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Lisa Grazzini and Alessandro Petretto

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Heterogeneous Capital Tax Competition in a Federation with Tax Evasion

Lisa Grazzini*and Alessandro Petretto†

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Abstract

In a federal country with two regions, consumers can decide not only the region where to invest, but also the type of capital investment. We analyse how such decision is affected by two sources of asymmetry: a first type of capital is taxed at a regional level while a second one is taxed at a federal level, and for the latter a different degree of tax evasion may arise across regions. We show how tax evasion arising at a federal level affects not only the federal tax policy but also the regional tax policies both directly and indirectly because of vertical tax competition. In particular, we show under which conditions a decrease in the level of tax compliance on the second type of capital can lead to a reduction in its federal tax rate, and simultaneously to an increase in the regional tax rate on the other type of capital investment.

Keywords: Fiscal federalism; Tax Competition, Tax evasion.
JEL Classification: H2; H41; H71; H77.

*Department of Economics and Management, University of Florence, Via delle Pandette 9, 50127 Florence, Italy. lisa.grazzini@unifi.it.

†Department of Economics and Management, University of Florence, Via delle Pandette 9, 50127 Florence, Italy. alessandro.petretto@unifi.it.

1 Introduction

Capital is generally recognized as highly mobile, and in a federal country capital flows across regions are significantly influenced by tax policies chosen at a federal and local level. However, in their activity to impose taxes on different types of tax bases, federal and local governments' decisions are constrained not only by the intensity of both horizontal and vertical tax competition which typically arise in federal set-ups, but also by dissimilar tax evasion opportunities across different regions.[1] These can be due to several reasons: For example, in regions that are dissimilar with respect to their historical and/or cultural background, citizens may have a different propensity to tax compliance or the efficiency in rasing local taxes may be dissimilar depending on different regional tax authorities.[2] Taxes can also be more easily evaded when they are issued by one level of government rather than another one (again for possible different levels of efficiency in raising taxes this time at one tier of government with respect to the other one). Further, taxes on some capital items can be more easily evaded with respect to others because of different degree of capital mobility which can be due also to different features of the diverse types of capital investments. Indeed, capital is not homogeneous in its nature: For example, capital investments can be in tangible capital, i.e. equipment, structures, and other material inputs, and/or in intangible capital, i.e. research and development, patents, copy rights, advertising, employee training and costumer relations. Such concerns, thus, give rise to one interesting question to be asked, namely how, in a federal country, the existence of two types of capital investment which are taxed by different tiers of government, and only the taxation of one of them can be evaded may affect the interaction between horizontal and vertical capital tax competition.

To answer to the above question we consider a simple model which describes a federal country divided into two identical regions where consumers can decide not only the region where to invest, but also the type of capital investment. In particular, we consider a set-up with two types of capital investment where one is taxed at a regional level (according to the source-based principle) while the other one is taxed at a federal level with a different degree of tax compliance across regions. Specifically, regional taxes on the first type of capital investments are used to finance a local public good, and no evasion may occur while a uniform federal tax on the second type of

[1] On the one hand, an horizontal externality occurs when governments at the same tier tax the same tax base, and each of them tends to act strategically by undercutting the level of capital taxation to encourage capital investments within its border. Since each government does not account for such a "tax base" effect, i.e. for the decreasing (increasing) in tax base that can damage (benefit) the others due to its own tax reduction (increase), capital tax rates tend to be fixed at an inefficiently low level. On the other hand, a vertical externality arises when governments at different tiers tax the same tax base, and again each of them tends to act strategically, but this time tax rates tend to be fixed at inefficiently high levels because neither tier of government considers the tax revenue loss suffered by the other tiers due to the shrinking of the same tax base, a "common poll" effect.

[2] For example, in Italy, the shadow economy referred to firms' activity is about 6.3% of the total value added. However, such value varies greatly across regions: Puglia and Umbria share the maximum value, 8.4%, while Provincia Autonoma of Bolzano has the lowest value, 4.0%. In general, the incidence of shadow economy is higher in southern regions (7.8%), it tends to be lower in central regions (6.6%), and it is still lower in northern regions (5.8% in regions in the North-East and 5.4% in regions in the North-West). Istat (2018).

capital investment is used to finance a national public good, and it can be evaded by individuals living in the two regions at a different extent. Accordingly, in our set-up, a standard horizontal tax competition across regions arises for the first type of capital while, for the second type of capital, vertical tax competition is not standard because there is not tax base overlap between the two tiers of government. Regional and federal governments do not share the same tax base given that they tax different types of capital investment, but of course both types of capital investment are financed by the same endowment of income. Our main result shows how evasion on federal taxation for one type of capital investment affects both federal and regional tax policies, and consequently the return to savings for consumers. In particular, we show under which conditions a decrease in the level of tax compliance on one type of capital can lead to a reduction in the federal tax rate on such type of capital investment, and simultaneously to an increase in the regional tax rate on the other type of capital investment.

In the economic literature, the interplay between horizontal and vertical tax externalities has been analysed in order to understand whether, in federal countries, equilibrium tax rates tend to be inefficiently high or low.[3] The seminal paper by Keen and Kotsogiannis (2002) considers a model with a federal country composed by identical states to examine under which conditions one type of externality dominates the other given that the two point in opposite directions. They show that, at equilibrium, inefficiently high or low state taxes arise depending on the relative elasticity of the supply of savings and the demand for capital, and on the extent to which the states tax rents.[4] When two federal countries are analysed, Janeba and Wilson (2004) show that more vertical tax competition may counter the inefficiencies due to horizontal tax competition in terms of public goods underprovision.[5] When, instead, a federation consists of two countries, one of which is unitary while the other one has a federal structure, being divided into two identical regions, Grazzini and Petretto (2007) show that, from a social point of view, the federal country may still set an inefficiently low tax rate, while the unitary country may instead choose an inefficiently high tax rate, at equilibrium.

However, the incentives for countries to compete for capital tax bases may also be affected by the possibility of tax evasion, and a specific line of research has concentrated its attention on the effects of tax evasion and auditing issues on the strategic behaviour of policy-makers.[6] Cremer and Gavhari (2000) analyse the effectiveness of tax coordination policies in an economic union when

[3]For recent surveys on capital tax competition see, for example, Keen and Konrad (2013), Genschel and Schwarz (2011), and Zodrow (2010).

[4]See also Keen and Kotsogiannis (2003) for a paper that analyses the same type of question, but in a set-up where policy-makers act as revenue-maximising Leviathans.

[5]Also Flochel and Madies (2002) analyse such counterdistortionary role in a Leviathan setting to show that when tax competition is more intense, public subsidies are more efficiently supplied at a federal level rather than at a regional level. Still in a Leviathan setting, Wrede (1996) examines the interaction between horizontal and vertical tax externalities to illustrate that uncoordinated Leviathans will not generally position themselves on the downward-sloping side of the Laffer curve for total tax revenue.

