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Monopolistic Competition with GAS Preferences

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Abstract

We study monopolistic competition equilibria with free entry and social planner solutions under symmetric Generalized Additively Separable preferences (that encompass known cases such as additive, homothetic, translog and other preferences). This setting can jointly produce competition and selection effects of entry, incomplete pass-through of cost changes and pricing to market. We characterize the inefficiencies of the market equilibrium under Gorman-Pollak preferences and show its optimality under implicit CES preferences. We propose a new specification of generalized translated power preferences for trade and macroeconomic applications.

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In this work we analyze monopolistic competition and optimal resource allocation for Generalized Additively Separable (GAS) preferences (Gorman, 1970; Pollak, 1972). This generalization of the Dixit and Stiglitz (1977) setting includes additive preferences, as well as homothetic preferences (as the translog specification) and other cases where demand depends on the own price and a common price aggregator. Through this microfoundation we can jointly match three phenomena that are empirically relevant in monopolistically competitive markets: competition effects due to entry (selection of the more efficient firms under heterogeneity), incomplete pass-through (lower markups for more productive firms) and pricing to market associated with income variations (possibly different across firms).[2] Moreover, we study the optimal allocation of resources and extend the optimality result under heterogeneous firms recently proved by Dhingra and Morrow (2019) for CES case to a more general class of preferences. These findings are useful to understand the consequences of trade liberalization, macroeconomic shocks and optimal policies in models based on monopolistic competition.

The classic Dixit-Stiglitz model at the basis of a large part of modern economic theories of imperfect competition is based on symmetric CES (Constant Elasticity of Substitution) preferences, whose demands functions depend on a power function of the price and on a common price index. The peculiar properties of this setting are well known. In particular, with homogeneous firms markups are constant, changes in market size create pure gains from variety (Krugman, 1980) and the market equilibrium is optimal (Dixit and Stiglitz, 1977). With heterogeneous firms, markups remain constant, changes in market size do not exert selection effects on the set of active firms (Melitz, 2003) and the equilibrium is still optimal (Dhingra and Morrow, 2019). Generalizations to homothetic preferences have emphasized competition effects of entry and inefficiency of the equilibria (Feenstra, 2003, 2018; Bilbiie *et al.*, 2012, 2019).

The original contribution of Dixit and Stigliz (1977) also explored the more general class of directly additive preferences whose direct utility can be written as:

$$U = \int_{\Omega} u(x(\omega))d\omega, \tag{1}$$

where Ω is the set of consumed goods and the consumption $x(\omega)$ of variety $\omega \in \Omega$ has subutility u. With these preferences the elasticity of substitution between a good and the others, which determines demand elasticity, depends only on its consumption level. Under monopolistic competition among homogeneous firms with free entry the markup is independent from income, and it changes with market size and marginal cost depending on the shape of the marginal subutility of consumption, while the equilibrium is in general inefficient. The analysis has been extended to heterogeneous firms in Zhelobodko *et al.* (2012), Bertoletti and Epifani (2014) and Dhingra and Morrow (2019).[3]

[2] For empirical evidence on these phenomena in trade settings see for instance De Loecker *et al.* (2016), Hsieh *et al.* (2016) and Simonovska (2015).

[3] Further applications are in Simonovska (2015), Mrázová and Neary (2018), Arkolakis *et al.* (2019) and others.

Recently, we have studied monopolistic competition for demand functions derived from indirectly additive preferences (Bertoletti and Etro, 2017a), namely when indirect utility can be written as:

$$V = \int_{\Omega} v(s(\omega))d\omega, \tag{2}$$

where $s(\omega) \equiv p(\omega)/E$ is the price of variety ω normalized by income E, with subutility v. In this case, demand elasticity depends only on the own normalized price. Under monopolistic competition the equilibrium markup is independent from market size and changes with income and marginal cost depending on the shape of the marginal subutility of prices, and the equilibrium is in general inefficient.[4]

Additive preferences belong to the more general type of GAS preferences whose demand system features a common aggregator of prices or quantities. We have introduced these preferences for monopolistic competition pricing in our companion paper (Bertoletti and Etro, 2017b), and here we consider in detail the free entry equilibria under symmetric preferences. According to Gorman (1970) there are two versions of GAS preferences. We call *Gorman-Pollak preferences* (henceforth GP preferences) a generalization of (1) and (2), which generates an elasticity of substitution depending on the product of price (or quantity) and the common aggregator.[5] Following Hanoch (1975), we define the other class of GAS preferences as *implicit CES preferences*: they feature an elasticity of substitution that is common across commodities, but can change through indifference curves, and therefore with the utility level, which is the relevant aggregator. We analyze monopolistic competition for these two classes of preferences and exemplify the results with a novel specification of "generalized translated power preferences" which nests directly additive, indirectly additive and homothetic versions, with demand functions that can be linear (as in Melitz and Ottaviano, 2008), perfectly rigid or perfectly elastic, and it is suitable for quantitative explorations.

It is convenient to distinguish the analysis of the monopolistic competition equilibrium with and without firms heterogeneity. Under homogeneous firms, the advantage of GP preferences is that they are more flexible on the impacts of productivity, income and market size (which are neutral on markups respectively under homotheticity, direct and indirect additivity). For instance, the homothetic GP preferences provide markups depending on the number of firms and can be exploited in endogenous entry models with competition effects to replicate the aggregate behavior of the economy. More in general, we emphasize conditions under which markups increase less than proportionally in costs, increase in income and decrease in market size: matching these facts jointly is empirically relevant for trade models but requires a departure from traditional

[4] Further applications are in Boucekkine *et al.* (2017) and, with heterogeneous firms, in Bertoletti *et al.* (2018) and Macedoni and Weinberger (2018).

[5] GP preferences include homothetic preferences whose demands depend on the own price and a common price aggregator (see also Matsuyama and Ushchev, 2017). In particular, we will provide the utility representation of translog preferences in terms of GP preferences.

microfoundations. With respect to the optimal allocation of resources, we find conditions under which GP preferences deliver either excess or insufficient entry, while the decentralized market equilibrium is optimal under implicit CES preferences, which extends the classic result by Dixit and Stiglitz (1977).

For the case of heterogeneous firms, we provide an extension of the Melitz (2003) setting with and without fixed costs of production, showing that both pricing and selection effects can be much richer than in standard models. Most important, we characterize the social planner choice of markup, measure of created firms and set of consumed goods, extending the results obtained by Dhingra and Morrow (2019) for directly additive preferences and by Bertoletti *et al.* (2018) for indirectly additive preferences. Under GP preferences and a Pareto distribution of unit costs across firms, as in Arkolakis *et al.* (2012, 2019), Bertoletti *et al.* (2018) and many other recent works in trade theory, we make further progress in deriving decentralized and optimal allocations. We also obtain a simple formula for the welfare gains from market expansion, which depends on the Pareto shape parameter and two elasticities of the utility function. In case of "generalized translated power preferences" we find that the equilibrium generates the optimal number of created firms, but typically too many goods are consumed and in excessive quantity for those with high marginal costs. For the class of implicit CES preferences, our extension of the Melitz (2003) model with arbitrary marginal cost distribution and positive fixed cost shows that opening up to costless trade (i.e., increasing the market size) generates selection effects and reduces markups as long as the demand elasticity is increasing in utility. Moreover, we confirm the optimality of the decentralized equilibrium for this entire class, generalizing a result established by Dhingra and Morrow (2019) only for explicit CES preferences.

This work contributes to a recent literature that has examined monopolistic competition settings beyond the classic Dixit-Stiglitz one. Some papers have already made progress with symmetric versions of general homothetic (Bilbiie *et al.*, 2012) and non-homothetic preferences (Bertoletti and Etro, 2016). However, it is only by exploiting the unique properties of the GAS preferences that we are able to derive more general results concerning both free-entry market equilibria and the optimal allocations. Early applications of GP preferences to macroeconomic and trade issues can be found in Etro (2018), Macedoni and Weinberger (2018) and Fally (2019). Arkolakis *et al.* (2019) have also explored demand systems nesting those generated by directly additive preferences and by some homothetic ones, though their focus is on quantifying the gains from trade liberalization in a multicountry model: GAS preferences overlap with theirs only for the directly additive class. Finally, we should remark that the literature mentioned above and the present work are limited to symmetric preferences: d'Aspremont and Dos Santos Ferreira (2016) and our companion paper have investigated the more general case of asymmetric preferences.

The remaining of this work is organized as follows. Section 1 studies GP preferences focusing first on homogeneous firms and then on heterogeneous ones. Section 2 studies implicit CES preferences again under both homogeneous and heterogeneous firms. Section 3 concludes. All proofs are in the Appendix.

1 Gorman-Pollak preferences

We consider monopolistic competition in a market with a population of L identical consumers with income/expenditure E. Their preferences can be represented by the following direct and indirect utility functions (Gorman, 1970, 1987, and Pollak, 1972):

$$U = \int_\Omega u(\xi x(\omega))d\omega - \phi(\xi) \quad \text{and } V = \int_\Omega v(\rho s(\omega))d\omega - \theta(\rho), \tag{3}$$

where $x(\omega)$ and $s(\omega)$ are consumption and (normalized) price of variety $\omega \in \Omega$, with respectively increasing and concave subutility u and decreasing and convex subutility v, and the aggregators ξ and ρ satisfy:

$$\phi'(\xi) = \int_\Omega u'(\xi x(\omega))x(\omega)d\omega \quad \text{and} \quad \theta'(\rho) = \int_\Omega v'(\rho s(\omega))s(\omega)d\omega, \tag{4}$$

for an increasing function $\phi(\xi)$ and a decreasing function $\theta(\rho)$. Before illustrating these preferences, it seems useful to anticipate that they are *directly additive* as in (1) when $\theta(\rho) = -\rho$, they are *indirectly additive* as in (2) when $\phi(\xi) = \xi$ and they are *homothetic* when $\theta(\rho) = -\ln \rho$ and $\phi(\xi) = \ln \xi$.

The role of ξ and ρ is to cancel out any direct cross effect on utility, as in the case of additive preferences, which is key to obtain demand systems depending only on one aggregator. Intuitively, ξ can be seen as generating the benefit of increasing the *effective* quantity of good ω to $\xi x(\omega)$ at the utility cost $\phi(\xi)$, which is equivalent to the possibility of reducing the inconvenience of consumption $\theta(\rho)$ at the cost of increasing the *effective* price of good ω to $\rho s(\omega)$.

Inverse and direct demands can then be easily computed from the Hotelling-Wold and Roy identities as:

$$s(\iota) = \frac{u'(\xi x(\iota))}{\int_\Omega u'(\xi x(\omega))x(\omega)d\omega} \quad \text{and} \quad x(\iota) = \frac{v'(\rho s(\iota))}{\int_\Omega v'(\rho s(\omega))s(\omega)d\omega}, \tag{5}$$

for $\iota \in \Omega$. Using (4)-(5) confirms that preferences are of the GAS type, in the sense that demands depend only on a common aggregator (Pollak, 1972). It also holds that $\rho = \phi'(\xi)$, $\xi = -\theta'(\rho)$, with $\rho\xi = -\theta'(\rho)\rho = \phi'(\xi)\xi$ the marginal utility of income (times E), and that the following relations link the direct and dual expressions of utility:

$$v'(u'(z)) = -z \quad \text{and} \quad u'(-v'(z)) = z, \tag{6}$$

$$\theta'\left(\phi'(z)\right) = -z \quad \text{and} \quad \phi'\left(-\theta'(z)\right) = z. \tag{7}$$

Let us define the following elasticities of the marginal subutilities:

$$\epsilon(z) \equiv -\frac{u''(z)z}{u'(z)} \quad \text{and} \quad \varepsilon(z) \equiv -\frac{v''(z)z}{v'(z)},$$

where differentiating (6) one can verify that $\varepsilon\left(u'(z)\right) = 1/\epsilon\left(z\right)$. Accordingly, inverse and direct (own) demand elasticities are given by $\epsilon\left(\xi x\right)$ and $\varepsilon\left(\rho s\right)$.

