pressure on futures prices rates of return, an effect which is somewhat smaller in absolute value in the high volatility regime. Fundamentalists' behaviour is more sensitive to regime shifts: destabilizing in regime 2 in the case of copper, it does not seem to affect the pricing of the remaining contracts.[12]

3.3 A Two-Step Time-Varying Transition Probabilities Markov Switching Estimation

In the standard Hamilton Markov switching model the state transition probabilities are assumed to be constant, a restriction which might distort the results. In order to assess the relevance of

[11] For instance, in the case of copper, the transition probability from a low to a high volatility regime, P_{12}, is 2 percent and the transition probability from a high to a low regime, P_{21}, is 6 percent. A low volatility duration of 45.29 weeks is associated with a high volatility one of 15.42 weeks.

[12] In the case of corn, Brent and natural gas futures, chartists only affect the corresponding price rates of growth in both regimes, the impact of fundamentalists being absent.

this problem we have re-estimated the model positing that the transition probabilities be affected by a series of variables, following the approach of Filardo (1994) and Diebold et al. (1994), among others.

As pointed out above, the matrix of transition probabilities will then be rewitten as

$$\begin{bmatrix} P_{11}(Q'_{t-1}\,\varphi_{11}) & P_{21}(Q'_{t-1}\,\varphi_{21}) \\ P_{12}(Q'_{t-1}\,\varphi_{12}) & P_{22}(Q'_{t-1}\,\varphi_{22}) \end{bmatrix} \tag{8'}$$

where it is assumed that $P_{12}(Q'_{t-1}\,\varphi_{12}) = 1 - P_{11}(Q'_{t-1}\,\varphi_{11})$ and $P_{21}(Q'_{t-1}\,\varphi_{21}) = 1 - P_{22}(Q'_{t-1}\,\varphi_{22})$. $Q_t = (1, q_{1t}, \dots, q_{n-1t})'$ is a $(n\,x\,1)$ vector of exogenous observable variables that may affect the transition probabilities and φ is a $(n\,x\,1)$ vector of coefficients obtained from the standard multinomial logit specification set out in equation (7) above. The selection of the variables that are likely to affect the transition probabilities is justified by a Principal Components Analysis of the Spearman robust correlation matrix between the (low volatility regime) one-step ahead predicted probabilities of the constant transition probabilities Markov switching estimates of Table 2.[13]

The PCA estimates of Table 3 suggest that two major factors affect the predicted probabilities, as the first and second principal components explain, respectively 36 and 20 percent of their correlation variability. The signs of the loadings are arbitrary (Joliffe, 1986, p. 54), they can be used, however, to provide a tentative financial/economic interpretation of the principal components. In the case of the first component the signs of the loadings are all positive and might quantify the impact of a macroeconomic indicator, such as the rate of change of the Dow Jones Industrial Average index, the

[13] A short overview of the standard Principal Components Analysis is set forth in the appendix.

Table 3. Principal Components Analysis of the Spearman Rank-Order Correlations Between the Low Volatility Regime One-Step Ahead Predicted Probabilities Estimates of Table 2.
02/01/2004 – 17/07/2022

Eigenvalues			
Number	Values	Proportion	Cumulative Proportion
1	2.19	0.36	0.36
2	1.20	0.20	0.56
3	0.87	0.14	0.71
4	0.66	0.11	0.82
5	0.56	0.09	0.91
6	0.52	0.08	1.00

Eigenvectors-Loadings						
	PC1	PC2	PC3	PC4	PC5	PC6
Copper	0.52	0.08	-0.23	-0.20	-0.29	-0.74
Cotton	0.42	-0.40	0.35	0.27	-0.60	0.31
Corn	0.44	-0.41	0.11	0.29	0.72	-0.13
Gold	0.44	0.04	-0.63	-0.28	0.07	0.56
Brent	0.33	0.42	0.64	-0.51	0.17	0.15
Nat. Gas	0.22	0.70	-0.04	0.68	0.00	0.06

shifts of which are likely to exert a homogeneous effect on commodity pricing.[14] The loadings of the second principal component, which we associate with the rate of change of the WTI futures contract 1 price, have differing signs. Large and positive in the case of natural gas and Brent, they are either small or negative in the case of the remaining contracts. The increase in Brent and natural gas prices can be given a straightforward economic explanation and reflects the co-movement of energy sector prices. At the same time oil price increases may raise production costs and depress the markets of the remaining commodity contracts.