[6]A related line of research concerns the implications of tax avoidance for tax competition (especially for multinational enterprises). See, for example, the survey by Zodrow (2010) for useful references.

there is the possibility of tax evasion, i.e. policy-makers can decide both the level of taxation and the audit probability (linked to the audit technology).[7] In a similar vein, Stöwhase and Traxler (2005) analyse how regional governments may use audit rates strategically, and the decentralized choice of them may be affected both by fiscal competition among regions and the type of fiscal equalization scheme (gross or net revenue sharing). The simultaneous presence of tax evasion and different specifications of an equalization scheme, vertical or horizontal, is examined by Grazzini and Petretto (2012) who show its impact on the overprovision of local public goods due to vertical fiscal externality.

Of course, reality tends to be particularly complex and, in the tax competition literature, the models used are highly stylized. In particular, "capital" is generally considered as a homogeneous and divisible good and "can be read as a metaphor for anything that is mobile internationally and generates real output where it is applied" (Keen and Konrad (2013), p. 8). Capital, however, is heterogeneous in its nature, and from an optimal taxation perspective, in a model with heterogeneous capital and constant returns to scale, for example, Auerbach (1979) shows that it is generally nonoptimal to tax uniformly different types of capital.[8] Within the literature on capital tax competition, the possibility to apply different tax rules to different types of capital tax base has been analysed with respect to preferential regimes that countries may use in order to discriminate usually in favour of those activities which are more mobile from an international viewpoint. Both at the OECD and EU level, such measures are considered socially undesirable because they are seen as harmful forms of tax competition which can make even worse the distortions due to tax competition. On such point of view there is not however an unanimous consensus. For example, on the one hand, in a model with two different tax bases and two symmetric countries whose governments maximise tax revenue, Keen (2001) shows that preferential regimes may be desirable when countries may use them to compete for the most mobile tax bases, thus protecting less mobile tax bases from erosion due to a too fierce tax competition. In other words, if countries had to set a uniform tax rate on both tax bases, tax revenue in each country would be lower with respect to the set-up with preferential tax regimes. Such result has been confirmed also for the case of asymmetric countries with respect to their population size by Bucovetsky and Haufler (2007). On the other hand, Janeba and Smart (2003) show that some restrictions on preferential tax regimes may be desirable, i.e. they need not be revenue decreasing, when aggregate tax bases are elastic, and not exogenously given as in Keen (2001). The same type of result is also obtained by Haupt and Peters (2005) when aggregate tax bases are exogenously fixed, but in each country investors have a home bias.

The plan of the paper is as follows. Section 2 describes the model, and Section 3 analyses the individual decision on consumption and saving taking into account the possibility to choose whether

[7] Notice that the effects of a lower audit probability are very similar to those due to a decrease in the statutory tax rate.

[8] For an analysis of optimal taxation in a set-up with two types of capital, i.e. tangible and intangible capital, see Hagen and Kanniainen (1995).

to invest in one type of capital or in the other one, and the location of the capital investment, i.e. in the region of residence or in the other one. Sections 4 and 5 examine how tax evasion can affect regional and federal tax policy, respectively. Finally, Section 6 contains some concluding remarks.

2 The model

Consider a federal country divided into two identical regions $i = A, B$, and suppose that in each region there is one individual $i = A, B$, with the following preferences:

$$U_i = U(C_i^1) + C_i^2 + g_i + \frac{G}{2}, \quad i = A, B, \tag{1}$$

where $U(.)$ is a well-behaved utility function, C_i^1, C_i^2 denote individual consumption in period 1 and 2, respectively, and g_i, G denote a local public good provided by the local government i and a national public good provided by the national government, respectively.[9] In the first period, in each region, every individual owns the same fixed endowment E of income, and she decides how much to consume, how much to invest, the type of investment, namely in a first type of tangible capital and/or in a second type of intangible capital, and where to invest. The individual first and second period budget constraint obtains as

$$E = C_i^1 + \sum_{j=A,B} k_i^j + \sum_{j=A,B} d_i^j, \quad i = A, B, \tag{2}$$

and

$$C_i^2 = \sum_{j=A,B} \left(1 + r^j - t^j\right) k_i^j + \sum_{j=A,B} [1 + s^j - (1 - \alpha^j)\tau] d_i^j, \quad i = A, B, \tag{3}$$

where k_i^A (d_i^A), k_i^B (d_i^B) denote the investment in the first (second) type of capital made by an individual living in region i in region A, B, respectively.[10] In each region $i = A$, B, r^i (s^i) denotes the gross remuneration of the first (second) type of capital earned by the capital investor, and t^i (τ) represents the regional (national) tax rate on the first (second) type of capital investment. In particular, we assume that the first type of capital is taxed at a regional level according to the source based principle while the second type of capital is taxed at a federal level. Contrary to the first type of capital investments, those in the second type of capital are characterized by the possibility to evade taxation, i.e. there exists a tax-gap between the tax base and the effective amount assessed by a national Fiscal Agency. Specifically, α^i, $0 \leq \alpha^i \leq 1$, $i = A, B$, denotes the rate of revenue loss due to tax evasion on the second type of capital in each region.[11] To simplify,

[9] The model could easily be rephrased in terms of a confederation made of two countries.

[10] Each type of capital is treated as an homogenous and divisible good without the possibility to distinguish the variants owned by the residents of the two regions (Keen and Konrad (2013)).

[11] In case our simple model were interpreted in terms of a confederation of states, it would describe in a highly stylized way the dissimilar propensity to evade capital taxation by different countries. Of course, reality is much more complicated because of the existence of bilateral tax treaties between countries, and accordingly different methods which can be used to avoid taxation.

α^i will be traited as an exogenous parameter but, of course, in an expected utility framework, it would be the result of a consumer choice on the optimal amount of revenue to be evaded given the probability to be discovered and the consequent fine. Our assumption of a positive value of evaded revenue implicitly refers to a set-up where, from the consumer's point of view, tax evasion turns out to be convenient on the basis of the above calculus.

To sum up, in our model, the tax treatment of both types of capital depends on the location of the investment: For the first type of capital, different tax treatments across regions are due to different tax rates while for the second type of capital, different effective tax treatments across regions are due to different tax evasion opportunities, notwithstanding a common federal tax rate.[12]

In each region, the same consumption good y^i, $i = A, B$, is produced by adopting the same technology which uses the total amount of both types of capital invested within its borders as inputs, K^i, $K^i = \sum_{j=A,B} k_j^i$, and D^i, $D^i = \sum_{j=A,B} d_j^i$, $i = A, B$. More specifically, in each region i, we assume the following decreasing returns to scale production function for the final good y^i:

$$y^i = f\left(K^i, D^i\right), \quad i = A, B, \tag{4}$$

and, thus, profits obtain as

$$\pi^i = y^i - r^i K^i - s^i D^i, \quad i = A, B, \tag{5}$$

where the price of the consumption good y^i is normalized to 1. The FOCs of this profit maximisation problem with respect to K^i and D^i obtains as

$$\begin{aligned} f_{K^i}(.) &= r^i, \\ f_{D^i}(.) &= s^i, \end{aligned}$$

and accordingly, the demand for the first and the second type of capital is given by $K^i = K^i(r^i, s^i)$ and $D^i = D^i(s^i, r^i)$, respectively.