When firms produce under a constant marginal cost c, variable profit can be expressed as:

$$\pi = \left[\frac{u'\left(\xi x\right)E}{\phi'\left(\xi\right)} - c\right]xL = \frac{\left(sE - c\right)v'(\rho s)L}{\theta'\left(\rho\right)}. \tag{8}$$

Under monopolistic competition each firm chooses its price or quantity taking as given the aggregators. Assuming that $\epsilon\left(\xi x\right)$ and $\varepsilon\left(\rho s\right)$ are respectively smaller and larger than unity, the following pricing rules of monopolistic competition emerge:

$$p = \frac{c}{1 - \epsilon\left(\xi x\right)} = \frac{\varepsilon\left(\rho s\right)c}{\varepsilon\left(\rho s\right) - 1}, \tag{9}$$

which depend on the aggregators through the relevant elasticity.[6]

The functional forms (3)-(4) have to satisfy some regularity conditions for preferences to behave well. These can be expressed in terms of the curvature measures:

$$\beta\left(\rho\right) \equiv -\frac{\theta''\left(\rho\right)\rho}{\theta'\left(\rho\right)} \quad \text{and} \quad \tilde{\beta}\left(\xi\right) \equiv -\frac{\phi''\left(\xi\right)\xi}{\phi'\left(\xi\right)}, \tag{10}$$

which can be either positive or negative and are such that $\beta(\phi'(z)) = 1/\tilde{\beta}(z)$, as can be verified differentiating (7). In particular, Fally (2018) has shown that, when ρ is well defined by (4) and $\varepsilon > 1$ holds, a sufficient condition is $\beta \leq 1$, and we will focus our discussion on this region (which actually encompasses all the additive and homothetic preferences of the GP type). Nevertheless, GP preferences can be well-behaved also for larger values of β.[7]

To verify that the GP preferences are directly additive when $\theta(\rho) = -\rho$ notice that in such a case $\xi = -\theta' = 1$, which delivers the same demand system as (1), implying $\beta = 0$. To verify that the GP preferences are indirectly additive when $\phi(\xi) = \xi$ notice that it must be the case that $\rho = \phi' = 1$, which delivers the same demand system as (2), implying $\beta \to \infty$. Finally, to verify that the GP preferences nest a homothetic family of preferences when $\phi(\xi) = \ln\xi$ and $\theta(\rho) = -\ln\rho$ notice that (4) implies that the aggregators must then be homogeneous of degree -1, so that from (5) demand ratios are homogeneous of degree 0, and $\beta = 1$.[8]

[6] As standard in the literature we assume constant marginal costs. Notice that with variable marginal costs prices would depend on demand aggregators even in case of CES preferences.

[7] For instance, one can prove that the indirect utility function defined by (3)-(4) is locally twice differentiable, decreasing and convex when $\beta\left(\rho\right) > \underline{\varepsilon} = \int_{\Omega}\varepsilon(\rho s(\omega))x(\omega)s\left(\omega\right)d\omega$. Notice that the income elasticity of demand:

$$\frac{\partial\ln x\left(\omega\right)}{\partial\ln E} = \frac{\beta\left(\rho\right)\left[\underline{\varepsilon} - \varepsilon\left(\rho s(\omega)\right) - 1\right] + \varepsilon\left(\rho s(\omega)\right)}{\underline{\varepsilon} - \beta\left(\rho\right)}$$

depends crucially on β and the average elasticity $\underline{\varepsilon}$.

[8] On the demand system of this homothetic family see Matsuyama and Ushchev (2017).

Obviously, GP preferences nest the CES case, whose demand system is given by:

$$s(\iota) = \frac{x(\iota)^{-\epsilon}}{\int_\Omega x(\omega)^{1-\epsilon} d\omega} \quad \text{and} \quad x(\iota) = \frac{s(\iota)^{-\varepsilon}}{\int_\Omega s(\omega)^{1-\varepsilon} d\omega}, \tag{11}$$

where $\varepsilon = 1/\epsilon > 1$ is constant. This homothetic specification emerges immediately whenever $u(z) = \frac{z^{1-\epsilon}}{1-\epsilon}$ and $v(z) = \frac{z^{1-\varepsilon}}{\varepsilon-1}$ and thus, from (4), $\xi = \left[\int x(\omega)^{1-\epsilon} d\omega\right]^{\frac{1}{1-\epsilon}}$ and $\rho = \left[\int s(\omega)^{1-\varepsilon} d\omega\right]^{\frac{1}{\varepsilon-1}}$ are power means.

Another homothetic example is given by the symmetric translog preferences of Feenstra (2003), which have been widely used in trade and macroeconomic applications. The translog demand function:

$$x(\iota) = \frac{\frac{1}{n} + \nu \left[\int_\Omega \ln s(\omega)^{\frac{1}{n}} d\omega - \ln s(\iota)\right]}{s(\iota)}, \tag{12}$$

with $\nu > 0$ parametrizing substitutability, comes from the following indirect utility:

$$V = \int_\Omega \left(\frac{\nu \ln \{\rho s(\omega)\}}{2} - \frac{1}{n} \right) \ln \{\rho s(\omega)\} \, d\omega + \ln \rho, \tag{13}$$

where $\ln \rho = \int_\Omega \ln s(\omega)^{-\frac{1}{n}} d\omega$, the aggregator ρ is the reciprocal of the geometric mean of normalized prices and $\theta(\rho) = -\ln \rho$ ensures homotheticity.

GP preferences extend well beyond known examples of additive and homothetic preferences: here we propose a novel specification that can be useful in applications.

Generalized translated power (GTP) preferences Consider the following representations of GTP preferences:

$$U = \int_\Omega \left(a\xi x(\omega) - \frac{(\xi x(\omega))^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) d\omega - \frac{\xi^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}} \; V = \int_\Omega \frac{(a - \rho s(\omega))^{1+\gamma}}{1+\gamma} d\omega - \frac{\rho^{1-\beta}}{\beta - 1}, \tag{14}$$

with $a, \gamma > 0$. The demand system is given by:

$$s(\iota) = \frac{a - [\xi x(\iota)]^{\frac{1}{\gamma}}}{\int_\Omega \left[a - [\xi x(\omega)]^{\frac{1}{\gamma}}\right] x(\omega) d\omega} \quad \text{and} \quad x(\iota) = \frac{[a - \rho s(\iota)]^{\gamma}}{\int_\Omega (a - \rho s(\omega))^\gamma s(\omega) d\omega},$$

Accordingly, these GTP preferences nest the cases of direct additivity for $\beta = 0$, homotheticity for $\beta \to 1$ and indirect additivity for $\beta \to \pm\infty$, as well as demand functions that are perfectly rigid for $\gamma \to 0$, linear for $\gamma = 1$ and perfectly elastic for $\gamma \to \infty$. While the indirectly additive case was used by Bertoletti *et al.* (2018), we are not aware of applications of the more general specification.

The interesting case with $\gamma = 1$ delivers a linear demand, as in generalized versions of the model of Melitz and Ottaviano (2008) without quasi-linearity. Two examples where the aggregators can be explicitly computed deserve to be mentioned. When $\beta \to 1$ these GTP preferences are homothetic and can be represented by the indirect utility:

$$V = \int_{\Omega} \frac{(a - \rho s(\omega))^2}{2} d\omega + \ln \rho, \tag{15}$$

with ρ satisfying $\int_{\Omega}(as(\omega)\rho - s(\omega)^2\rho^2)d\omega = 1$. When $\beta = -1$, we get the "quadratic" indirect utility:

$$V = \int_{\Omega} \frac{(a - \rho s(\omega))^2}{2} d\omega + \frac{\rho^2}{2}, \tag{16}$$

with $\rho = a \int_{\Omega} s(\omega)d\omega/[1 + \int_{\Omega} s(\omega)^2 d\omega]$. In both cases the utility specification depends on price means.

1.1 Monopolistic competition with homogeneous firms

We now consider the monopolistic competition equilibrium with homogenous firms to study its comparative statics and compare it with the optimal market structure. While results can be derived starting from the primal or the dual version of the preferences, it is convenient to focus on the indirect utility in (3).

If each firm has a constant marginal cost c, the symmetric equilibrium price p and the aggregator ρ satisfy the following conditions (assuming $\varepsilon > 1$):

$$p = \frac{\varepsilon(\rho s) c}{\varepsilon(\rho s) - 1} \quad \text{where } \theta'(\rho) = nv'(\rho s) s, \tag{17}$$

which depend on the number of firms n, on the marginal cost c and on income E. The second-order condition for profit maximization requires that $2\varepsilon(z) > \zeta(z)$, where $\zeta(z) = -zv'''(z)/v''(z)$.

The comparative statics of the equilibrium price depends on the shape of ε as well as on the behavior of the aggregator ρ. As well known, the elasticity depends only on the number of firms under homotheticity. For instance, under the translog preferences of Feenstra (2003), the elasticity of demand can be computed from (12) as $\varepsilon = 1 + \nu n$ under symmetry, which implies markups decreasing with the number of firms. As another example, consider the homothetic GTP preferences represented by (15). In this case we obtain the elasticity $\varepsilon = \rho s/(a - \rho s)$ implying the price rule:

$$p = \frac{c + aE/\rho}{2}.$$

For a given aggregator, this price exhibits incomplete pass-through of the marginal cost and pricing to market (a markup increasing in income). However, the aggregator, which is linear with respect to income, must satisfy $1 = n(a - \rho s)\rho s$, therefore we can also solve for ρs and for the elasticity $\varepsilon(n)$ in function of the number of firms (assumed large enough: $\sqrt{n} > 2/a$). With this we obtain the equilibrium price:

$$p = \frac{c}{2}\left(1 + \frac{a}{2\sqrt{\frac{a^2}{4} - \frac{1}{n}}}\right), \tag{18}$$

which is decreasing in the number of firms, linear in the marginal cost and does not directly depend on income, as one should expect under homotheticity. This is a case where substitutability between goods increases with the number of firms, leading to markups decreasing with entry.[9]

When we depart from homotheticity and additivity, we can obtain competition effects of n, incomplete pass-through of c and pricing to market of E at the same time. As an example, let us consider the quadratic GTP preferences represented by (16), where we remind that $\beta = -1$. The price rule is the same as in the last example, because the elasticity is the same for a given aggregator. However, imposing symmetry we now derive the equilibrium price:

$$p = \frac{c}{2}\left(1 + \sqrt{1 + \frac{4E^2}{nc^2}}\right). \tag{19}$$

This price is still decreasing in the number of firms (competition effect), but now it increases less than proportionally in the marginal cost (incomplete pass-through) and increases in income (pricing to market). The reason is that substitutability between goods increases with the number of firms, but it also decreases with productivity and consumer income.

1.1.1 Free entry equilibrium

Let us assume that entry requires a fixed cost of production $F > 0$. The free-entry equilibrium implies expressions for price, the number of firms and the individual consumption satisfying (17) and:

$$n = \frac{EL}{\varepsilon(\rho s) F} \quad \text{and} \quad x = \frac{[\varepsilon(\rho s) - 1] F}{cL}, \tag{20}$$

which are not independent, since the budget constraint requires $pnx = E$. Once again, the analysis simplifies if we assume homotheticity, in which case by (17) the equilibrium value of the effective price ρs and the number of firms are positively related. Accordingly, the marginal cost is neutral on markups, while an increase of EL (or a reduction of F) increases the number of firms less than proportionally and decreases markups if and only if $\varepsilon'(z) > 0$ (as in the GPT case where $\varepsilon(z) = \gamma z/(a - z)$), with neutrality in the CES case (11).[10]

[9] This is the same qualitative result as under homothetic translog preferences (Feenstra, 2003; Bilbiie *et al.*, 2012).

[10] Notice that the neutrality of market size under CES preferences holds also under variable costs. For instance, consider the variable cost function cq^δ, where the marginal cost is increasing with respect to the quantity q, decreasing or constant depending on whether δ is above, below or equal to one. Then, the monopolistic competition price with free entry is independent from EL, but it increases (decreases) in the fixed cost F if $\delta > (<) 1$.