These findings are not unduly surprising, given the pivotal role played by oil in the World economy. Its price impacts on both the level of aggregate activity and the corresponding

[14] Alternative variables were used in tentative Markov switching time-varying transition probabilities analyses, such as the rate of change of the Nasdaq and of the FTSE stock indexes, of the euro/USD exchange rate or of the Bloomberg commodity prices index, with little success. It should finally be noticed that qualitatively similar, if less satisfactory, results are obtained by applying the PCA analysis to the correlation matrix of the corresponding commodity futures prices rates of return.

inflationary dynamics, and influences the monetary decisions of central banks.[15] As for the DJIA rate of return, it provides a leading indicator of financial markets expectations on the behaviour of the US economy, with a crucial impact on international finance and portfolio management.

In Table 4 are set forth the Markov switching estimates of equation (4'), obtained with the time-varying transition probabilities procedure of equations (7), (8) and (9).

Here too the quality of fit is satisfactory and the results are qualitatively similar to those obtained with the constant transition probabilities estimates of Table 2. Likelihood Ratio tests for the time-varying specification of the transition probabilities ($LR_{CTP/TVTP}$) reject the constant transition probabilities specification, with the exception of the cotton futures contract.[16] In the same way, the null that the one-step ahead predicted regime probabilities obtained, respectively, with the constant and time-varying transition probabilities approaches are equal is always almost universally rejected (see the PRB statistics of row 8 of Table 4).

It should be noticed that, since $P_{i1}(Q'_{t-1}\varphi) = 1 - P_{i2}(Q'_{t-1}\varphi),\ i = 1, 2$, the factors that bring about an increase (decrease) in $P_{i1}(Q'_{t-1}\varphi)$, will bring about a decrease (increase) in $P_{i2}(Q'_{t-1}\varphi)$. The impact of the variables in Q_{t-1} is provided by the coefficients $\varphi_{ijk}, i, j = 1, 2$, $k = WTI, DJIA$.

The economic effect of an increase in WTI prices is far from homogeneous, being expansionary for the energy sector only. Oil price hikes do bring about a deterioration of the terms of trade of oil importing countries along with domestic inflationary pressures and an overall contractionary effect, with specular effects on oil exporting

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[15] The benchmark oil price used in our study is the West Texas Intermediate (WTI) spot price for delivery in Cushing, Oklahoma. It is prized for its low sulphur content and high API gravity (For more details see Stevens, 2005).

[16] If the Dow-Jones Index is replaced by the Nasdaq, however, the test fails to reject the time-varying regime specification also in the case of cotton futures contracts.

countries. Portfolio management of financial agents is affected too, with subsequent turmoil inducing shifts in the prices of assets with which oil futures are correlated.

The empirical findings do corroborate these hypotheses. φ_{11WTI} is negative in the energy consuming copper industry (decreasing the probability of shifting from a low to a low

Table 4. Markov Switching Time-Varying Transition Probabilities Estimates 02/01/2004 – 17/07/2022

$$r_{ft} = \theta_{0s_t} + \theta_{1s_t} r_{ft-1} + \theta_{2s_t}\left(\overline{\overline{f_{t-1}}} - f_{t-1}\right) + e_{rfs_tt} \qquad (4')$$

$$\begin{bmatrix} P_{11}(Q'_{t-1}\,\varphi_{11}) & P_{21}(Q'_{t-1}\,\varphi_{21}) \\ P_{12}(Q'_{t-1}\,\varphi_{12}) & P_{22}(Q'_{t-1}\,\varphi_{22}) \end{bmatrix} \qquad (8'')$$