Finally, rents (5) arising in region i from the production of the consumption good y^i are supposed to be fully taxed at the local level,[13] so that, for each region, the local public budget constraint obtains as

$$g_i = t_i K^i + \pi^i, \quad i = A, B,$$

while the national public budget constraint obtains as

$$G = \tau \sum_{j=A,B} (1 - \alpha^j) D^j.$$

Individual and public decisions are taken according to a three-stage game. At the first stage of the game, the federal government decides the optimal national tax rate on the second type of capital. At the second stage, in each region, the local government chooses its tax rate on the first

[12] The model abstracts from the possibility that horizontal and/or vertical intergovernmental transfers could be implemented to offset the distortions due to both horizontal and vertical tax competition (for such an analysis, see for example, Kelders and Koethenbuerger (2010) and Kotsogiannis (2010)).

[13] See Keen and Kotsogiannis (2002) and Grazzini and Petretto (2007) for an analogous assumption.

type of capital invested within its border, taking as given the tax rate chosen by the other region on the same type of capital, and the federal tax rate on the second type of capital investments. At the third stage of the game, individuals in each region take their consumption and investment decisions.

3 Individual decision on consumption and saving

Let us solve the game by backward induction, and firstly consider the third stage of the game where each agent $i = A, B$, maximises (1) subject to (2) and (3). The FOCs with respect to C_i^1, C_i^2, k_i^A, k_i^B, d_i^A, and d_i^B obtain as

$$\begin{array}{lc} C_i^1: & \frac{\partial U}{\partial C_i^1} = \mu, \\ C_i^2: & \gamma = 1, \\ k_i^j: & \gamma\left(1 + r^j - t^j\right) = \mu, \\ d_i^j & \gamma\left[1 + s^j - (1-\alpha^j)\tau\right] = \mu, \quad i, j = A, B, \end{array} \tag{6}$$

where μ and γ denote the Lagrangean multiplier associated to the first and second period individual budget constraint, respectively. The net return to savings denoted by ρ, which differs from the cost of capital for firms because of taxation, obtains as follows

$$\rho = r^i - t^i = s^i - (1 - \alpha^i)\tau, \quad i = A, B.$$

This is a standard arbitrage condition which, however, refers to an asymmetric tax competition set-up given the different level of tax evasion in the two regions A and B.[14] Assuming full employment of capital allows us to obtain the market clearing condition:

$$\sum_{j=A,B} K^j(\rho + t^j, \rho + (1-\alpha^j)\tau) + \sum_{j=A,B} D^j(\rho + (1-\alpha^j)\tau, \rho + t^j) = \Gamma(\rho), \tag{7}$$

where $\Gamma(\rho)$ denotes total savings in both types of capital, with $\Gamma'(\rho) \geqslant 0$.[15] Accordingly, the previous equation determines the net return to savings as a function of the two regional tax rates on the first type of capital, the federal tax rate on the second type of capital, and the different degree of tax evasion in the two regions:

$$\rho = \rho(t^A, t^B, (1-\alpha^A)\tau, (1-\alpha^B)\tau). \tag{8}$$

Differentiating (7) with respect to t^i and ρ yields

$$\frac{\partial \rho}{\partial t^i} = \frac{K^i_{r^i} + D^i_{r^i}}{\Gamma' - \sum_{j=A,B}(K^j_{r^j} + K^j_{s^j}) - \sum_{j=A,B}(D^j_{s^j} + D^j_{r^j})}, \quad i = A, B, \tag{9}$$

where $\Gamma' \geqslant 0$, $K^i_{r^i}, D^i_{s^i} < 0$, and $K^i_{s^i} = D^i_{r^i} > 0$, $i = A, B$, for regularity conditions from the profit function. In order to analyse the effects due to the existence of two types of capital investment,

[14] For a treatment of symmetric vs. asymmetric horizontal tax competiton see Keen and Konrad (2013).

[15] Derivatives are denoted by a prime for functions of one argument.

the taxation of one of which can be partly evaded, in what follows we make the following plausible assumptions on the magnitude of the response of the demand for each type of capital to changes in their gross remuneration:

Assumption 1: $\left|K^i_{r^i}\right| > D^i_{r^i}$ and $\left|D^i_{s^i}\right| > K^i_{s^i}$, $i = A, B$, i.e. the direct effect of the gross remuneration of each type of capital on its demand is higher than the indirect effect of it on the demand for the other type of capital;

Assumption 2: $\left|K^i_{r^i}\right| > K^i_{s^i}$, and $\left|D^i_{s^i}\right| > D^i_{r^i}$, $i = A, B$, i.e. the direct effect of the gross remuneration of the first (second) type of capital on its demand is higher than the indirect effect of the gross remuneration of the second (first) type of capital on the demand for the first (second) type of capital.

From (9), it is easy to check that an increase in the regional taxation of the first type of capital has a negative effect on the net return to savings:

$$-1 < \frac{\partial \rho}{\partial t^i} < 0, \quad i = A, B. \tag{10}$$

Similarly, differentiating (7) with respect to τ, and ρ yields

$$\frac{\partial \rho}{\partial \tau} = \frac{\sum_{j=A,B}(1-\alpha^j)K^j_{s^j} + \sum_{j=A,B}(1-\alpha^j)D^j_{s^j}}{\Gamma' - \sum_{j=A,B}(K^j_{r^j} + K^j_{s^j}) - \sum_{j=A,B}(D^j_{s^j} + D^j_{r^j})}, \tag{11}$$

where it is easy to check that

$$-1 < \frac{\partial \rho}{\partial \tau} < 0, \tag{12}$$

i.e. an increase in the federal taxation of the second type of capital has a negative effect on the net return to savings.

To investigate the effects of tax evasion, we now concentrate our attention on how the net remuneration to savings depends on the different degree of tax evasion in the two regions. In this respect, we can state the following

Lemma 1. $0 < \frac{\partial \rho}{\partial \alpha^i} < \tau, \quad i = A, B.$

Proof. See the Appendix.□

In line with intuition, an increase in the degree of tax evasion has a positive effect on the net remuneration of all sources of capital. However, notice that this result does not take into account how the regional governments can react to different degrees of tax evasion when choosing their tax policy on the first type of capital. As we show in what follows, even if tax evasion can only arise at a federal level on the second type of capital investments, it also affects regional tax choices on the first type of capital investments, and accordingly the net return to savings for consumers at the end of the game.