For the general case, the GP preferences offer a variety of possible comparative statics results. In the Appendix we derive the sign of the impacts on prices and number of firms as follows:

$$sign\left\{\frac{\partial \ln p}{\partial \ln L}\right\} = sign\left\{\frac{-\varepsilon'(z)}{\varepsilon(z)-\beta(\rho)}\right\} = sign\left\{\frac{\partial \ln n}{\partial \ln L} - 1\right\}, \tag{21}$$

$$sign\left\{\frac{\partial \ln p}{\partial \ln E}\right\} = sign\left\{\frac{-\varepsilon'(z)\beta\left(\rho\right)}{\varepsilon(z)-\beta(\rho)}\right\} = sign\left\{\frac{\partial \ln n}{\partial \ln E} - 1\right\}, \tag{22}$$

$$sign\left\{\frac{\partial \ln p}{\partial \ln c} - 1\right\} = sign\left\{\frac{\varepsilon'(z)\left[\beta\left(\rho\right)-1\right]}{\varepsilon(z)-\beta(\rho)}\right\} = sign\left\{\frac{\partial \ln n}{\partial \ln c}\right\}. \tag{23}$$

Given the wide range of possible results, here we summarize the most relevant cases as follows:[11]

Proposition 1. *Under Gorman-Pollak preferences and homogeneous firms with $\beta\left(\rho\right) \leqslant 1$ and $\varepsilon'(z) > 0$, the equilibrium of monopolistic competition with free entry implies that an increase of market size decreases prices and increases the number of firms less than proportionally, an increase of income increases prices and the number of firms more than proportionally if and only if $\beta\left(\rho\right) < 0$, and the pass-through of changes in the marginal cost is incomplete if $\beta\left(\rho\right) < 1$.*

It is easy to verify that market size is neutral on markups if $\beta \to \infty$ (indirect additivity), income is neutral if $\beta = 0$ (direct additivity) and costs are neutral on markups if $\beta = 1$ (homotheticity). Besides these neutralities, the important message from Proposition 1 is that moving beyond homothetic or additive preferences it is possible to jointly match competition effects (markup reductions following an expansion of market size), incomplete pass-through (markup reductions following a cost increase) and pricing to market (a markup increase following an increase of income). In particular, when $\varepsilon'(z) > 0$ and $\beta\left(\rho\right) < 0$ we have at the same time competition effects, incomplete pass-through and pricing to market. One can also verify that only under those assumptions we obtain these three comparative static results together.

1.1.2 Optimality

With this characterization of market equilibrium, we can now derive the optimal allocation of resources to verify whether monopolistic competition generates excess or insufficient entry of firms. For this purpose, it is convenient to define the elasticity of the indirect subutility:

$$\eta\left(z\right) \equiv -\frac{v'(z)z}{v(z)} > 0. \tag{24}$$

[11] Notice that the comparative static results of L and E in Proposition 1 only require the weaker condition $\beta(\rho) < \varepsilon(z)$. If $\beta(\rho) > \varepsilon(z)$, with $\beta(\rho)$ sufficiently large to satisfy the regularity conditions of preferences, and retaining the assumption that $\varepsilon'(z) > 0$, the impact of market size would be reversed while prices would increase with respect to income and pass-through would be incomplete. Of course, the results of Proposition 1 reverse when $\varepsilon'(z) < 0$.

The optimal allocation is characterized in what follows by the relevant first-order conditions, and it turns out that the sign of $\eta'(z) \propto 1+\eta(z)-\varepsilon(z)$ is critical in comparing it to the equilibrium.

Proposition 2. *Under Gorman-Pollak preferences with homogeneous firms, the optimal allocation satisfies:*

$$p^* = \left[1+\frac{1}{\eta(\rho^* s^*)}\right]c, \quad n^* = \frac{EL}{[1+\eta(\rho^* s^*)]F} \quad \text{and} \quad x^* = \frac{\eta(\rho^* s^*)F}{cL}, \tag{25}$$

where $\theta'(\rho^*) = n^* v'(\rho^* s^*) s^*$. *Assuming* $\beta(\rho) \leqslant 1$, *the equilibrium is characterized by excess entry if* $\eta'(z) > 0$ *everywhere, and by insufficient entry if* $\eta'(z) < 0$ *everywhere.*

The intuition for these results relies on the fact that the marginal rate of transformation between number of goods and prices $-\frac{\partial V/\partial n}{\partial V/\partial s} = \frac{s}{n\eta}$ is inversely related to the elasticity η, thus a higher elasticity implies lower gains from variety and makes it convenient to reduce the number of goods provided by the social planner and increase their consumption by a lower price. Moreover, η is proportional to the expenditure/utility ratio of each commodity, and, when this is increasing, the market is setting prices too high, thus attracting entry of too many firms compared to the social planner.[12]

We can illustrate the results in few examples. As implied by Bertoletti and Etro (2016) and Bilbiie *et al.* (2019), one can verify that the translog preferences generate excess entry. As another example, let us reconsider the GTP preferences with $\gamma = 1$ analyzed above. In this case $\eta(z) = 2\varepsilon(z)$ is increasing, therefore we must have excess entry. This can be verified in the homothetic version (15), which provides $p = \frac{a+\rho s}{2\rho s}c$ and therefore the price:

$$p^* = \frac{c}{2}\left(1+\frac{a}{\frac{a}{2}+\sqrt{\frac{a^2}{4}-\frac{1}{n}}}\right). \tag{26}$$

This expression is everywhere below the equilibrium price, confirming that the model delivers excess entry in equilibrium. In all these cases the optimal price can be restored with an appropriate taxation.

1.2 Monopolistic competition with heterogeneous firms

We now consider the extension of the celebrated Melitz (2003) model in a closed economy to GP preferences. We assume that there is a common (and sunk) entry cost $F_e > 0$ and a fixed cost of production $F \geq 0$. After entry, firms draw their constant marginal costs from a continuous distribution $G(c)$ on the support $[0, \bar{c}]$,[13] and then decide whether to produce and pay the fixed cost of

[12] The comparison can be easily expressed in terms of the elasticity of the direct subutility $\tilde{\eta}(z) = u'(z)z/u(z)$, with excess entry if this is decreasing, as in Dixit and Stiglitz (1977).

[13] Here $\bar{c}$ is supposed to be large enough to never become binding (this requires positive fixed costs, i.e., $F > 0$, in the case of preferences without finite choke prices).

production or not. As above, it is convenient to focus on the dual representation of preferences in (3).

1.2.1 Free entry equilibrium

Only the most efficient firms with marginal costs which belong to the interval $[0, \widehat{c}]$ are *ex-post* active, where the cut-off $\widehat{c}$ identifies the marginal firm that obtains zero profits if active. When there are no fixed costs of production ($F = 0$), this is the firm whose unit cost is given by aE/ρ, where a is the normalized, effective choke-off price such that $v(z) = 0$ for $z \geq a$ (if $a \to \infty$, as with CES preferences, all firms would be active). When there are positive fixed costs of production ($F > 0$), the variable profits of the marginal firm are equal to them in a market equilibrium.

Given $\widehat{c}$ and the measure of entrant firms N, the measure of firms that will be *ex-post* active, whose goods are actually consumed, is given by $n = G(\widehat{c})N$. Accordingly, we can rewrite consumers' utility as follows:

$$V = N \int_0^{\widehat{c}} v(\rho s(c))\, dG(c) - \theta(\rho), \tag{27}$$

where the aggregator satisfies:

$$\theta'(\rho) = N \int_0^{\widehat{c}} v'(\rho s(c))\, s(c)\, dG(c), \tag{28}$$

and $s(c) = p(c)/E$ is the normalized price of a c-firm. The variable profits of such a firm are:

$$\pi(c) = (p(c) - c)\, x(c)\, L,$$

and the equilibrium pricing rule $p(c)$ satisfies:

$$p(c) = \frac{\varepsilon(\rho s(c))\, c}{\varepsilon(\rho s(c)) - 1}, \tag{29}$$

with $x(c) = \frac{v'(s(c)\rho)}{\theta'(\rho)}$ denoting the individual demand for a c-variety. We assume that the pricing rule is uniquely defined by (29), which is certainly the case if $\varepsilon(z)$ is monotonic. Since $\pi'(c) < 0$, the cut-off $\widehat{c}$ satisfies the zero profit condition:

$$(p(\widehat{c}) - \widehat{c})\, v'(\rho s(\widehat{c}))\, L = \theta'(\rho)\, F, \tag{30}$$

which in the limit case of $F = 0$ identifies the cut-off $\widehat{c} = aE/\rho$ as the minimum price that drives demand to zero.

The measure of the entrant firms, instead, is determined by the free-entry condition of zero expected profit:

$$\mathbb{E}\{\pi(c)\} = \int_0^{\widehat{c}} (p(c) - c)\, x(c)\, L dG(c) - G(\widehat{c})F = F_e. \tag{31}$$

Combining (30) and (31) we can also see that the cut-off $\widehat{c}$ has to satisfy (assuming $F > 0$):

$$\int_0^{\widehat{c}} \frac{\pi(c)}{\pi(\widehat{c})} dG(c) = \frac{F_e}{F} + G(\widehat{c}), \tag{32}$$

which emphasizes that equilibrium firm selection depends on the ratio between average and marginal (variable) profits as well as on the ratio between entry and fixed costs. Rewriting (31) by using (29) and the budget constraint we obtain the mass of created firms:

$$N = \frac{EL}{\bar{\varepsilon}\left[F_e + FG(\widehat{c})\right]}, \tag{33}$$

where

$$\bar{\varepsilon} = \left[\int_0^{\widehat{c}} \frac{1}{\varepsilon(\rho s\,(c))} \frac{x\,(c)\,s\,(c)}{\int_0^{\widehat{c}} x\,(c)\,s\,(c)\,dG\,(c)} dG\,(c)\right]^{-1} \tag{34}$$

is the harmonic average of demand elasticities weighted by the market shares. We assume that a unique equilibrium exists. This is the case under CES preferences (11), as long as $F > 0$, which implies that $\bar{\varepsilon} = \varepsilon$ is a constant and that both cut-off and markups are independent from income and population: as is well-known from Melitz (2003), opening up to costless trade (i.e., a larger market size) does not induce any selection effects in this case and does not affect the prices of the active firms.[14]

The number of goods created and consumed are determined by intuitive conditions (which below will be compared to the optimality conditions). The average elasticity of demand $\bar{\varepsilon}$ drives average profitability, and therefore determines the measure of firms created in equilibrium, while the marginal cost cut-off $\widehat{c}$ depends on the ratio between average and marginal profitability, and markups change with the marginal cost of each good depending on the elasticity of demand $\varepsilon(z)$. The selection effects of market size, income, fixed costs and entry cost on $\hat{c}$ and the consequent impact on prices can be obtained in principle by differentiating (32)-(34). In the following proposition we summarize the equilibrium and state properties that apply to special classes of GP preferences:

PROPOSITION 3. *Under Gorman-Pollak preferences and heterogeneous firms the equilibrium of monopolistic competition with free entry is given by a pricing rule $p(c)$ satisfying (29), an aggregator ρ satisfying (28), a measure of firms N satisfying (33)-(34), and a cut-off $\hat{c}$ satisfying (30). Changes in income are neutral on the selection of firms and on pricing under direct additivity, changes in market size are neutral on the selection of firms and on pricing under indirect additivity, and changes in total income drive selection effects and pricing under homotheticity.*

[14] The absence of selection effects is not confined to the Melitz case with constant marginal cost of production. One can verify that under CES preferences market size neutrality holds also under increasing marginal costs, as when variable costs are given by cq^{δ}, where c is an idiosyncratic parameter and $\delta > 1$. However, in this case changes in the ratio F_e/F would affect prices through their impact on $\widehat{c}$.

To obtain more detailed results on the effects occurring under GP preferences, we now consider more restrictive technological conditions.

Pareto distribution without fixed costs Let us focus on the case of a Pareto distribution of the unit costs, i.e.:

$$G(c) = \left(\frac{c}{\bar{c}}\right)^{\kappa} \tag{35}$$

with shape parameter $\kappa > 1$, assuming that there are no fixed costs of production after entry. These assumptions are the same as those used in a variety of recent general equilibrium trade models, as those by Arkolakis *et al.* (2019) under direct additivity and by Bertoletti *et al.* (2018) under indirect additivity, therefore we extend their settings to the entire type of GP preferences.