	Copper		**Cotton**		**Corn**		**Gold**		**Brent**		**Nat.Gas**	
	L	**H**	**L**	**H**	**L**	**H**	**L**	**H**	**L**	**H**	**L**	**H**
s_t	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**
$P_{s_t,not\ s_t}$	0.04	0.08	0.03	0.05	0.04	0.07	0.01	0.03	0.03	0.11	0.05	0.07
θ_{0s_t}	0.00 (1.44)	0.00 (0.02)	0.00 (1.58)	-0.00 (-0.79)	0.00 (0.63)	0.00 (0.25)	0.00 (2.44)	0.00 (0.34)	0.00 (1.62)	-0.03 (-2.3)	-0.00 (-0.30)	0.00 (0.32)
θ_{1s_t}	0.33 (5.05)	-0.02 (-0.14)	0.37 (5.30)	0.26 (2.27)	0.08 (0.79)	0.32 (3.78)	0.34 (5.27)	0.23 (1.73)	0.20 (2.11)	0.88 (2.82)	0.35 (4.45)	0.14 (1.42)
θ_{2s_t}	0.12 (1.98)	-0.29 (-2.24)	0.14 (2.75)	-0.04 (-0.42)	-0.12 (-1.24)	0.04 (0.57)	0.12 (1.98)	0.09 (0.68)	-0.06 (-0.55)	0.99 (2.45)	0.09 (1.32)	-0.03 (-0.32)
$log\sigma_t^2$	-3.84 (-110.5)	-3.07 (-48.3)	-3.84 (-94.3)	-3.11 (-55.9)	-4.01 (-46.9)	-3.18 (-70.5)	-4.18 (-14.2)	-3.42 (-53.7)	-3.48 (-112.5)	-2.47 (-26.6)	-3.32 (-79.7)	-2.59 (-54.5)
PRB	0.73 [[0.46]	-5.02 [0.00]	-2.40 [0.02]	-5.03 [0.00]	22.41 [0.00]	25.60 [0.00]	4.10 [0.00]	-4.10 [0.00]	127.05 [0.00]	0.19 [0.85]	5.03 [0.00]	-5.03 [0.00]
φ_{11c}	5.20 (6.40)		3.23 (5.90)		2.96 (5.21)		5.69 (7.41)		6.44 (4.53)		5.27 (5.02)	
φ_{11WTI}	-35.92 (-3.20)		-15.35 (-1.13)		-26.39 (-1.75)		-1.40 (-0.10)		63.97 (2.93)		56.07 (3.72)	
φ_{11DJIA}	115.37 (3.99)		8.06 (0.40)		60.17 (2.66)		72.29 (2.84)		-34.95 (-1.20)		-126.08 (-2.93)	
φ_{21c}	-2.63 (-4.91)		-4.23 (--6.80)		-2.94 (-5.87)		-3.60 (-5.83)		-2.29 (-3.37)		-2.89 (-6.74)	
φ_{21WTI}	10.64 (1.28)		17.52 (0.93)		21.22 (2.61)		14.58 (2.12)		13.84 (1.19)		-17.41 (-2.31)	
φ_{21DJIA}	24.05 (1.69)		-58.9 (-1.99)		-22.19 (-1.20)		-5.05 (-0.14)		7.28 (0.57)		25.77 (1.56)	
Function value	2082.121		2058.289		1961.564		2469.298		1620.452		1479.119	
$LR_{CTP/TVTP}$	21.398 [0.00]		6.184 [0.18]		8.848 [0.06]		12.064 [0.01]		11.422 [0.02]		14.566 [0.00]	
AR(1)	0.00 [0.97]		0.18 [0.67]		0.29 [0.59]		0.29 [0.59]		0.01 [0.94]		0.01 [0.92]	
AR(2)	1.66 [0.43]		0.54 [0.76]		1.23 [0.54]		0.88 [0.64]		0.05 [0.98]		2.16 [0.34]	
ARCH(1)	0.32 [0.57]		0.22 [0.64]		0.55 [0.46]		0.00 [0.98]		1.53 [0.21]		1.58 [0.21]	
ARCH(2)	0.96 [0.62]		0.30 [0.86]		3.64 [0.16]		0.33 [0.85]		6.71 [0.03]		2.16 [0.34]	