Further, from (10), it is easy to check that

$$\frac{\partial r^i}{\partial t^i} > 0, \qquad \frac{\partial r^i}{\partial t^{-i}} < 0, \quad i = A, B, \tag{13}$$

and from (12), it follows that

$$-\alpha^i < \frac{\partial s^i}{\partial \tau} = \frac{\partial \rho}{\partial \tau} + 1 - \alpha^i < 1 - \alpha^i, \quad i = A, B. \tag{14}$$

While results in (13) are standard, those in (14) are novel, because, for example, an increase in the federal capital tax rate does not necessarily lead to an increase in the cost of capital as it is the case for regional capital taxation, for which $\frac{\partial r^i}{\partial t^i} > 0$, $i = A, B$. An increase in the federal tax rate can lead to an increase or a decrease in the cost of the second type of capital depending on the different degree α^i, $i = A, B$, of tax evasion in the two regions. In particular, $\frac{\partial s^i}{\partial \tau}$, $i = A, B$, tends to be positive (negative), the lower (higher) the value of the degree of tax evasion, α^i . In the extreme cases, (14) implies that when $\alpha^i = 0$ then $\frac{\partial s^i}{\partial \tau} > 0$, and when $\alpha^i = 1$ then $\frac{\partial s^i}{\partial \tau} < 0$, $i = A, B$.

In each region, demand for each type of capital depends on both the regional capital tax rates and the levels of regional tax evasion on the second type of capital: $K^i = K^i(r^i, s^i) = (\rho + t^i, \rho + (1 - \alpha^i)\tau)$ and $D^i(\rho + (1 - \alpha^i)\tau, \rho + t^i)$. By deriving such demands for capital with respect to the regional tax rates, t^i, we obtain the following

$$\frac{\partial K^i}{\partial t^i} = K^i_{r^i}\left(\frac{\partial \rho}{\partial t^i} + 1\right) + K^i_{s^i}\frac{\partial \rho}{\partial t^i} < 0, \qquad \frac{\partial K^{-i}}{\partial t^i} = \left(K^{-i}_{r^{-i}} + K^{-i}_{s^{-i}}\right)\frac{\partial \rho}{\partial t^i} > 0, \quad i = A, B,$$

and

$$\frac{\partial D^i(.)}{\partial t^i} = D^i_{s^i}\frac{\partial \rho}{\partial t^i} + D^i_{r^i}\left(\frac{\partial \rho}{\partial t^i} + 1\right) > 0, \qquad \frac{\partial D^{-i}}{\partial t^i} = \left(D^{-i}_{s^{-i}} + D^{-i}_{r^{-i}}\right)\frac{\partial \rho}{\partial t^i} > 0, \quad i = A, B.$$

In words, an increase in the regional tax rate in one region leads to a decrease (increase) in the amount of the first (second) type of capital invested in that region, and simultaneously to an increase in the amount of both types of capital invested in the other region.

Further, by deriving the demand for capital $K^i(.)$ and $D^i(.)$ with respect to the federal tax rate, τ, we obtain the following

$$\frac{\partial K^i}{\partial \tau} = \left(K^i_{r^i} + K^i_{s^i}\right)\frac{\partial \rho}{\partial \tau} + (1 - \alpha^i)K^i_{s^i} > 0, \quad i = A, B,$$

and

$$\frac{\partial D^i}{\partial \tau} = D^i_{s^i}\left(\frac{\partial \rho}{\partial \tau} + 1 - \alpha^i\right) + D^i_{r^i}\frac{\partial \rho}{\partial \tau}, \quad i = A, B.$$

Assumption 2 implies that $\frac{\partial K^i}{\partial \tau} > 0$, i.e. an increase in the national tax rate on the second type of capital leads to an increase in the first type of capital invested in each region of the same country. Instead, the sign of $\frac{\partial D^i}{\partial \tau}$ can be either positive or negative. In particular, $\frac{\partial D^i}{\partial \tau} \gtreqless 0 \Longleftrightarrow \alpha^i \gtreqless 1 + \left(1 + \frac{D^i_{r^i}}{D^i_{s^i}}\right)\frac{\partial \rho}{\partial \tau}$. In other words, when the degree of tax evasion on the second type of capital is sufficiently low (high), an increase in the federal tax rate on it leads to a reduction (increase) in the amount of income invested in such type of capital investment.

Finally, by deriving the demand for capital $K^i(.)$ and $D^i(.)$ with respect to the regional degree of tax evasion on national taxation, α^i, we obtain the following

$$
\begin{aligned}
\frac{\partial K^i}{\partial \alpha^i} &= \left(K^i_{r^i} + K^i_{s^i}\right)\frac{\partial \rho}{\partial \alpha^i} - \tau K^i_{s^i} < 0, && (15)\\
\frac{\partial K^{-i}}{\partial \alpha^i} &= \frac{\partial \rho}{\partial \alpha^i}\left(K^{-i}_{r^{-i}} + K^{-i}_{s^{-i}}\right) < 0, \quad i = A, B, && (16)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial D^i}{\partial \alpha^i} &= D^i_{s^i}\left(\frac{\partial \rho}{\partial \alpha^i} - \tau\right) + D^i_{r^i}\frac{\partial \rho}{\partial \alpha^i} > 0, && (17)\\
\frac{\partial D^{-i}}{\partial \alpha^i} &= \frac{\partial \rho}{\partial \alpha^i}\left(D^{-i}_{s^i} + D^{-i}_{r^{-i}}\right) < 0, \quad i = A, B, && (18)
\end{aligned}
$$

where $\frac{\partial D^i}{\partial \alpha^i} > 0$ because of Lemma 1. This means that an increase in the degree of tax evasion on the second type of capital in a region leads to a decrease (increase) in the amount of the first (second) type of capital invested in the same region while it leads to a decrease in the amount of both types of capital invested in the other region.

4 The effects of tax evasion on regional tax policies

Let us now consider the second stage of the game. Suppose that inside each region i, $i = A, B$, a local government has to choose the tax rate on the first type of capital by taking as given the choice made by the other region on the same type of capital, and the federal tax rate on the second type of capital. Accordingly, in each region i, the local government maximizes the indirect utility function of a representative agent with respect to the regional tax rate on the first type of capital:

$$
\begin{aligned}
&\max_{t^i} \quad V_i = U\left(E - S_i(\rho)\right) + (1+\rho)S_i(\rho) + t^i K^i(\rho + t^i, \rho + (1-\alpha^i)\tau) + && (19)\\
&+\pi^i(\rho + t^i, \rho + (1-\alpha^i)\tau) + \frac{1}{2}\tau \sum_{j=A,B}(1-\alpha^j)D^j(\rho + (1-\alpha^j)\tau, \rho + t^j), \quad i = A, B, && (20)
\end{aligned}
$$

where $S_i(\rho) \equiv \sum_{j=A,B} k^j_i + \sum_{j=A,B} d^j_i$ denotes the individual savings of an agent living in region i, and recall that $\rho = \rho(t^A, t^B, (1-\alpha^A)\tau, (1-\alpha^B)\tau)$ from (8). The FOCs of this problem obtain as

$$
\begin{aligned}
F^i(t^i, t^{-i}, \alpha^i, \alpha^{-i}, \tau) \equiv{}& \frac{\partial \rho}{\partial t^i}S_i(\rho) + \left\{K^i + t^i\left[K^i_{r^i}\left(\frac{\partial \rho}{\partial t^i} + 1\right) + K^i_{s^i}\frac{\partial \rho}{\partial t^i}\right]\right\} - \left[K^i\left(\frac{\partial \rho}{\partial t^i} + 1\right) + D^i\frac{\partial \rho}{\partial t^i}\right] + \\
&+\frac{1}{2}\tau\left\{(1-\alpha^i)\left[D^i_{s^i}\frac{\partial \rho}{\partial t^i} + D^i_{r^i}\left(\frac{\partial \rho}{\partial t^i} + 1\right)\right] + (1-\alpha^{-i})\frac{\partial \rho}{\partial t^i}\left(D^{-i}_{s^{-i}} + D^{-i}_{r^{-i}}\right)\right\} = 0, \quad i = A, B. \quad (21)
\end{aligned}
$$