Under these conditions the cut-off $\widehat{c}$ is determined by the choke price at which demand is null:

$$\widehat{c} = \frac{aE}{\rho}. \tag{36}$$

The price rule (29) still applies, but it can be rewritten as:

$$p(c) = \frac{\varepsilon(p\,(c)\,a/\widehat{c})c}{\varepsilon(p\,(c)\,a/\widehat{c}) - 1}, \tag{37}$$

which depends on $\widehat{c}$ and therefore on ρ in a simple way, as well as on the marginal cost c. In particular, we obtain:

$$\frac{\partial \ln p}{\partial \ln c} = \frac{\varepsilon(z) - 1}{\varepsilon(z) - 1 + \frac{\varepsilon'(z)z}{\varepsilon(z)}} \quad \text{and} \quad \frac{\partial \ln p}{\partial \ln \widehat{c}} = \frac{\varepsilon'(z)z}{\varepsilon(z)\,[\varepsilon(z) - 1]}\frac{\partial \ln p}{\partial \ln c}. \tag{38}$$

This implies that less efficient firms (with higher marginal costs) have smaller markups than more efficient firms if and only if $\varepsilon'(z) > 0$, in which case selection effects reducing $\widehat{c}$ are going to reduce markups. In what follows we will assume $\varepsilon'(z) > 0$, since it is empirically plausible that more efficient firms set higher markups (De Loecker *et al.*, 2016) and that the exit of less efficient firms is associated to a reduction in markups of all the active firms.[15]

Let us also define b by $\varepsilon\,(b) \equiv 1$; namely, b is the effective normalized price $p\,(0)\,a/\widehat{c}$ set by the most efficient firm with $c = 0$ (assuming that it is well-defined by the first-order condition for profit maximization). The distribution of the effective normalized prices among the active firms on the support $[b, a]$ can be computed as:

$$\begin{aligned} F_{\rho s}(z) &= \Pr\{\rho s(c) \leq z, c \leq \widehat{c}\} \\ &= \frac{G(h(z)E/\rho)}{G(aE/\rho)}, \end{aligned}$$

[15] This plausible property is equivalent to the so-called sub-convexity of the demand function: see Mrázová and Neary (2019).

where $h(z) = z\left[1 - 1/\varepsilon(z)\right]$, with $h' > 0$. The distribution $F_{\rho s}$ would depend in general on the expenditure level and it might also depend, through ρ, on the market size, but under our assumption of a Pareto distribution it reads as:

$$F_{\rho s}(z) = \left(\frac{h(z)}{a}\right)^{\kappa}, \tag{39}$$

which depends neither on market size nor on the expenditure level. The intuition is that the inframarginal price adjustments due to variations in the threshold $\widehat{c}$, in turn due to changes in E or in L/F_e, are exactly compensated by the process of entry/exit in terms of the effective normalized prices. Let us define the constant:

$$\Psi \equiv -\int_b^a v'(z)\, z dF_{\rho s}(z) > 0, \tag{40}$$

which only depends on the preferences and the parameter κ. Since we can rewrite the average demand elasticity as:

$$\bar{\varepsilon} = \Psi\left[\int_b^a \frac{-v'(z)\, z}{\varepsilon(z)} dF_{\rho s}(z)\right]^{-1},$$

we obtain that this is also a constant, and in particular that it is independent from market size and income as well as from the entry cost. The consequence is that the measure of entrants N is linear with respect to EL/F_e for all GP preferences.

Moreover, we can also use these results to obtain the comparative statics of the model with respect to shocks to the market size and the income of consumers. In particular, using (33) and (36), we can rewrite (28) as:[16]

$$\theta'(\rho)\,\rho = -\Psi n, \tag{41}$$

which implies that $\widehat{c}$ has to satisfy the following equation:

$$\theta'\left(\frac{aE}{\widehat{c}}\right) = \frac{-\Psi L}{\bar{\varepsilon} a F_e}\frac{\widehat{c}^{\kappa+1}}{\bar{c}^{\kappa}}. \tag{42}$$

This formula shows that a rich array of selection effects of E and L/F_e arises depending on the nature of preferences and technological conditions, which in turn affect pricing as mentioned above (through (38)) and the measure of consumed variety $n = N\left(\widehat{c}/\bar{c}\right)^{\kappa}$. In the Appendix we derive closed form solutions for our specification of GTP preferences (14), where $\bar{\varepsilon} = \kappa + 1$, so that the mass of firms is:

$$N = \frac{EL}{(\kappa+1)F}$$

[16] Notice that this says that Ψ is equal to the equilibrium marginal utility of income divided by n/E.

independently from the nature of preferences. Moreover, the equilibrium prices can be derived as:

$$p(c) = \frac{\gamma c + \widehat{c}}{\gamma + 1} \text{ with } \hat{c} = \left[\frac{\overline{c}^{\kappa} (1+\gamma)^{\gamma+1} F_e}{\kappa \gamma^{\gamma} a^{\gamma+\beta} B(\kappa, \gamma+2) L E^{\beta}} \right]^{\frac{1}{1+\kappa-\beta}},$$

implying incomplete pass-through and effects of income and market size depending on κ and β.

More generally, we can obtain the comparative statics for the cut-off by differentiating (42) to get:

$$\frac{\partial \ln \widehat{c}}{\partial \ln E} = \frac{-\beta(\rho)}{1+\kappa-\beta(\rho)} \quad \text{and} \quad \frac{\partial \ln \widehat{c}}{\partial \ln L} = \frac{-1}{1+\kappa-\beta(\rho)}, \tag{43}$$

which depend on the curvature of $\theta(\rho)$ and on the Pareto shape parameter κ. Accordingly, selection effects (which reduce prices given our assumption that $\varepsilon' > 0$) are caused by a rise of market size if $\beta \leq 1$, and also by a rise of income if $0 < \beta \leq 1$, but the latter has an anti-selective impact if $\beta < 0$ (which induces an increase in prices). In addition, changes of L and E affect the measure of consumed varieties as follows:

$$\frac{\partial \ln n}{\partial \ln E} = \frac{(1-\beta(\rho))(\kappa+1)}{1+\kappa-\beta(\rho)} \quad \text{and} \quad \frac{\partial \ln n}{\partial \ln L} = \frac{1-\beta(\rho)}{1+\kappa-\beta(\rho)}, \tag{44}$$

where both elasticities are positive for $\beta < 1$.

A wide range of comparative statics results can emerge from the model. Before summarizing those which apply to the empirically relevant cases, it is useful to consider some special classes of preferences. Consider the case of *direct additivity*, in which $\theta = -\rho$ (and therefore $\beta = 0$). Then we can compute the measure of consumed varieties $n = \rho/\Psi$, and the equilibrium cut-off:

$$\widehat{c} = \left[\frac{\overline{c}^{\kappa} a \overline{\varepsilon} F_e}{\Psi L} \right]^{\frac{1}{\kappa+1}}. \tag{45}$$

While the expenditure level does not affect the set of active firm and equilibrium prices, it increases proportionally the measure of consumed varieties. On the contrary, an increase in L/F_e exerts a selection effect, increasing less than proportionally the measure of active firms and decreasing prices (as in Bertoletti and Epifani, 2014, and Arkolakis *et al.*, 2019).

In the case of *indirect additivity* in which $\phi = \xi$ and $\rho = 1$ (so that $\beta \to \infty$), we compute from (42)-(41) the mass of consumed varieties $n = \frac{E^{\kappa+1} L}{\overline{\varepsilon} F_e} \left(\frac{a}{\overline{c}}\right)^{\kappa}$, and from (36):

$$\widehat{c} = aE \tag{46}$$

consistently with Bertoletti *et al.* (2018). Population does not affect the cost threshold, which is linearly increasing in income, while the measure of consumed goods is proportional to population, and increases more than proportionally with income. Accordingly, prices are neutral with respect to market size while they increase with income, which generates an anti-selection effect.

Consider finally the case of *homotheticity* in which $\theta = -\ln \rho$, and therefore $\beta = 1$. Then we have $n = \Psi^{-1}$, which is independent from both income and population. An increase of EL/F_e, instead, exerts a selection effect on the set of the active firms:

$$\widehat{c} = \left[\frac{\overline{c}^{\kappa} \overline{\varepsilon} F_e}{\Psi E L} \right]^{\frac{1}{\kappa}}, \tag{47}$$

leaving unchanged the measure of consumed goods. Similar results emerge with other specifications of homothetic preferences which have been examined elsewhere (see Feenstra, 2018 and Arkolakis *et al.*, 2019), suggesting the generality of these properties under homotheticity.

Let us go back to the general comparative statics in (43)-(44). To fix ideas, we state the following result on price and selection effects which derives immediately from the conditions above:

Proposition 4. *Under Gorman-Pollak preferences, a Pareto distribution of marginal costs without fixed costs of production, with $\beta(\rho) \leqslant 1$ and $\varepsilon'(z) > 0$, the equilibrium of monopolistic competition with free entry implies that an increase of market size is associated with price reductions and selection effects of the more efficient firms, an increase of income is associated with price increases and entry of less efficient firms if and only if $\beta(\rho) < 0$, and both generate an increase in the mass of consumed goods if $\beta(\rho) < 1$.*

Within the range of cases that can emerge, only GP preferences satisfying $\varepsilon'(z) > 0$ and $\beta(\rho) < 0$ imply equilibria consistent with empirically plausible effects in trade models: more efficient firms set higher markups (see De Loecker *et al.*, 2016, for related evidence), a market expansion induces a selection of more efficient firms (Hsieh *et al.*, 2016) and an increase in income leads each firm to increase its price (see Simonovska, 2015). One can verify that these three effects cannot jointly occur under any other specification, suggesting its usefulness to replicate empirically plausible results in monopolistic competition applications.

Welfare analysis We briefly investigate the impact on welfare of shocks to the decentralized economy.[17] Let us define the constant:

$$\Xi \equiv \int_b^a v(z)\, dF_{\rho s}(z),$$

which again only depends on preferences and the parameter κ. Then we can write equilibrium welfare as:

$$V = \Xi n - \theta(\rho).$$

[17] This has been the focus of the recent literature on trade liberalization (Arkolakis *et al.*, 2012, 2019; Bertoletti *et al.*, 2018; Feenstra, 2018; Fally, 2019).

Accordingly, there are only two channels by which welfare gains could materialize, that is either by changing the number of consumed varieties, or by affecting the price aggregator $dV = \Xi dn - \theta'(\rho)\, d\rho$.

As an example consider a shock to the market size. With directly additive preferences an increase of L generates both an increase of n and a welfare improving rise of ρ associated to the alledged selection effect (as in Arkolakis *et al.*, 2019). Instead, with indirectly additive preferences the whole welfare improvement comes only from the increase in the measure of consumed varieties n (as in Bertoletti *et al.*, 2018). Finally, when preferences are homothetic we know that the mass of consumed goods is fixed, since the linear impact on N is exactly offset by the selection effect of a reduction of $\widehat{c}$, which is associated to a welfare improving increase of ρ (as in Feenstra, 2018).

In general, the gains from globalization derive in part from an increase in the measure of the consumed varieties and in part from a reduction of the inconvenience of consumption made possible by the selection of cheaper goods. For $\beta(\rho) < 1$, using (41) we have $\Psi dn = -\theta'(\rho)[1-\beta(\rho)]\, d\rho$, and therefore:

$$dV = \left(\Xi + \frac{\Psi}{1-\beta(\rho)}\right) dn. \tag{48}$$

For instance, the impact of a market expansion on the measure of consumed varieties given by (44) can be immediately translated into a welfare effect by this formula. In particular, to obtain a (local) measure of the corresponding equivalent variation of income (dW), notice that $dV = -\theta'(\rho)\, \rho d\ln E$. Therefore we can compute:

$$d\ln W = \frac{1 + \bar{\eta}^{-1}[1-\beta(\rho)]}{1+\kappa-\beta(\rho)} d\ln L, \tag{49}$$

where $\bar{\eta} \equiv \frac{\Psi}{\Xi}$ is a weighted average of the elasticity $\eta(z)$.[18] The higher is this measure, the smaller is the equivalent variation of income needed to match the welfare gains provided by a larger market. Notice that we obtain $d\ln W = \frac{1}{\kappa} d\ln L$ for $\beta \to 1$, as in Arkolakis *et al.* (2012): indeed, under homotheticity a market expansion does not change the mass of consumed varieties, and there are only gains from selection.