Notes. *PRB*: t test of the null Ho: $\mu_{TVTP} = \mu_{CTP}$, where μ_{TVTP} and μ_{CTP} are the mean values of the one-step ahead predicted regime probabilities obtained, respectively with the time-varying and constant transition probabiliies approaches; .$LR_{CTP/TVTP}$: *LR* test of the null hypothesis that $\varphi_{11DJIA} = \varphi_{11WTI} = \varphi_{21DJIA} = \varphi_{21WTI} = 0$, distributed as a chi-square with 4 degrees of freedom.

volatility regime, i.e. increasing the probability of moving from a low to a high volatility regime). Conversely, φ_{11WTI} is positive and highly significant in the case of Brent oil and natural gas, increasing the probability of shifting from a low to a low volatility regime, (i.e. decreasing the probability of shifting from a low to a high volatility regime), an effect which can be attributed to a generalized energy saving impact of higher oil prices. The φ_{21WTI} coefficients are significant and positive in the case of corn and gold, negative in the natural gas estimates. An increase in the WTI futures prices rate of growth increases the probability of shifting from a high to a low volatility regime, which confirms its overall dampening effect on the prices of the contracts of this paper. The negative sign of φ_{21WTI} in the case of natural gas, i.e. a decrease (an increase) in the transition probability of shifting from a high to a low volatility regime whenever the rate of change of WTI prices rises (declines), is probably due to financial contagion between these two markets.

Shifts in the rate of change of the Dow Jones Industrial Average price index reflect short term market expectations: a positive (negative) shift in the stock price index rate of change will be associated with a decrease (increase) in the probability of shifting from a low to a high volatility regime in the case of three contracts, since the coefficient of φ_{11DJIA} is positive (and thus φ_{12DJIA} is negative), with the exception of natural gas, where this transition probability is likely to rise, the coefficient of φ_{11DJIA} being negative. Gas futures contracts would play in this case the role of a safe haven (hedging) asset. In the case of cotton and Brent the transition probability is not affected by stock index prices.

The estimates of the φ_{21DJIA} coefficients are less informative, being significantly different from zero in two cases only. The probability shift from a high to a low volatility regime is positively and negatively correlated with the stock index price rate of change in the copper and cotton markets respectively, corresponding to contagious and hedging pricing opportunities.

In Table 5 are reported, for each commodity, the correlation coefficients between the low volatility (regime 1) one-step ahead predicted regime probability and the standard deviation and lagged rate of return of the corresponding futures contract. As expected, we find a large negative and significant correlation coefficient between the low volatility probabilities and the weekly standard deviations. We detect, moreover, a significant positive correlation of the regime probabilities with one-period lagged futures price rates

Table 5. Correlation Between Low Volatility (Regime 1) One-Step Ahead Predicted Regime Probabilities and Weekly Futures Returns and Standard Deviations 02/01/2004 –17/07/2022

	Copper	Cotton	Corn	Gold	Brent	Nat.Gas
Constant Transition Probability Estimation						
r_{ft-1}	0.02 (0.67)	0.15 (4.61)	0.01 (0.22)	0.08 (2.39)	0.12 (3.38)	-0.06 (-1.80)
$\sqrt{\sigma_t^2}$	-0.67 (-27.63)	-0.64 (-25.79)	-0.64 (-25.92)	-0.68 (-28.44)	-0.70 (-21.31)	-0.71 (-30.99)
Time-Varying Transition Probability Estimation						
r_{ft-1}	0.09 (2.87)	0.15 (4.56)	0.01 (0.38)	0.07 (2.30)	0.38 (11.78)	-0.04 (-1.37)
$\sqrt{\sigma_t^2}$	-0.61 (-23.61)	-0.64 (-25.95)	-0.66 (-27.34)	-0.67 (-28.02)	-0.64 (-24.40)	-0.66 (-26.99)

of return of cotton, gold and Brent in the case of the constant transition probabilities estimation procedure, and with the futures returns of most contracts, with the exception of corn and natural gas, in the case of the time-varying transition probabilities estimation. A possible interpretation would be that of associating regime 1 with a bullish market (positive futures returns and low price volatility). The positive correlation of the predicted regime 1 probabilities with the one-period lagged futures prices rates of change might reflect the pressure of trend followers' speculation.