Each term in (21) has a straightforward interpretation as in the standard literature on capital tax competition, with the difference that, in our set-up, regional taxation affects individual decisions not only with reference on how much to invest and in which region to invest, but also on whether to invest in the first or in the second type of capital whose taxation can be evaded at a federal level. Accordingly, the tax rate on the first type of capital chosen at a regional level affects individual decisions not only through the channel of the gross remuneration on the first type of capital, r^i, but also via the gross remuneration on the second type of capital, s^i.

More specifically, the first term in (21) represents the negative effect on the net remuneration to individual savings due to an infinitesimal increase in the regional tax rate on the first type of capital, t^i. The second term describes the sum of the direct and the indirect impact on regional tax revenue of an infinitesimal increase in t^i. As expected, the direct effect is positive while the indirect effect is negative when $t^i > 0$. This latter term captures the effect on the total amount of the first type of capital invested in region i of an increase in t^i both via the gross remuneration of the first type of capital, $K^i_{r^i}\left(\frac{\partial\rho}{\partial t^i}+1\right)$, and via the gross remuneration of the second type of capital, $K^i_{s^i}\frac{\partial\rho}{\partial t^i}$. Together both effects describe the capital flight arising from region i toward the other region following an infinitesimal increase in t^i, the other region benefits in terms of an increase in its tax ravenue, i.e. a positive *horizontal externality*. However, since such positive externality is not taken into account by region i, the latter perceives a negative indirect effect. The third term represents the change in terms of rent tax revenue due to an infinitesimal increase in t^i which increases the cost of the first type of capital, but simultaneously it decreases the cost of the second type of capital in region i. From the point of view of the region i, the first part of such an effect is negative, i.e. $-K^i\left(\frac{\partial\rho}{\partial t^i}+1\right) < 0$, while the second part is positive, i.e. $-D^i\frac{\partial\rho}{\partial t^i} > 0$. Thus, the total effect will be negative (positive) when the effect through the first type of capital is higher (lower) than the effect through the second type of capital. For example, in case such total effect is negative, it also describes a *horizontal externality* given by the fact that the reduction in rent tax revenue in region i corresponds to an increase in rent tax revenue in the other region. Finally, the fourth term describes a *vertical externality* that arises on the federal tax revenue following an infinitesimal change in the regional tax rate. This term is given by the sum of two terms that are both positive: The first (second) one describes the increase in the federal tax revenue following a shift from investments in the first type of capital towards investments in the second type of capital in region i $(-i)$ due to an infinitesimal increase in t^i.

Conditions (21) define each region's reaction function:

$$t^i = t^i(t^{-i}, \alpha^i, \alpha^{-i}, \tau), \quad i = A, B, \tag{22}$$

so that each regional tax rate on the first type of capital depends on the tax rate chosen by the other region, the federal tax rate, and the degree of tax evasion arising in both regions for the second type of capital. A Nash equilibrium of the game played by the regions is given by the solution to the system of the above reaction functions. Accordingly, by taking into account (22) into (8), at the Nash equilibrium, the net remuneration of capital obtains as

$$\rho = \rho(t^i(.), t^{-i}(.), \alpha^i, \alpha^{-i}, \tau) \quad i = A, B. \tag{23}$$

We are now in a position to analyse how tax evasion on federal taxation may affect regional taxation, at the second stage of the game. In order to pursue such an aim, let us firstly examine how the regional tax rate chosen in one region affects the one chosen in the other region. In this respect, we can state the following[16]

Proposition 1. $\frac{\partial t^{-i}}{\partial t^i} > 0$, $i = A, B$.

Proof. See the Appendix.□

Proposition 1 confirms the standard result on strategic complementarity between regional capital tax rates obtained in a set-up with horizontal tax competition. What is worth noticing is that such result also holds in the present set-up where individuals can invest in two alternative types of capital, and horizontal tax competition only arises with respect to the first type while federal taxation on the second type of capital can be partly evaded.

To proceed in our analysis on how tax evasion on the second type of capital may affect regional tax policies on the first type of capital, rewrite the optimality condition (21) as follows

$$
\begin{aligned}
F^i(t^i, t^{-i}, \alpha^i, \alpha^{-i}, \tau) \equiv \frac{\partial \rho}{\partial t^i}\left[S_i(\rho) - K^i(\rho + t^i, \rho + (1-\alpha^i)\tau) - D^i(\rho + (1-\alpha^i)\tau, \rho + t^i)\right] + \\
+ t^i\left[K^i_{r^i}\left(\frac{\partial \rho}{\partial t^i} + 1\right) + K^i_{s^i}\frac{\partial \rho}{\partial t^i}\right] + \\
+ \frac{1}{2}\tau\left\{(1-\alpha^i)\left[D^i_{s^i}\frac{\partial \rho}{\partial t^i} + D^i_{r^i}\left(\frac{\partial \rho}{\partial t^i} + 1\right)\right] + (1-\alpha^{-i})\left[D^{-i}_{s^{-i}}\frac{\partial \rho}{\partial t^i} + D^{-i}_{r^{-i}}\frac{\partial \rho}{\partial t^i}\right]\right\} = 0, \quad i = A, B.
\end{aligned}
\tag{24}
$$

At the Nash equilibrium, by taking into account (22), let us differentiate both first order conditions in (24) with respect to α^i:

$$
\begin{aligned}
\frac{\partial F^{-i}}{\partial \alpha^i} + \frac{\partial F^{-i}}{\partial t^i}\frac{dt^i}{d\alpha^i} + \frac{\partial F^{-i}}{\partial t^{-i}}\frac{dt^{-i}}{d\alpha^i} &= 0, \\
\frac{\partial F^i}{\partial \alpha^i} + \frac{\partial F^i}{\partial t^i}\frac{dt^i}{d\alpha^i} + \frac{\partial F^i}{\partial t^{-i}}\frac{dt^{-i}}{d\alpha^i} &= 0, \quad i = A, B.
\end{aligned}
$$

By solving this system of equations with respect to $\frac{dt^i}{d\alpha^i}$ and $\frac{dt^{-i}}{d\alpha^i}$, the following two equations obtain

$$
\frac{dt^i}{d\alpha^i} = \frac{1}{\Omega^i}\left(\frac{\partial F^i}{\partial t^{-i}}\frac{\partial F^{-i}}{\partial \alpha^i} - \frac{\partial F^{-i}}{\partial t^{-i}}\frac{\partial F^i}{\partial \alpha^i}\right), \quad i = A, B, \tag{25}
$$

$$
\frac{dt^{-i}}{d\alpha^i} = -\frac{1}{\Omega^i}\left(\frac{\partial F^i}{\partial t^i}\frac{\partial F^{-i}}{\partial \alpha^i} - \frac{\partial F^{-i}}{\partial t^i}\frac{\partial F^i}{\partial \alpha^i}\right), \quad i = A, B. \tag{26}
$$

where

$$
\Omega^i \equiv \frac{\partial F^i}{\partial t^i}\frac{\partial F^{-i}}{\partial t^{-i}} - \frac{\partial F^i}{\partial t^{-i}}\frac{\partial F^{-i}}{\partial t^i}. \tag{27}
$$

To perform our analysis, first we sign Ω^i in the following

Lemma 2. $\Omega^i < 0$, $i = A, B$.