In the Appendix we compute the average elasticity under GTP preferences as $\bar{\eta} = \frac{(1+\gamma)(1+\kappa)}{\gamma} > 1$. Then, a back of the envelop computation allows to put this result in perspective comparing the implications of different assumptions on preferences. Let us adopt a standard calibration with $\kappa = 5$ (as in Arkolakis *et al.*, 2019, and Bertoletti *et al.*, 2018) and $\gamma = 1$ (which delivers a linear demand and a 50% pass-through of cost changes), and consider as a benchmark the welfare gains under homotheticity (that is when $\beta \to 1$). Then, indirect additivity ($\beta \to -\infty$) implies welfare gains from market expansions that are 42% of those under the assumption of homotheticity, while direct additivity

[18] Note that

$$\bar{\eta} = \int_b^a \frac{\eta(z)\, v(z)}{\int_b^a v(z)\, dF_{\rho s}(z)} dF_{\rho s}(z),$$

which is a sort of normalized marginal utility of income.

($\beta = 0$) implies welfare gains that are 90% as under homotheticity. Increasing the demand elasticity (i.e. rising γ) increases the measure of the welfare gains.

1.2.2 Optimality

We conclude our analysis of GP preferences by considering the social planner problem with heterogeneous firms, which allows us to generalize results obtained by Dhingra and Morrow (2019) for the case of directly additive preferences, and by Bertoletti *et al.* (2018) for the case of indirectly additive preferences. It is well known that a condition for optimality is that the markup must be common across commodities to insure that the marginal rate of substitution between any two goods equals the ratio of marginal costs (the social marginal rate of transformation).[19] Therefore, we set $p = mc/E$, where m is the common markup. Then, we can write the social planner problem as follows:

$$\max_{N,\widehat{c},m} \left\{ V = N \int_0^{\widehat{c}} v\left(\rho \frac{mc}{E}\right) dG(c) - \theta\left(\rho\right) \right\} \tag{50}$$

$$s.t. \;\; N \int_0^{\widehat{c}} cx(c) L dG(c) = EL - N\left[F_e + FG(\widehat{c})\right],$$

$$\theta'\left(\rho\right) = N \int_0^{\widehat{c}} v'\left(\rho \frac{mc}{E}\right) \frac{mc}{E} dG\left(c\right),$$

$$x(c) = \frac{v'\left(\rho \frac{mc}{E}\right)}{N \int_0^{\widehat{c}} v'\left(\rho \frac{mc}{E}\right) \frac{mc}{E} dG\left(c\right)},$$

where the three constraints are respectively the resource constraint, the definition of the aggregator and the individual demand associated with GP preferences.

The solution depends on the average elasticity of the sub-utility:

$$\bar{\eta}(m, \rho, \widehat{c}) = \int_0^{\widehat{c}} \eta\left(\frac{\rho mc}{E}\right) \frac{v(\frac{\rho mc}{E})}{\int_0^{\widehat{c}} v\left(\frac{\rho mc}{E}\right) dG(c)} dG(c) \tag{51}$$

and can be summarized as follows:

Proposition 5. *Under Gorman-Pollak preferences and heterogeneous firms, the solution of the social planner problem is given by a pricing rule $p(c) = m^* c$, an aggregator ρ^*, a measure of firms N^* and a cut-off $\hat{c}^*$ satisfying:*

$$m^* = 1 + \frac{1}{\bar{\eta}^*},$$

$$N^* = \frac{EL}{\left(\bar{\eta}^* + 1\right)\left[F_e + FG(\widehat{c}^*)\right]},$$

[19] The proof ot this is similar to the one in Bertoletti *et al.* (2018, Appendix A) for the case of indirectly additive preferences.

and

$$\int_0^{\widehat{c}^*} \frac{v\left(\frac{m^*\rho^*}{E}c\right)}{v\left(\frac{m^*\rho^*}{E}\widehat{c}^*\right)} dG(c) = \frac{F_e}{F} + G(\widehat{c}^*)$$

with positive fixed costs, and $\widehat{c} = \frac{aE\bar{\eta}^*}{\rho(\bar{\eta}^*+1)}$ *without fixed costs, where* $\theta'(\rho^*) = \frac{m^*N^*}{E}\int_0^{\widehat{c}^*} v'\left(\frac{m^*c}{E}\rho^*\right) c dG(c)$ *and* $\bar{\eta}^* = \bar{\eta}(m^*, \rho^*, \widehat{c}^*)$ *is the average subutility elasticity.*

Intuitively, $\bar{\eta}^*$determines the measure of goods to be introduced in the economy before knowing the marginal cost at which they can be produced, while the optimal threshold for actual production $\widehat{c}^*$ depends on the ratio between the average incremental surplus and the one generated by the marginal firm. The comparison with the market equilibrium is rather simple: in the latter case product creation depends on the average demand elasticity (which determines the expected profits) and firm selection depends on the ratio between average and marginal actual profitability.

Again, further results can be obtained from additional assumptions on preferences and technology. In particular, CES preferences (11) imply that $\eta = \varepsilon - 1$ is constant and:

$$\frac{\pi(c)}{\pi(c')} = \frac{cx(c)}{c'x(c')} = \left(\frac{c}{c'}\right)^{1-\varepsilon} = \frac{v(\rho mc)}{v(\rho mc')},$$

for any c and c' of active firms ($c' > c$), which in turn confirms that the equilibrium of the Melitz model is efficient, as already known from Dhingra and Morrow (2019).

Further progress can be made under the assumption of a Pareto distribution. This implies a constant average elasticity $\bar{\eta}$ independently from the nature of preferences:

PROPOSITION 6. *Under Gorman-Pollak preferences with heterogeneous firms and a Pareto distribution of marginal costs, the optimal allocation implies* $\bar{\eta}^* = \kappa$.

Remarkably, the optimal markup depends only on the shape parameter of the Pareto distribution and decreases with it. The optimal number of firms decreases with the same parameter directly, but also with the optimal cut-off (when there are fixed costs) which depends on the shape parameter. The equilibrium is inefficient because low-cost firms choose too high prices and high-cost firms choose too low prices, but without further assumptions comparisons between equilibrium and optimal measures of goods created and consumed are ambiguous.

In the Appendix we derive closed form solutions for our specification of GTP preferences (14) without fixed costs, both for the decentralized equilibrium and the optimal allocation, showing that the number of created firms is optimal, but too many goods are consumed if:

$$\beta < \frac{\gamma \log\left(1+\frac{1}{\gamma}\right)}{\log\left(1+\frac{1}{\kappa}\right)}.$$

This holds under both direct and indirect additivity, as well as for any $\beta \leqslant 0$ or a positive but small enough β. Under homotheticity ($\beta = 1$) excess entry emerges if and only if $1+\frac{1}{\kappa} < (1+\frac{1}{\gamma})^{\gamma}$, a condition which holds if κ is sufficiently large (for instance if $\kappa \geq \gamma \geq 1$).

In a recent interesting work, Macedoni and Weinberger (2018) have employed these GPT preferences augmented with heterogeneity in quality rather than cost. Also in this case entry tends to be inefficient, with goods of too low quality provided in the market, and quality standards can be used to improve the allocation of resources. They also estimate the model to evaluate the welfare impact of actual quality standards.

2 Implicit CES preferences

In this section we consider monopolistic competition based on demand systems derived from implicit CES preferences (Gorman, 1970). These represent the other class of GAS preferences, which also belongs to the implicitly additive type studied by Hanoch (1975).[20] They generalize the CES case by having an elasticity of substitution that is common across goods but can possibly change across indifference curves. As far as we know, they have never been employed to analyze monopolistic competition, though, as we will show, they preserve some of the convenient properties of the explicit CES preferences, such as common markups across goods and static efficiency, while providing a more flexible comparative statics. In particular, implicit CES preferences can generate competition and selection effects due to entry which can amplify gains from trade in multi-country models *à la* Melitz (2003) and business cycle propagation in dynamic entry models *à la* Bilbiie *et al.* (2012).

We focus on preferences represented by the following direct and indirect utilities:

$$U = \left[\int_{\Omega} x(\omega)^{1-\epsilon(U)} d\omega\right]^{\frac{1}{1-\epsilon(U)}} \quad \text{and} \quad V = \left[\int_{\Omega} s(\omega)^{1-\varepsilon(V)} d\omega\right]^{\frac{1}{\varepsilon(V)-1}}, \tag{52}$$

where, differently from the explicit CES case, $\varepsilon(z) = 1/\epsilon(z) > 1$ is a function of the utility level. As long as this is not constant, preferences are non-homothetic since relative demands change with the utility level. Here, the regularity condition that (52) has to satisfy is that marginal utilities (disutilities) need to be

[20] General implicit additivity requires either a direct or an indirect utility function that is implicitly defined by an additive specification, and delivers demand systems depending on up to two aggregators, one of which is the utility itself (they include a homothetic family popularized by Kimball, 1995): see Bertoletti and Etro (2017b) for details.

positive (negative), which is guaranteed when $\epsilon'(U) < 0$ ($\varepsilon'(V) > 0$): see Fally (2018).

The demand system can be easily derived as follows:

$$s(\iota) = \frac{x(\iota)^{-\epsilon(U)}}{U^{1-\epsilon(U)}} \quad \text{and} \quad x(\iota) = \frac{s(\iota)^{-\varepsilon(V)}}{V^{\varepsilon(V)-1}},$$

which clearly confirms that these preferences belong to the GAS type. A firm producing with marginal cost c has variable profits:

$$\pi = \left[\frac{x^{-\epsilon(U)}E}{U^{1-\epsilon(U)}} - c\right] xL = \frac{(sE - c)}{V^{\varepsilon(V)-1}} s^{-\varepsilon(V)} L. \tag{53}$$

Its profit-maximizing price satisfies:

$$p = \frac{c}{1 - \epsilon(U)} = \frac{\varepsilon(V)\, c}{\varepsilon(V) - 1}, \tag{54}$$

therefore the markup is the same for all firms and changes with the utility index.

2.1 Monopolistic competition with homogeneous firms

Let us consider the free entry equilibrium when every firm has marginal cost c and pays a positive fixed cost of production F. It is standard to verify that the free entry equilibrium implies the price (54) and:

$$n = \frac{EL}{\varepsilon(V)\, F}, \tag{55}$$

where, using the implicit definition of the indirect utility, its equilibrium level V satisfies:

$$V = \frac{\varepsilon(V) - 1}{c} \left(\frac{E}{\varepsilon(V)}\right)^{\frac{\varepsilon(V)}{\varepsilon(V)-1}} \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon(V)-1}},$$

which we assume to have a unique solution. Whenever $\varepsilon'(V) > 0$ marginal cost changes are incompletely passed to prices, and an increase of income or market size increases utility, reduces markups and raises less than proportionally the number of goods provided. This delivers the following:

Proposition 7. *Under implicit CES preferences the equilibrium of monopolistic competition with free entry of homogeneous firms implies that an increase in utility (due to higher income or market size) is associated with a markup reduction and a less than proportional increase of the number of firms if* $\varepsilon'(V) > 0$.

Also in this case it is interesting to evaluate the optimal allocation of resources. This solves the problem:

$$\max_{n,s} \{V\} \quad s.t. \;\; V = \frac{n^{\frac{1}{\varepsilon(V)-1}}}{s},$$

$$(EL - nF)s \geq cL,$$

where the first constraint is the definition of utility after imposing symmetry and the second is the resource constraint. It is easy to verify that the first-order conditions for the solution can be rewritten as:

$$p^* = \frac{\varepsilon(V^*)\, c}{\varepsilon(V^*) - 1} \text{ and } n^* = \frac{EL}{\varepsilon(V^*)\, F}, \tag{56}$$

where the utility satisfies $V^* = (E/p^*)n^{*\frac{1}{\varepsilon(V^*)-1}}$, implying the same system of equations as in the equilibrium. Thus, we can conclude with:

PROPOSITION 8. *Under implicit CES preferences the equilibrium of monopolistic competition with free entry of homogeneous firms is optimal.*

This extends to the class of implicit CES preferences a result which is well-known since Spence (1976) and Dixit and Stiglitz (1977) to hold for the case of explicit CES preferences. The intuition relies on the fact that markups are constant across goods, which preserves the equality between price ratios and marginal rates of substitution between goods.[21]

2.2 Monopolistic competition with heterogeneous firms

We now consider the free entry equilibrium with firms differing in marginal costs as in Melitz (2003), and in our earlier analysis of GP preferences. Since implicit CES preferences lack a finite choke price, without fixed costs of production all the goods would be demanded. Therefore, it is interesting to focus on the relevant case in which firms face a positive fixed cost to produce, as in the original Melitz model.