3.4 Interpretation of the Regime Probabilities

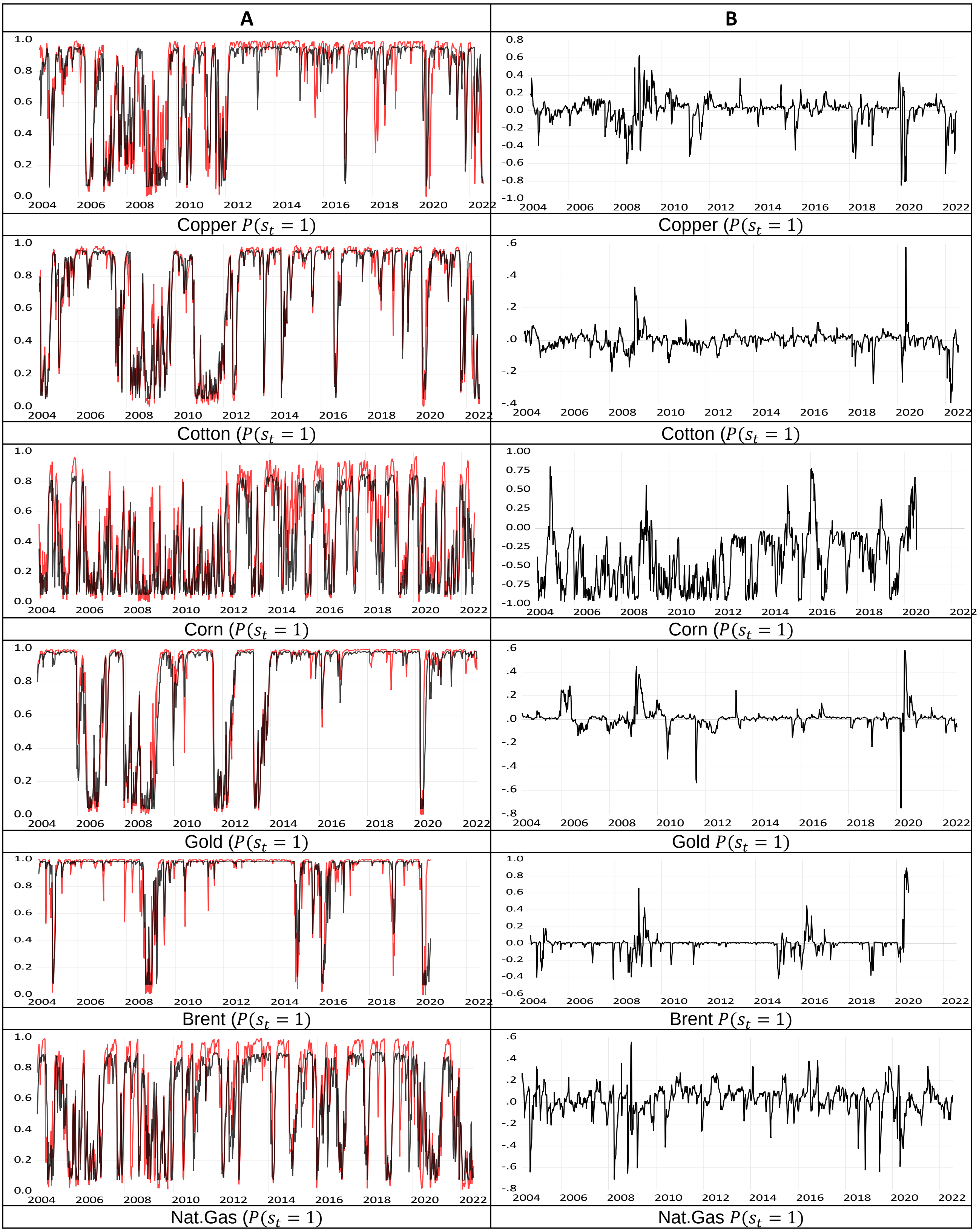
A visual inspection of the regime 1 one-step ahead predicted probabilities of Figure 2, panel A, identifies major downward shifts in the 2008-2009, 2011 and 2020 time periods. The first two are related to the the US financial run on commodities, which followed the subprime crisis, and to the consequences of the Lehman collapse credit freeze. The third dip is probably due to the contractionary effects European banking and public debt crises. The last downward sihift is related to the COVID pandemic and its deflationary impact. In the remaining time periods we find preciously little simultaneity between the dynamics of the various futures contracts regime probabilities. There seems to be little scope for treating commodities as a homogeneous financial asset. Commodities have to be traded in isolation and not as a group.