[16] For the sake of tractability, second cross derivatives are assumed negligible in the proofs where they appear.

Proof. See the Appendix.□

We are now able to examine how national tax evasion on the second type of capital affects regional taxation on the first type of capital. The following definition will be useful for the next proposition and corollary:

$$\widetilde{D}^i_{s^i} \equiv \left| \frac{D^i_{s^i}}{1+\frac{1}{\frac{\partial \rho}{\partial t^i}}} \right|, \quad i = A, B.$$

We can thus state the following

Proposition 2. If $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then $\frac{\partial t^i}{\partial \alpha^i} > 0$ and $\frac{\partial t^{-i}}{\partial \alpha^i} > 0$, $i = A, B$.

Proof. See the Appendix.□

Proposition 2 shows under which condition an increase in the degree of tax evasion on the second type of capital investments leads to an increase in both regional tax rates on the first type of capital.[17] At a first sight, this result may seem counter-intuitive because one could expect that regional governments would decrease their taxation on the first type of capital to offset a capital flight towards investments in the second type of capital which should be favoured by greater evasion possibilities. On the contrary, when $D^i_{r^i}$ is sufficiently high, i.e. an increase in the cost of the first type of capital leads to a large rise in the second type of capital investments (there is a high degree of substitution between the two types of capital in the production function), regional governments choose to increase their taxation in order to counter the reduction in their tax base. The reason behind this result is thus a consequence of our peculiar set-up with horizontal tax competition between regions and vertical interaction between the regional and the central governments where, contrary to the standard literature, individuals not only decide on where to allocate capital investments but also on whether to invest in the first or in the second type of capital. This result is however only a partial one because at this stage of the game regional governments do not take into account how the federal government will react to a change in the degree of evasion on its tax base, and how a variation in the federal tax rate may affect the regional ones. This will be catched up in the next section where we solve the first stage of the game.

To deep our understanding of the channels through which tax evasion on the second type of capital may affect regional tax policies, we can now analyse how tax evasion affects the net remuneration of capital at the Nash equilibrium of the second stage of the game. By differentiating (23) with respect to α^i, the total effect of tax evasion on the net remuneration of capital obtains as

$$\frac{d\rho}{d\alpha^i} = \frac{\partial \rho}{\partial \alpha^i} + \sum_{j=A,B} \frac{\partial \rho}{\partial t^j}\frac{\partial t^j}{\partial \alpha^i}, \quad i = A, B. \tag{28}$$

[17] Notice that in order that the condition underlying Proposition 2, $D^i_{r^i} > \widetilde{D}^i_{s^i}$, is satisfied together with assumption 2, i.e. $D^i_{r^i} < \left|D^i_{s^i}\right|$, we assume that $-\frac{1}{2} < \frac{\partial \rho}{\partial t^i} < 0$ which implies that $\widetilde{D}^i_{s^i} < \left|D^i_{s^i}\right|$, i.e. $\left|\frac{\frac{\partial \rho}{\partial t^i}}{\frac{\partial \rho}{\partial t^i}+1}\right| < 1$. In other words, in what follows a smaller interval with respect to the one in (10) is assumed.

Notice that $\frac{d\rho}{d\alpha^i}$ describes the effect of an infinitesimal increase in tax evasion of region i, $i = A, B$, on the net remuneration of capital in (23) both *directly*, i.e. $\frac{\partial\rho}{\partial\alpha^i}$ (see Lemma 1), and *indirectly* via the change in both regional tax rates on the first type of capital, i.e. $\sum_{j=A,B} \frac{\partial\rho}{\partial t^j} \frac{\partial t^j}{\partial\alpha^i}$, $i = A, B$, where $\frac{\partial\rho}{\partial t^i}$ obtains from (9) and $\frac{\partial t^j}{\partial\alpha^i}$, $i, j = A, B$ obtains from (25) and (26).

We are now in a position to state the following

Corollary 1. If $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then $0 < \frac{d\rho}{d\alpha^i} < \frac{\partial\rho}{\partial\alpha^i}$, $i = A, B$.

Proof. See the Appendix.□

Corollary 1 shows that, under the same condition underlying Proposition 2, the result on the positive effect of tax evasion on the net remuneration of capital already obtained at the third stage of the game (Lemma 1) is confirmed at the Nash equilibrium of the second stage of the game. However, this Corollary also shows that the magnitude of this effect is lower at the second stage of the game with respect to the third one, $\frac{d\rho}{d\alpha^i} < \frac{\partial\rho}{\partial\alpha^i}$, because of the counter negative indirect effect of tax evasion on the net remuneration of capital through the change in regional tax rates. Indeed, an increase in tax evasion leads to a rise in regional tax rates (see Proposition 2) which, in its turn, has a negative effect on the net remuneration of capital from (10), i.e. $\sum_{j=A,B} \frac{\partial\rho}{\partial t^j} \frac{\partial t^j}{\partial\alpha^i} < 0$.

To conclude on the second stage of the game, now we examine the effect on the regional tax rate of an infinitesimal increase in the federal tax rate. To this aim we state the following

Proposition 3. If $\left|\frac{\partial\rho}{\partial\tau}\right| > 1 - \alpha^i$ then $\frac{dt^i}{d\tau} > 0, i = A, B$.

Proof. See the Appendix.□

Proposition 3 shows under which condition there is strategic complementarity between the federal tax rate and the regional tax rates: A higher federal tax rate drives capital towards investments in the first type of capital rising regional tax bases, and regional governments respond by increasing their own tax rate.[18] Notice that the result in Proposition 3 according to which regional tax rates on the first type of capital increase in response to an increase in the federal tax rate on the second type of capital is in line with the one in Proposition 2 which stated that regional tax rates increase in response to a decrease in tax evasion on the second type of capital. In both circumstances, as in the case of horizontal tax competition (see Proposition 1), regions benefit from a positive externality, which is however due to a vertical interaction between regional and federal tax policies. In particular, such vertical interaction does not reflect completely standard vertical tax competition because in our setup there is not tax base overlap between different levels of governments, i.e. regional and federal governments do not share the same tax base given that they tax different types of capital investments.