For a given utility V, which here is the relevant aggregator, a c-firm using price p faces variable profits given by $\pi = \frac{(p-c)}{V^{\varepsilon(V)-1}} \left(\frac{p}{E}\right)^{-\varepsilon(V)} L$. Its optimal price $p(c)$ satisfies always the rule:

$$p(c) = \frac{\varepsilon(V)c}{\varepsilon(V) - 1}, \tag{57}$$

which implies a common markup across firms, which decreases with the utility level if and only if $\varepsilon' > 0$.

Let us write individual demand and variable profits for a c-firm as:

$$x(c) = \frac{(p(c)/E)^{-\varepsilon(V)}}{V^{\varepsilon(V)-1}},$$

[21] Interestingly, the optimality of the free entry equilibrium in a static setting does not extend automatically to a dynamic one with endogenous entry outside of the steady state. Whenever the markups change over time, for instance due to aggregate shocks, the allocation of resources to the entry process is suboptimal. Optimality can be restored with taxation of dividends or entry fees, which should vary with aggregate variables (see Etro, 2018, and Bilbiie *et al.*, 2019, for related discussions).

and

$$\pi\left(c\right)=\frac{\left[p(c)-c\right]}{V^{\varepsilon(V)-1}}\left(\frac{p(c)}{E}\right)^{-\varepsilon(V)}L.$$

The monotonicity of $\pi\left(c\right)$ with respect to c allows us to determine the threshold $\hat{c}$:

$$\pi(\hat{c})=F. \tag{58}$$

Free entry requires:

$$\int_0^{\hat{c}}[\pi(c)-F]dG(c)=F_e. \tag{59}$$

The equilibrium measure of entrant firms can be derived from the budget constraint by using (57) and (59) as:

$$N=\frac{EL}{\varepsilon(V)[FG(\hat{c})+F_e]}, \tag{60}$$

and the equilibrium level V computed by the utility expression:

$$V=\left[NE^{\varepsilon(V)-1}\int_0^{\hat{c}}p(c)^{1-\varepsilon(V)}dG(c)\right]^{\frac{1}{\varepsilon(V)-1}}, \tag{61}$$

where the measure of firms, prices and the cut-off satisfy the equilibrium conditions above. We assume that there is a unique equilibrium, as it is the case in the Melitz model.

Combining the equilibrium conditions (58) and (59), one can obtain the following relation between the threshold $\hat{c}$ and the equilibrium utility level V:

$$\int_0^{\hat{c}}\left(\frac{c}{\hat{c}}\right)^{1-\varepsilon(V)}dG(c)=\frac{F_e}{F}+G(\widehat{c}), \tag{62}$$

which is crucial to analyze selection effects. It is easy to verify that with explicit CES preferences there are none: in particular, an increase in utility associated to an increase in EL does not affect $\hat{c}$, while increasing proportionally the measure of consumed goods (as is well known, it takes costly trade to induce selection effects in the Melitz model). Consider now the case of variable elasticity: as long as utility increases, there must be a reduction (increase) in the cut-off $\widehat{c}$ if $\varepsilon(V)$ is increasing (decreasing) in utility. We immediately obtain:

PROPOSITION 9. *Under implicit CES preferences and heterogeneous firms the equilibrium of monopolistic competition with free entry is given by a pricing rule $p(c)$ satisfying (57), a utility V satisfying (61), a measure of firms N satisfying (60), and a cut-off $\hat{c}$ satisfying (62).* A*n increase in utility (due to higher income or market size) is associated with a markup reduction and a selection of the more efficient firms if $\varepsilon'(V)>0$.*

The case where the elasticity of substitution increases with utility is one where the pro-competitive effects of trade liberalization are far from being elusive. When consumer income increases, this model delivers a generalized reduction in all markups and prices, implying countercylical markups in a dynamic extension.[22] For this reason, the model could be usefully applied to macroeconomic analysis with variable markups that amplify the propagation of the business cycle (as in Cavallari and Etro, 2018) after an appropriate estimate of the elasticity function.

We can finally analyze the social planner problem as we have done previously for the GP preferences. Again, the optimal markup must be constant across goods, say m. Then, the problem can be written as:

$$\max_{N,\widehat{c},m}\{V\} \quad s.t. \;\; V=\left[N\int_0^{\widehat{c}}\left(\frac{mc}{E}\right)^{1-\varepsilon(V)}dG(c)\right]^{\frac{1}{\varepsilon(V)-1}},$$

$$N\int_0^{\widehat{c}}cx(c)LdG(c)=EL-N\left[F_e+FG(\widehat{c})\right],$$

$$x(c)=\frac{\left(\frac{mc}{E}\right)^{-\varepsilon(V)}}{V^{\varepsilon(V)-1}}.$$

where we emphasized the resource constraint and the demand functions. In the Appendix we show that this problem delivers optimal values $(N^*,\widehat{c}^*,m^*,V^*)$ that correspond to the ones of the unique equilibrium:

PROPOSITION 10. *Under implicit CES preferences the equilibrium of monopolistic competition with free entry of heterogeneous firms is optimal.*

The important work by Dhingra and Morrow (2019) has recently proved the optimality of the equilibrium of the Melitz model in a closed economy with CES preferences and heterogeneous firms. Their result naturally extends to the entire class of implicit CES preferences.

We conclude this section noticing that preferences as these can be exploited for a variety of applications. One can endogenize the quality of goods generating richer implications for differences between firms and selection effects. The case of costly trade can be addressed as in Melitz (2003) generating new forms of selection effects: in particular, opening up to trade can reduce markups, fostering gains from trade liberalization. Moreover, one could introduce implicit CES preferences in flexible price macroeconomic models: while optimality would be lost due to changes in markup across periods, the propagation of shocks would be affected by this same variability, and indeed amplified when the demand elasticity is increasing, especially in case of endogenous entry *à la* Bilbiie *et al.* (2012).[23]

[22] Of course, the opposite results are associated to the case where $\varepsilon'(V)<0$.

[23] See also Cavallari and Etro (2018) and Etro (2018) for models with homogeneous firms and Ghironi and Melitz (2005) for an early model with heterogeneous firms.

3 Conclusion

Most of modern economic theory with monopolistic competition is based on CES microfoundations that heavily limit its empirical relevance. Our contribution aims at introducing more general microfoundations with empirically relevant implications. Applications to trade and macroeconomics could fruitfully employ GAS preferences to explore the role of demand in affecting gains from trade, the impact of aggregate shocks and the policies needed to restore efficiency.

Future research may also study monopolistic competition under more general conditions on the demand side (e.g. demands featuring more than one aggregator, as in the case of implicit additivity, or without explicit aggregators), as well as on the supply side (including for instance variable marginal costs), and under heterogeneity between consumers.

References

d'Aspremont, Claude and Rodolphe Dos Santos Ferreira, 2016, Oligopolistic vs. Monopolistic Competition: Do intersectoral effects matter?, *Economic Theory*, 62, 1, 299-324

d'Aspremont, Claude and Rodolphe Dos Santos Ferreira, 2017, The Dixit–Stiglitz Economy with a 'Small Group' of Firms: A simple and robust equilibrium markup formula, *Research in Economics*, 71, 4, 729-39

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson and Andrés Rodríguez-Clare, 2019, The Elusive Pro-Competitive Effects of Trade, *Review of Economic Studies*, 86, 1, 46-80

Arkolakis, Costas, Arnaud Costinot and Andrés Rodriguez-Clare, 2012, New Trade Models, Same Old Gains?, *American Economic Review*, 102, 1, 94-130

Bertoletti, Paolo and Paolo Epifani, 2014, Monopolistic Competition: CES Redux?, *Journal of International Economics*, 93, 2, 227-38

Bertoletti, Paolo and Federico Etro, 2016, Preferences, Entry and Market Structure, *The RAND Journal of Economics*, 47, 4, 792-821

Bertoletti, Paolo and Federico Etro, 2017a, Monopolistic Competition when Income Matters, *The Economic Journal*, 127, 603, 1217-43

Bertoletti, Paolo and Federico Etro, 2017b, Monopolistic Competition, as You Like It, WP 08/2017, Venice, Ca' Foscari University

Bertoletti, Paolo, Federico Etro and Ina Simonovska, 2018, International Trade with Indirect Additivity, *American Economic Journal: Microeconomics*, 10, 2, 1-57

Bilbiie, Florin, Fabio Ghironi and Marc Melitz, 2012, Endogenous Entry, Product Variety, and Business Cycles, *Journal of Political Economy*, 120, 2, 304-45

Bilbiie, Florin, Fabio Ghironi and Marc Melitz, 2019, Monopoly Power and Endogenous Variety: Distortions and Remedies, *American Economic Journal: Macroeconomics*, in press

Boucekkine, Raouf, Hélène Latzer and Mathieu Parenti, 2017, Variable Markups in the Long-Run: A Generalization of Preferences in Growth Models, *Journal of Mathematical Economics*, 68C, 80-6

Cavallari, Lilia and Federico Etro, 2018, Demand, Markups and the Business Cycle, DISEI WP 2018-30, University of Florence

De Loecker, Jan, Pinelopi Goldberg, Amit Khandelwal and Nina Pavcnik, 2016, Prices, Markups, and Trade Reform, *Econometrica*, 84, 2, 445-510

Dhingra, Swathi and John Morrow, 2019, Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity, *Journal of Political Economy*, 127, 1, 196-232

Dixit, Avinash and Joseph Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, *The American Economic Review*, 67, 297-308

Etro, Federico, 2018, Macroeconomics with Endogenous Markups and Optimal Taxation, *Southern Economic Journal*, 85, 2, 378-406

Fally, Thibault, 2018, Integrability and Generalized Separability, mimeo, University of California at Berkeley

Fally, Thibault, 2019, Generalized Separability and the Gains from Trade, *Economics Letters*, 178, 8-12

Feenstra, Robert, 2003, A Homothetic Utility Function for Monopolistic Competition Models without Constant Price Elasticity, *Economics Letters*, 78, 1, 79-86

Feenstra, Robert, 2018, Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity, *Journal of International Economics*, 110(C), 16-27

Ghironi, Fabio and Marc Melitz, 2005, International Trade and Macroeconomic Dynamics with Heterogenous Firms, *Quarterly Journal of Economics*, 120, 865-915

Gorman, William Moore (Terence), 1970, Conditions for Generalized Additive Separability, in Charles Blackorby and Anthony F. Shorrocks Eds., 1995, *Collected Works of W.M. Gorman: Separability and Aggregation*, Oxford: Oxford University Press, Vol. 1, 186-207

Gorman, William Moore (Terence), 1987, Separability, *The New Palgrave: A Dictionary of Economics*, 4, London: Macmillan Press, 305-11

Hanoch, Giona, 1975, Production and Demand Models with Direct or Indirect Implicit Additivity, *Econometrica*, 43, 3, 395-419

Hsieh, Chang-Tai, Nicholas Li, Ralph Ossa and Mu-Jeung Yang, 2016, Accounting for the New Gains from Trade Liberalization, NBER WP 22069

Kimball, Miles, 1995, The Quantitative Analytics of the Basic Neomonetarist Model, *Journal of Money, Credit and Banking*, 27, 1241-1277

Krugman, Paul, 1980, Scale Economies, Product Differentiation, and the Pattern of Trade, *The American Economic Review*, 70, 950-9

Macedoni, Luca and Ariel Weinberger, 2018, Quality Heterogeneity and Misallocation: the benefits of raising your standards, mimeo, University of Oklahoma

Matsuyama, Kiminori and Philip Ushchev, 2017, Beyond CES: Three Alternative Classes of Flexible Homothetic Demand Systems, CEPR Discussion Paper 12210

Melitz, Marc, 2003, The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, *Econometrica*, 71, 6, 1695-725

Melitz, Marc and Gianmarco Ottaviano, 2008, Market Size, Trade, and Productivity, *Review of Economic Studies*, 75, 1, 295-316

Mrázová, Monika and Peter Neary, 2018, Selection Effects with Heterogeneous Firms, *Journal of the European Economic Association*, in press

Mrázová, Monika and Peter Neary, 2019, IO for Export(s), Department of Economics, Discussion Paper 868, Oxford