As for panel B, the divergence between the constant and time-varying transition probabilities estimates of the one-step ahead predicted volatility regimes is due to the improved sensitivity to news of the latter and to the corresponding greater reaction of its one-step ahead volatility regime predictions. An expansionary shock should result in a short term increase in the one-step ahead regime 1 (low) probability obtained with the time-varying transition probabilities with respect to the corresponding probability obtained with the constant transition probabilities approach. The differential should then increase (be positive).

A contractionary shock should be associated with an decrease in the one-step ahead (low) regime 1 probability and a corresponding decrease (negative value) of the differential between the time-varying and constant regime probabilities. A close analysis of the graphs of Figure 2, panel B, does not seem to contradict these conclusions.

Figure 2. Markov Switching One-Step Ahead Predicted Regime 1 Probabilities Obtained with Time-Varying and Constant Transition Probabilities Estimates. 02/01/2004 – 17/07/2022

Levels (Panel A) **Differences (Panel B)**



Note. Predicted regime 1 probabilities, corresponding to constant (black) and time-varying (red) transition probabilities

4. Conclusion

Markov switching analysis exploits the nonlinearities of the time series to identify the nature and the timing of implicit regime shifts. Our research, building on a HAM framework, uses the time-varying parameterization of the transition probabilities estimates in order to link these shifts to exogenous variables, which are selected with a two-step procedure.

The rates of change of the price of the WTI oil futures contract 1 and of the DJIA stock index turn out to be the main drivers of the shifts. Their inclusion in the estimation improves the overall explanatory power of the estimation procedure. These variables provide plausible additional information on the dynamics of the six major commodity contracts, which can be used for portfolio management purposes. The DJIA index corroborates the paramount impact of the US business cycle, whereas the relevance of WTI oil futures prices points to the growing impact of geo-political factors upon commodity pricing.

Bibliography

Alvarez-Plata, P, Schrooten, M., (2006). The Argentinean Currency Crisis: A Markov-Switching Model Estimation, The Developing Economies, 44, p. 79-91.

Bazzi, M., Blasques, F., Koopman, S.J, Lucas, A., (2017). Time Varying Transition Probabilities for Markov Regime Switching Models, Journal of Time Series Analysis, 38, p. 458-478.

Bekaert, G., Harvey, C.R., (1995), Time-Varying World Market Integration, Journal of Finance, 50, 403-444.

Brock W.A., Dechert, W.D., Scheinkman, J.A., (1987), A test for independence based on the correlation dimension, Department of Economics, University of Winsconsin-Madison, SSRI Working Paper n. 8702.

Brock, W.A., Hommes, C.H., (1997). A rational route to randomness, Econometrica, 65, p. 1059-1095.

Brock, W.A., and Hommes, C.H., (1998), Heterogeneous beliefs and routes to chaos in a simple asset pricing model, Journal of Economic Dynamics and Control, 22, p. 1235-1274.

Brunetti, C., Scotti, C., Mariano, R.S., Tann, A.H.H., (2008), Markov Switching GARCH Models of Currency Turmoil in Southern Asia, Emerging Markets Review, 9, p. 164-178.

Chia, W.-M., Li, M., Zheng, H., (2014). Regime Switching Models in the Foreign Exchange Market. In Dieci R., He X.-Z., Hommes C. (eds), Nonlinear economic dynamics and financial modelling. Springer, Heidelberg, p. 201-223.

Diebold, F.X., Lee, J.-H., Weinbach, G.C., (1994), Regime Switching and Endogenous Transition Probailities, in Non-stationary Time Series Analysis, ed. Hargreaves, C., Oxford, U.K., Oxford University Press.