[18] Notice that the condition underlying Proposition 3 requires that the absolute value of the effect of the federal tax rate on the net remuneration of capital is sufficiently large. Similarly to what already noticed above with respect to the condition underlying both Proposition 2 and Corollary 1, in what follows, we consider a smaller interval with respect to the one in (12) by assuming the condition in Proposition 3, i.e. $-1 < \frac{\partial\rho}{\partial\tau} < -(1 - \alpha^i)$.

5 The effects of tax evasion on the federal tax policy

Let us now move to the first stage of the game where the federal government maximises a social welfare function which is given by the sum of the indirect utility functions of the residents of the two regions. In particular, the federal government problem obtains as

$$\max_{\tau} \sum_{j=A,B} \left[U\left(E - S_j(\rho)\right) + (1+\rho)S_j(\rho) + t^j K^j(\rho + t^j, \rho + (1-\alpha^j)\tau) + \pi^j(\rho + t^j, \rho + (1-\alpha^j)\tau)\right] + \tag{29}$$

$$+\tau \sum_{j=A,B} (1-\alpha^j) D^j(\rho + (1-\alpha^j)\tau, \rho + t^j),$$

where recall that $\rho = \rho(t^i(.), t^{-i}(.), \alpha^i, \alpha^{-i}, \tau)$, $i = A, B$, from (23). The FOC of this problem with respect to τ is given by

$$G(t^i, t^{-i}, \alpha^i, \alpha^{-i}, \tau) \equiv \frac{d\rho}{d\tau} S_j(\rho) + \sum_{j=A,B} \frac{\partial t^j}{\partial \tau} K^j + \sum_{j=A,B} t^j \left[K^j_{r^j}\Theta^i + K^j_{s^j}\Gamma^i\right] + \tag{30}$$

$$-\left[\Theta^i \sum_{j=A,B} K^j + \Gamma^i \sum_{j=A,B} D^j\right] + \sum_{j=A,B} (1-\alpha^j) D^j + \tau \sum_{j=A,B} \left(1-\alpha^j\right)\left[D^j_{s^j}\Gamma^i + D^j_{r^j}\Theta^i\right] = 0,$$

where

$$\begin{aligned} \Theta^i &\equiv \frac{d\rho}{d\tau} + \frac{\partial t^i}{\partial \tau}, \quad i = A, B, \\ \Gamma^i &\equiv \frac{d\rho}{d\tau} + 1 - \alpha^i, \quad i = A, B, \end{aligned}$$

and

$$\frac{d\rho}{d\tau} = \frac{\partial \rho}{\partial \tau} + \sum_{j=A,B} \frac{\partial \rho}{\partial t^j}\frac{\partial t^j}{\partial \tau} < 0, \tag{31}$$

because of (12), (10), and Proposition 3.

Each term in (30) can be given a simple interpretation analogous to the ones already provided for the regional governments at the second stage of the game with the difference that of course now the maximisation problem is the one of the federal government with respect to the tax rate on the second type of capital, and the net remuneration of capital is (23). In particular, notice that the federal tax policy affects individual decisions concerning which type of capital investment is more profitable. The first term in (30) is equivalent to the first term in (24) except that it is expressed with respect to an infinitesimal change in the federal tax rate τ instead of an infinitesimal change in the regional tax rate t^i. The second and the third term represent a *vertical externality* on regional tax revenue from the first type of capital taxation. In particular, the second term describes the direct effect on regional tax revenue of an increase in the federal tax rate. This effect is positive because the federal and regional tax rates are strategic complements (see Proposition 3). The third term represents the indirect effect on regional tax revenue of an increase in τ, i.e. a tax-base effect. Specifically, such effect is negative (positive) when $\Theta^i > (<)0$ and $\Gamma^i < (>)0$. The fourth term describes a *vertical externality* on regional rent taxation due to an infinitesimal increase in the federal tax rate The first part of such an effect is negative (positive) while the second part is positive (negative) when $\Theta^i > (<)0$ and $\Gamma^i < (>)0$, and thus the total effect will be positive or negative depending on which effect prevails on the other. Finally, the fifth and the sixth term represent the direct and the indirect effect on the federal tax revenue of an increase in the federal tax rate, respectively. When $\tau > 0$, the direct effect is always positive while the indirect effect is positive (negative) when $\Theta^i > (<)0$ and $\Gamma^i < (>)0$.

We are now in a position to analyse how tax evasion on the second type of capital affects the federal tax rate. In this respect, we state the following

Proposition 4. If i) $1 - \alpha^i < \left|\frac{d\rho}{d\tau}\right| < \frac{\partial t^i}{\partial \tau}$ and ii) $1 - \tau < \left|\sum_{j=A,B} \frac{\partial \rho}{\partial t^j}\frac{\partial t^j}{\partial \tau}\right| < 1$ then $\frac{\partial \tau}{\partial \alpha^i} < 0$, $i = A, B$.

Proof. See the Appendix.□

Proposition 4 shows under which conditions tax evasion on the second type of capital negatively affects the federal tax rate, taking into account the fact that the federal government acts as a Stackelberg leader with respect to the regional governments.[19,20] Suppose for example that tax evasion on the second type of capital increases. On the one hand, this leads to a decrease in the federal tax base (a direct effect). On the other hand, an increase in tax evasion also leads to an increase in the federal tax base because the federal government anticipates that an increase in tax evasion leads to an increase in regional tax rates, and thus to an increase in the federal tax base due to a capital flight from the first type of capital investments towards the second type of capital investments (an indirect effect). Accordingly, notwithstanding a decrease in the federal tax base due to an increase in tax evasion, the federal government decreases its tax rate in response to an increase in tax evasion because of the anticipation of the capital flight from regional towards federal tax base.

The result in Proposition 4 allows us also to analyse the effects of tax evasion on regional taxation by taking into account not only the *positive direct* effect arising at the second stage of the game, but also the *negative indirect* effect that arises at the first stage of the game because of vertical tax competition via the strategic complementarity between the federal and the regional tax rate (see Proposition 3). At the second stage of the game, Proposition 2 has shown a *positive direct effect* according to which an increase in tax evasion on the second type of capital leads to an increase in the regional tax rate on the first type of capital. At the first stage of the game, Proposition 4

[19] In (31), notice that $\left|\frac{d\rho}{d\tau}\right| = \left|\frac{\partial \rho}{\partial \tau} + \sum_{j=A,B} \frac{\partial \rho}{\partial t^j}\frac{\partial t^j}{\partial \tau}\right| > \left|\sum_{j=A,B} \frac{\partial \rho}{\partial t^j}\frac{\partial t^j}{\partial \tau}\right|$ because $\frac{\partial \rho}{\partial \tau} < 0$ from (12) and $\sum_{j=A,B} \frac{\partial \rho}{\partial t^j}\frac{\partial t^j}{\partial \tau} < 0$ from 10 and Lemma 3 Accordingly, conditions (i) and (ii) of Proposition 3 imply that we are considering the case when $1 - \tau < 1 - \alpha^i$, i.e. $\tau > \alpha^i$, $i = A, B$.