Pollak, Robert, 1972, Generalized Separability, *Econometrica*, 40, 3, 431-53

Simonovska, Ina, 2015, Income Differences and Prices of Tradables, *Review of Economic Studies*, 82, 4, 1612-56

Spence, Michael, 1976, Production Selection, Fixed Costs, and Monopolistic Competition, *Review of Economic Studies*, 43, 2, 217-35

Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti and Jacques-François Thisse, 2012, Monopolistic Competition in General Equilibrium: Beyond the CES, *Econometrica*, 80, 6, 2765-84

Appendix A: Proofs

Proof of Proposition 1. We can combine the equilibrium relations to obtain the following equation for $z = \rho s$:

$$f \equiv \theta' \left(\frac{z \left[\varepsilon \left(z \right) - 1 \right] E}{\varepsilon \left(z \right) c} \right) - \frac{v' \left(z \right) cL}{\left[\varepsilon \left(z \right) - 1 \right] F} = 0.$$

This gives:

$$\begin{aligned} \frac{\partial f}{\partial z} &= \frac{\theta'' E}{c} \left(\frac{\varepsilon \left[\varepsilon - 1 \right] + z\varepsilon' \varepsilon - z\varepsilon' \left[\varepsilon - 1 \right]}{\varepsilon^2} \right) - \frac{cL}{F} \frac{v'' \left[\varepsilon - 1 \right] - \varepsilon' v'}{\left[\varepsilon - 1 \right]^2} \\ &= \frac{\left(2\varepsilon - \zeta \right) \theta' \left(\rho \right) \left(\varepsilon - 1 \right) \left(\varepsilon - \beta \right)}{z}, \end{aligned}$$

which is negative under the assumption $\beta \leqslant 1 < \varepsilon$. Since:

$$\frac{\partial f}{\partial L} = - \frac{v' c}{\left(\varepsilon - 1 \right) F} > 0, \quad \frac{\partial f}{\partial E} = \theta'' \frac{\rho}{E} \quad \text{and} \quad \frac{\partial f}{\partial c} = \frac{\theta'}{c} \left(\beta - 1 \right),$$

using (38), we get the comparative statics:

$$sign \left\{ \frac{\partial \ln p}{\partial \ln L} \right\} = sign \left\{ \frac{-\varepsilon'(z)}{\varepsilon(z) - \beta(\rho)} \right\} = sign \left\{ \frac{\partial \ln n}{\partial \ln L} - 1 \right\},$$

$$sign \left\{ \frac{\partial \ln p}{\partial \ln E} \right\} = sign \left\{ \frac{-\varepsilon'(z) \beta \left(\rho \right)}{\varepsilon(z) - \beta(\rho)} \right\} = sign \left\{ \frac{\partial \ln n}{\partial \ln E} - 1 \right\},$$

and

$$sign \left\{ \frac{\partial \ln p}{\partial \ln c} - 1 \right\} = sign \left\{ \frac{\varepsilon'(z) \left[\beta \left(\rho \right) - 1 \right]}{\varepsilon(z) - \beta(\rho)} \right\} = sign \left\{ \frac{\partial \ln n}{\partial \ln c} \right\},$$

which immediately implies the results under the assumptions that $\varepsilon'(z) > 0$ and $\beta(\rho) \leqslant 1 < \varepsilon(z)$. $\square$

Proof of Proposition 2. The optimal allocation problem is given by:

$$\max_{n,s}\left\{V=nv\left(\rho s\right)-\theta\left(\rho\right)\right\}$$

$$s.t.\quad\left(EL-nF\right)s\geq cL\quad\text{and}\quad nv'\left(\rho s\right)s=\theta'\left(\rho\right),$$

where the first constraint is the resource constraint and the second one defines the aggregator under symmetry. The two first-order conditions characterize the solution in the text, which is assumed to exist and be unique.

The symmetric equilibrium normalized s and effective $z=\rho s$ price satisfy the following relation between s and z:

$$s=\frac{\varepsilon\left(z\right)}{\varepsilon\left(z\right)-1}\frac{c}{E},$$

while optimality alternatively requires the relation:

$$s=\frac{1+\eta\left(z\right)}{\eta\left(z\right)}\frac{c}{E}.$$

The first function lies everywhere above (below) the second in the space (z,s) if $\eta'(z)\propto 1+\eta(z)-\varepsilon(z)>(<)\,0$ everywhere.

Both in the equilibrium and in the optimal allocation $\{s,\rho,n\}$ must satisfy the aggregator definition $\theta'\left(\rho\right)=nv'\left(\rho s\right)s$ and the zero profit condition $(p-c)EL=npF$ (notice that last expression implies a positive relation between p and n). Eliminating the number of firms these conditions can be summarized by:

$$v'\left(z\right)(sE-c)L=F\theta'\left(\frac{z}{s}\right),$$

which provides a second relation between s and z. If the latter increases in the space (z,s) excess (insufficient) entry occurs if $\eta'(z)>(<)\,0$ everywhere, because the pricing relation imply that the equilibrium value of s must be above (below) s^*, and then n must be above (below) n^*. But by total differentiation we get:

$$\frac{ds}{dz}=\frac{\varepsilon(z)-\beta(\rho)}{\left[\frac{p}{p-c}-\beta(\rho)\right]\rho}>0$$

under the assumption $\beta(\rho)\leqslant 1<\varepsilon(z)$ (the denominator is positive since $p>c$ to satisfy the zero profit condition), which concludes the proof. □

Proof of Proposition 3. Defining the equilibrium value of the normalized aggregator as:

$$\tilde{\rho}=\frac{\rho}{E},$$

the market equilibrium $\{p(c),\tilde{\rho},N,\widehat{c}\}$ can be rewritten through the following equations:

$$\theta'\left(\tilde{\rho}E\right)E=N\int_0^{\widehat{c}}v'\left(\tilde{\rho}p\left(c\right)\right)p\left(c\right)dG\left(c\right),\tag{63}$$

$$p(c) = \frac{\varepsilon\left(\tilde{\rho}p\left(c\right)\right)c}{\varepsilon\left(\tilde{\rho}p\left(c\right)\right)-1} \tag{64}$$

$$\left(p(\widehat{c})-\widehat{c}\right)v'\left(\tilde{\rho}p\left(\widehat{c}\right)\right)L = \theta'\left(\tilde{\rho}E\right)F. \tag{65}$$

$$N = \frac{EL}{\overline{\varepsilon}\left[F_e + FG(\widehat{c})\right]}, \tag{66}$$

where

$$\overline{\varepsilon} = \left[\int_0^{\widehat{c}} \frac{1}{\varepsilon(\tilde{\rho}p\left(c\right))} \frac{v'\left(\tilde{\rho}p\left(c\right)\right)p\left(c\right)}{\int_0^{\widehat{c}} v'\left(\tilde{\rho}p\left(c\right)\right)p\left(c\right)dG\left(c\right)} dG\left(c\right)\right]^{-1}. \tag{67}$$

Under our assumptions, (64) implies that the pricing rule is uniquely determined by $\tilde{\rho}$, and this in turn implies, by (67), that $\overline{\varepsilon}$ only depends on $\widehat{c}$ and $\tilde{\rho}$. Combining (63) and (66) we reduce the equilibrium system to

$$\theta'\left(\tilde{\rho}E\right)\overline{\varepsilon}\left[F_e + FG(\widehat{c})\right] = L\int_0^{\widehat{c}} v'\left(\tilde{\rho}p\left(c\right)\right)p\left(c\right)dG\left(c\right), \tag{68}$$

$$\left(p(\widehat{c})-\widehat{c}\right)v'\left(\tilde{\rho}p\left(\widehat{c}\right)\right)L = \theta'\left(\tilde{\rho}E\right)F. \tag{69}$$

Suppose that $F > 0$. When preferences are directly additive $\theta'\left(\rho\right) = -1$ and, therefore, conditions (68) and (69) determine $\tilde{\rho}$ and $\widehat{c}$ independently from income E. Thus neither pricing nor firm selection are affected by income E (and N is linear with respect to E). When preferences are indirectly additive $\rho = 1$ and $\tilde{\rho} = 1/E$. Then, by (64) pricing only depends on income and (68) and (69) reduce to a single equation which determine the threshold $\widehat{c}$ as a function of E that does not depend on market size L (and N is linear with respect to L). Finally, when preferences are homothetic $\theta'\left(\tilde{\rho}E\right) = -1/\tilde{\rho}E$ and, accordingly, conditions (68) and (69) determine $\tilde{\rho}$ and $\widehat{c}$ as a function of EL.

Suppose now that $F = 0$: then by (69) $\widehat{c} = a/\tilde{\rho}$ and (68) determines $\tilde{\rho}$, which is independent from E under direct additivity, and depends only on EL when preferences are homothetic. Finally, under indirect additivity of preferences $\tilde{\rho} = 1/E$ and $\widehat{c} = aE$, which is again independent from L, concluding the proof. □

Proof of Proposition 4. Immediate from the comparatives statics in the text and the fact that, under $\varepsilon'(z) > 0$, prices defined by (37) are increasing in $\widehat{c}$. □

Proof of Proposition 5. Combining the constraints of the social planner problem we obtain the following expression for the markup:

$$m = \frac{EL}{EL - N\left[F_e + FG(\widehat{c})\right]}. \tag{70}$$

Using this, the social planner problem reduces to:

$$\max_{N,\widehat{c}}\left\{N\int_0^{\widehat{c}} v\left(\frac{\rho cL}{EL - N\left[F_e + FG(\widehat{c})\right]}\right)dG(c) - \theta\left(\rho\right)\right\},$$

where the aggregator must satisfy the definition in the text, but its changes do not affect the objective function.

If there are positive fixed costs of production (namely, if $F > 0$), the two first-order conditions can be solved for:

$$N = \frac{EL}{[\bar{\eta}(m, \rho, \widehat{c}) + 1]\,[F_e + FG(\widehat{c})]}, \tag{71}$$

and:

$$\int_0^{\widehat{c}} \frac{v\left(\frac{\rho mc}{E}\right)}{v\left(\frac{\rho m\widehat{c}}{E}\right)} dG(c) = \frac{F_e}{F} + G(\widehat{c}),$$

where $\bar{\eta}$ is an "average" of the elasticities $\eta(z)$ given by (51), whose weights are the corresponding shares of the incremental social benefit. Using (70) and (71) we can also rewrite the optimal markup as:

$$m = 1 + \frac{1}{\bar{\eta}(m, \rho, \widehat{c})}. \tag{72}$$

When there are no fixed costs of production (namely, if $F = 0$) the social planner problem simplifies to:

$$\max_{N, \widehat{c}} \left\{ N \int_0^{\widehat{c}} v\left(\frac{cL\rho}{EL - NF_e}\right) dG(c) - \theta(\rho) \right\},$$

and in this case it is always optimal to consume any good that provides positive subutility, so that the optimal cut-off must satisfy:

$$\widehat{c} = \frac{a\,(EL - NF_e)}{\rho L}.$$

Given this, the planner problem simplifies further to:

$$\max_{N} \left\{ N \int_0^{\frac{a(EL - NF_e)}{\rho L}} v\left(\frac{cL}{EL - NF_e}\rho\right) dG(c) - \theta(\rho) \right\},$$

whose first-order condition gives:

$$N = \frac{EL}{[\bar{\eta}(m, \rho, \widehat{c}) + 1]\,F_e}. \tag{73}$$

This is consistent with the result under positive fixed cost, and implies the same markup expression as there, i.e., $m = 1 + 1/\bar{\eta}(m, \rho, \widehat{c})$. Replacing in the earlier expression for the cut-off, we finally have:

$$\widehat{c} = \frac{aE\bar{\eta}(m, \rho, \widehat{c})}{\rho\,[\bar{\eta}(m, \rho, \widehat{c}) + 1]}. \tag{74}$$

which concludes the proof. $\square$

Proof of Proposition 6. Integration by parts delivers:

$$\int_0^{\widehat{c}} v'\left(\rho s(c)\right)\rho s(c) dG(c) = -\int_0^{\widehat{c}} v\left(s(c)\right)\left[g(c)+cg'(c)\right]dc,$$

which allows us to rewrite the average elasticity as:

$$\bar{\eta}(m,\rho,\widehat{c}) = \frac{\int_0^{\widehat{c}} v\left(\rho\frac{mc}{E}\right)\left[g(c)+cg'(c)\right]dc}{\int_0^{\widehat{c}} v\left(\rho\frac{mc}{E}\right)g(c)dc}, \tag{75}$$

where the role of cost distribution in shaping the optimal markup emerges more clearly. Under the assumption of a Pareto distribution $g'(c) = (\kappa-1)g(c)/c$, therefore we obtain $\bar{\eta}(m,\rho,\widehat{c}) = \kappa$. □