Engel, C., Hakkio, C.S., (1996), The Distribution of Exchange Rates in the EMS, International Journal of Finance and Economics, 1, p. 55-67.

Filardo, A.J., (1994), Business Cycles and Their Transitional Dynamics, Journal of Business & Economic Statistics, 12, p. 299-308.

Filardo, A.J., Gordon, S.F., (1998), Business Cycle Duration, Journal of Econometrics, 85, p. 99-123.

Frankel, J.A., Froot, K.A., (1986) Understanding the US Dollar in the Eighties: The Expectations of Chartists and Fundamentalists, Economic Record, 62, 24-38.

Kao, C.-W., Kleykamp, D., Wan J.-Y., (2016), Nonlinear Oil Price Dynamics and the Impact of Heterogeneous Agents, available at DOI: 10.3966/054696002015060097003

Hamilton, J.D., (1989), A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle,. Econometrica, 57, p. 357-384.

Hommes, C., (2005), Heterogeneous Agents Models in Economics and Finance, TI 2005-056/I , Tinbergen Institute Discussion Papers, CeNDEF, Department of Quantitative Economics, University of Amsterdam.

Joliffe, I.T. (1986), Principal Component Analysis, John Wiley & Sons, New York.

Nakov, A., Nuño, G., (2011), Saudi ARAMCO and the oil market. ECB Working Paper, 1354.

Omar, A.M.A., Wisniewski, T.P., Nolte, S. (2017). Diversifying away the risk of war and cross-border political crisis, Energy Economics, 64, p. 494-510

Psaradakis, Z., Sola, M., (2017), Markov-Switching Models with State-Dependent Time-Varying Transition Probabilities, BWPEF 1702, Birbeck College, London.

Psaradakis, Z., Sola, M., Spagnolo. F., Spagnolo, N., (2013), Some Cautionary Results Concerning Markov-Switching Models with Time-Varying Transition Probabilities, Department of Economics, Mathematics and Statistics, Birkbeck College, University of London.

Reitz, S., Westerhoff, F.H., (2007), Commodity Price Cycles and Heterogeneous Speculators: A STAR-GARCH Model, Empirical Economics, 33, p. 231-244.

Santabárbara, D., (2017), The Oil Market: Recent Developments and Outlook. Banco de España, Analytical Articles Economic Bulletin 3/2017.

Shleifer A., Vishny, R., (1997), The Limits of Arbitrage, Journal of Finance, 52, p. 35-55.

Stevens, P. (2005). Oil markets, Oxford Review of Economic Policy, 1, p. 19-42.

Westerhoff, F., (2004), Greed, fear and stock market dynamics, Physica A: Statistical Mechanics and its Applications, 343, issue C, p. 635-642.

Westerhoff, F., Reitz, S., (2005), Commodity Price Dynamics and the Nonlinear Market Impact of Technical Traders: Empirical Evidence for the US Corn Market, Physica A: Statistical Mechanics and its Applications, 349, p. 641-648

Appendix 1

Principal Components Analysis

The standard Principal Components Analysis can be summarized as follows.

The principal component transformation of the (z x 1) column vector U_t of one-step ahead (regime 1) predicted volatilities P_{lt} reads as

$$V_t = B'(U_t - U_t^*)$$

where U^* is the vector of sample means and $\Delta = (\delta_{lm})$ is the (z x z) sample covariance matrix. B is a (z x z) orthogonal matrix whose lth column b_l is the *lth* eigenvector of Δ i.e. the *lth* vector of principal components loadings. V_t is a (z x 1) vector of principal components where the *lth* principal component $v_{lt} = b_l'(U_t - U_t^*)$ has zero mean and variance δ_l, the *lth* eigenvalue of Δ. Appropriately normalized, it quantifies the fraction of the variance of the one-step ahead predicted regime 1 probabilities explained by the *lth* princial component. (In the same way, the sum of the first *m* normalized eigenvectors measures how much of this variance is explained by the first *m* principal components.)[17]

[17] We follow the standard procedure and take principal components of the correlation matrix, rescaling all variables to have unit sample variance.