Proof of Proposition 7. Immediate from the analysis in the text for $\varepsilon(V) > 1$ after noticing that $\frac{\partial \ln p}{\partial \ln V} = \frac{-\varepsilon'(V)V}{[\varepsilon(V)-1]\varepsilon(V)} = \frac{1}{\varepsilon(V)-1}\frac{\partial \ln n}{\partial \ln V}$ is negative whenever $\varepsilon'(V) > 0$. □

Proof of Proposition 8. Using the constraint as an equality, the social planner problem simplifies to:

$$\max_s\{V\} \quad s.t. \;\; V = \frac{\left(\frac{EL}{F}-\frac{cL}{sF}\right)^{\frac{1}{\varepsilon(V)-1}}}{s},$$

whose optimality condition:

$$\frac{cL\left(\frac{EL}{F}-\frac{cL}{sF}\right)^{\frac{1}{\varepsilon(V)-1}-1}}{s^3F\left[\varepsilon(V)-1\right]} = \frac{\left(\frac{EL}{F}-\frac{cL}{sF}\right)^{\frac{1}{\varepsilon(V)-1}}}{s^2}$$

simplifies to:

$$s = \frac{c\varepsilon(V)}{E\left[\varepsilon(V)-1\right]}.$$

Combined with the constraint $n = \frac{EL}{F} - \frac{cL}{sF}$ it provides the expressions in the text. □

Proof of Proposition 9. By total differentiation of (62) we have:

$$\frac{d\widehat{c}}{dV} = \frac{\varepsilon'(V)\int_0^{\hat{c}}\left(\frac{c}{\hat{c}}\right)^{1-\varepsilon(V)}\log\left(\frac{c}{\hat{c}}\right)dG(c)}{\left[\varepsilon(V)-1\right]\hat{c}^{\varepsilon(V)}\int_0^{\hat{c}} c^{1-\varepsilon(V)}dG(c)} \lesseqgtr 0 \text{ if } \varepsilon'(V) \gtreqless 0$$

which immediately implies the result. □

Proof of Proposition 10. Let us consider the social planner problem. By combining the resource constraint, the demand function and the implicit definition of V we obtain that the markup must satisfy:

$$m = \frac{EL}{EL - N\left[F_e + FG(\widehat{c})\right]},$$

and the problem reduces to:

$$\max_{N,\widehat{c}}\left\{V=\frac{EL-N\left[F_e+FG(\widehat{c})\right]}{L}\left[N\int_0^{\widehat{c}}c^{1-\varepsilon(V)}dG(c)\right]^{\frac{1}{\varepsilon(V)-1}}\right\}.$$

The first-order condition with respect to N gives:

$$N=\frac{EL}{\varepsilon(V)[FG(\hat{c})+F_e]},$$

which implies the markup:

$$m=\frac{\varepsilon(V)}{\varepsilon(V)-1}.$$

This reduces the above problem to:

$$\max_{\widehat{c}}\left\{V=[\varepsilon(V)-1]\left[\left(\frac{E}{\varepsilon(V)}\right)^{\varepsilon(V)}L\int_0^{\widehat{c}}\frac{c^{1-\varepsilon(V)}}{FG(\hat{c})+F_e}dG(c)\right]^{\frac{1}{\varepsilon(V)-1}}\right\},$$

whose first-order condition with respect to $\widehat{c}$ satisfies:

$$\widehat{c}^{1-\varepsilon(V)}\left[FG(\hat{c})+F_e\right]=\int_0^{\widehat{c}}c^{1-\varepsilon(V)}dG(c)F,$$

which is equivalent to the equilibrium condition (62). □

Appendix B: Generalized translated power preferences

We consider the specification of GPT preferences for the model with heterogeneous firms under a Pareto distribution of unit costs. We reproduce the indirect utility function:

$$V=\int_\Omega\frac{(a-\rho s(\omega))^{1+\gamma}}{1+\gamma}d\omega-\frac{\rho^{1-\beta}}{\beta-1}.$$

This specification becomes directly additive for $\beta=0$, indirectly additive for $\beta\to\infty$, and homothetic for $\beta\to 1$. The demand functions are perfectly rigid for $\gamma\to 0$, linear for $\gamma=1$ and perfectly elastic for $\gamma\to\infty$.

Given the individual demand $x=[a-\rho s]^\gamma\rho^\beta$, where

$$\rho^{-\beta}=\int_\Omega(a-\rho s(\omega))^\gamma s(\omega)d\omega,$$

it is easy to verify that a $c-$firm adopts the following pricing rule:

$$p(c)=\frac{\gamma c+\widehat{c}}{\gamma+1}\quad\text{with}\quad\widehat{c}=\frac{aE}{\rho},\tag{76}$$

which resembles the one obtained under indirect additivity by Bertoletti *et al.* (2018), except for the presence of the aggregator ρ, that drives the properties of this more general model (notice that $\varepsilon(z) = \gamma z/(a-z)$ with $\varepsilon'(z) > 0$).

Exploiting the properties of the Pareto distribution, we can compute the expected profits as follows:

$$\begin{aligned}\mathbb{E}\{\pi(c)\} &= \int_0^{\hat{c}} \pi(c)dG(c) = \\ &= \frac{\kappa\gamma^\gamma\rho^{\beta+\gamma}EL}{\overline{c}^\kappa}\left(\frac{1}{(1+\gamma)E}\right)^{1+\gamma}\int_0^{\hat{c}}(\hat{c}-c)^{\gamma+1}c^{\kappa-1}dc.\end{aligned}$$

The latter expression can be integrated by substitution (using $t = c/\hat{c}$) to get:

$$\begin{aligned}\mathbb{E}\{\pi(c)\} = \frac{\kappa\gamma^\gamma\hat{c}^{\gamma+\kappa+1}\rho^{\beta+\gamma}EL}{\overline{c}^\kappa}\left(\frac{1}{(1+\gamma)E}\right)^{1+\gamma}\int_0^1 t^{\kappa-1}(1-t)^{1+\gamma}dt \\ = \frac{\kappa\gamma^\gamma a^{\gamma+\kappa+1}E^{\kappa+1}L}{\overline{c}^\kappa(1+\gamma)^{1+\gamma}\rho^{\kappa-\beta+1}}B(\kappa,\gamma+2),\end{aligned}$$

where $B(z,h) = \int_0^1 t^{z-1}(1-t)^{h-1}dt$ is the Euler Beta function (such that $B(z+1,h) = zB(z,h)/(z+h)$ and $B(z,h+1) = hB(z,h)/(z+h)$). By using

$$1 = \rho^\beta N\int_0^{\hat{c}}(a-\rho s(c))^\gamma s(c)dG(c)$$

and the free entry condition we can solve for the aggregator as:

$$\rho = \left[\frac{\kappa\gamma^\gamma a^{\gamma+\kappa+1}E^{\kappa+1}L}{\overline{c}^\kappa(1+\gamma)^{\gamma+1}F_e}B(\kappa,\gamma+2)\right]^{\frac{1}{1+\kappa-\beta}}.$$

This allows us to obtain the cut-off:

$$\hat{c} = \left[\frac{\overline{c}^\kappa(1+\gamma)^{\gamma+1}F_e}{\kappa\gamma^\gamma a^{\gamma+\beta}B(\kappa,\gamma+2)LE^\beta}\right]^{\frac{1}{1+\kappa-\beta}}, \tag{77}$$

as a function of both income E and per capita entry cost F_e/L. Using the definition of the aggregator we can compute:

$$1 = N\int_0^{\hat{c}}\left[a-\frac{\rho}{E}p(c)\right]^\gamma\frac{\rho^\beta}{E}p(c)dG(c),$$

or

$$\overline{c}^\kappa(1+\gamma)^{\gamma+1}E^{\gamma+1} = \kappa\gamma^\gamma\widehat{c}^{\gamma+1}\rho^{\beta+\gamma}N\int_0^{\hat{c}}\left(1-\frac{c}{\widehat{c}}\right)^\gamma\left(c^{\kappa-1}+\gamma\frac{c^\kappa}{\widehat{c}}\right)dc.$$

Integrating by substitution we obtain:

$$\overline{c}^\kappa(1+\gamma)^{\gamma+1}E^{\gamma+1} = \kappa\gamma^\gamma\widehat{c}^{\kappa+\gamma+1}\rho^{\beta+\gamma}N\left[\int_0^1 t^{\kappa-1}(1-t)^\gamma dt+\gamma\int_0^1 t^\kappa(1-t)^\gamma dt\right],$$

which can be solved for the measure of entrant firms as:

$$N = \frac{EL}{(\kappa+1)\,F_e}$$

after substituting for ρ and using the properties of the Beta function. We obtain therefore an average demand elasticity $\bar{\varepsilon} = \kappa + 1$ which depends only on the Pareto parameter (and implies that the equilibrium measure of created goods is the same as the optimal one). It is then easy to compute the measure of consumed goods $n = NG(\widehat{c})$ as:

$$n = \frac{\overline{c}^{\frac{\kappa(\beta-1)}{\kappa-\beta+1}}\,(\gamma+1)^{\frac{(\gamma+1)\kappa}{\kappa-\beta+1}}\left(\frac{L}{F_e}\right)^{\frac{1-\beta}{\kappa-\beta+1}} E^{\frac{(\kappa+1)(1-\beta)}{1+\kappa-\beta}}}{(\kappa+1)\left[\kappa\gamma^{\gamma}a^{\beta+\gamma}B(\kappa,\gamma+2)\right]^{\frac{\kappa}{1+\kappa-\beta}}}.$$

This is constant whenever preferences are homothetic ($\beta = 1$), linear with respect to E when preferences are directly additive ($\beta = 0$) and more than proportional in income when they are indirectly additive ($\beta \to \infty$). Equilibrium welfare reads as:

$$V = \frac{\kappa\gamma^{\gamma+1}a^{\gamma+1}B\left(\kappa,\gamma+2\right)}{\left(1+\gamma\right)^{2+\gamma}}n + \frac{\rho^{1-\beta}}{1-\beta}.$$

Since under GTP preferences we have $h(z) = \frac{(\gamma+1)z-a}{\gamma}$ and we can compute $F_{\rho s}\left(z\right) = \left[\frac{(\gamma+1)z}{a\gamma} - \frac{1}{\gamma}\right]^{\kappa}$, we can also derive:

$$\bar{\eta} = \frac{(1+\gamma)(1+\kappa)}{\gamma}$$

which provides the following equivalent variation for the welfare gains from a market expansion (49):

$$d\ln W = \frac{1+\frac{\gamma(1-\beta)}{(\gamma+1)(\kappa+1)}}{1+\kappa-\beta}d\ln L. \tag{78}$$

Moving to the social planner problem, we can refer to the results in the text and compute the optimal value of the aggregator as:

$$\rho^{*} = \left[\frac{\kappa^{\kappa+2}a^{\kappa+\gamma+1}LE^{\kappa+1}}{(\gamma+1)\,\overline{c}^{\kappa}(1+\kappa)^{\kappa+1}F_e}B(\kappa,\gamma+2)\right]^{\frac{1}{1+\kappa-\beta}},$$

and the optimal the cut-off:

$$\widehat{c}^{*} = \left[\frac{(\gamma+1)\,\overline{c}^{\kappa}(1+\kappa)^{\beta}F_e}{\kappa^{\beta+1}a^{\gamma+\beta}B(\kappa,\gamma+2)LE^{\beta}}\right]^{\frac{1}{1+\kappa-\beta}}, \tag{79}$$

which differs from the equilibrium one in general. Considering the region with $\beta \leq 1$, this implies that too many goods are consumed if:

$$\beta < \frac{\gamma \log\left(1 + \frac{1}{\gamma}\right)}{\log\left(1 + \frac{1}{\kappa}\right)}.$$

which holds for a small enough β.