[20] Notice that if $\frac{\partial t^i}{\partial \tau} < 1$ (more plausible case) the two conditions of Proposition 3 can be rewritten as follows: $1 - \tau < \left|\sum_{j=A,B} \frac{\partial \rho}{\partial t^j}\frac{\partial t^j}{\partial \tau}\right| < \left|\frac{d\rho}{d\tau}\right| < \frac{\partial t^i}{\partial \tau}$.

shows that an increase in tax evasion on the second type of capital also leads to a decrease in the national tax rate which, in its turn, leads to a decrease in the regional tax rate because of strategic complementarity (see Proposition 3). Thus, a *negative indirect effect* arises: An increase in tax evasion leads to a decrease in the regional tax rate through the strategic complementarity between the federal and the regional tax rates. We can thus conclude that tax evasion on the second type of capital positively (negatively) affects the regional tax rate on the first type of capital when the *direct* effect arising at the second stage of the game dominates (is dominated by) the *indirect* effect which arises at the first stage of the game, but it is transmitted to the second stage through vertical tax competition.

6 Concluding remarks

The aim of this paper has been to analyse the implications of the existence of both heterogeneous capital and tax evasion for the interplay between horizontal and vertical tax competition in a federal country. In particular, we have analysed a federal country where consumers can decide both the region where to invest, and whether to invest in one or another type of capital. Each type of capital is assumed to be taxed by a different level of government with a different degree of tax compliance: The first type of capital is taxed at a regional level (according to the source-based principle) and no evasion may occur while the second type of capital is taxed at a federal level, and it can be evaded by individuals living in the two regions at a different extent. Our aim has been to study how, in a federal country with these two sources of asymmetry, tax evasion at a federal level affects both federal and regional tax policies. Our main result shows under which conditions an increase in tax evasion on the second type of capital arising at a federal level negatively affects the federal tax rate and positively affects the regional tax rate.

Finally, notice that our analysis has been performed in a simple model that describes the interaction between horizontal and vertical tax externalities with two types of capital investments and tax evasion. Such simple set-up could be extended in several directions. For example, extensions to broader set-ups could include asymmetric regions with respect to population, investors' home bias, and different degrees of tax evasion between the two types of capital.

7 Appendix

Proof of Lemma 1.

Differentiating (7) with respect to α^i, $i = A$, B, and ρ yields

$$\frac{\partial \rho}{\partial \alpha^i} = \frac{-\tau\left(K^i_{s^i} + D^i_{s^i}\right)}{\Gamma' - \sum_{j=A,B}(K^j_{r^j} + K^j_{s^j}) - \sum_{j=A,B}(D^j_{s^j} + D^j_{r^j})}, \quad i = A, B, \tag{32}$$

where it is easy to check that $0 < \frac{\partial \rho}{\partial \alpha^i} < \tau, \quad i = A, B.\square$

Proof of Proposition 1.

To evaluate how a change in t^i affects the tax rate in the other region, t^{-i}, rewrite the optimality condition (21) for region $-i$ as follows

$$\begin{aligned}
&F^{-i}(t^{-i}, t^i, \alpha^{-i}, \alpha^i, \tau) \equiv \\
&\equiv \frac{\partial \rho}{\partial t^{-i}} \left[S_{-i}(\rho) - K^{-i}(\rho + t^{-i}, \rho + (1-\alpha^{-i})\tau) - D^{-i}(\rho + (1-\alpha^{-i})\tau, \rho + t^{-i}) \right] + \\
&+ t^{-i} \left[K^{-i}_{r^{-i}} \left(\frac{\partial \rho}{\partial t^{-i}} + 1 \right) + K^{-i}_{s^{-i}} \frac{\partial \rho}{\partial t^{-i}} \right] + \\
&+ \frac{1}{2}\tau \left\{ (1-\alpha^{-i}) \left[D^{-i}_{s^{-i}} \frac{\partial \rho}{\partial t^{-i}} + D^{-i}_{r^{-i}} \left(\frac{\partial \rho}{\partial t^{-i}} + 1 \right) \right] + (1-\alpha^i) \left[D^i_{s^i} \frac{\partial \rho}{\partial t^{-i}} + D^i_{r^i} \frac{\partial \rho}{\partial t^{-i}} \right] \right\} = 0, \\
&\qquad i = A, B. \quad (33)
\end{aligned}$$

Since equation (33) implicitly defines the reaction function $t^{-i} = t^{-i}(t^i, \alpha^{-i}, \alpha^i, \tau)$, $i = A, B$, we can evaluate how a change in t^i affects the tax rate in the other region, t^{-i}, i.e. $\frac{\partial t^{-i}}{\partial t^i} = -\frac{\partial F^{-i}/\partial t^i}{\partial F^{-i}/\partial t^{-i}}$. By following a procedure familiar in the tax competition literature (Andersson *et al.* (2004)), i.e. assuming $\partial F^{-i}/\partial t^{-i} < 0$ by the second order condition of the problem in (19), then *sign* $\frac{\partial t^{-i}}{\partial t^i} =$ *sign* $\frac{\partial F^{-i}}{\partial t^i}$. Thus, from (33), it is easy to check that

$$\frac{\partial F^{-i}}{\partial t^i} = \frac{\partial \rho}{\partial t^{-i}} \frac{\partial \rho}{\partial t^i} \left[\frac{dS_{-i}}{d\rho} - \left(K^{-i}_{r^{-i}} + K^{-i}_{s^{-i}} \right) - \left(D^{-i}_{s^{-i}} + D^{-i}_{r^{-i}} \right) \right] > 0, \quad i = A, B. \tag{34}$$

because of (10), and assumption 2. Thus, $\frac{\partial t^{-i}}{\partial t^i} > 0$.□

Proof of Lemma 2.

To show that $\Omega^i < 0$, let us rewrites (24) and (34), respectively, as follows

$$\frac{\partial F^i}{\partial t^i} = \gamma^i - \widetilde{\gamma}^i, \quad i = A, B, \tag{35}$$

and

$$\frac{\partial F^i}{\partial t^{-i}} = \frac{\partial \rho}{\partial t^i} \frac{\partial \rho}{\partial t^{-i}} \widehat{\gamma}^i, \quad i = A, B, \tag{36}$$

where

$$\gamma^i \equiv \left(\frac{\partial \rho}{\partial t^i} \right)^2 \widehat{\gamma}^i, \tag{37}$$

$$\widehat{\gamma}^i \equiv \frac{dS_i}{d\rho} - \left(K^i_{r^i} + D^i_{r^i} \right) - \left(K^i_{s^i} + D^i_{s^i} \right), \tag{38}$$

$$\widetilde{\gamma}^i \equiv \frac{\partial \rho}{\partial t^i} \left(K^i_{r^i} + D^i_{r^i} \right) - K^i_{r^i} \left(\frac{\partial \rho}{\partial t^i} + 1 \right) - K^i_{s^i} \frac{\partial \rho}{\partial t^i}. \tag{39}$$

Then, by using (35) and (36), (27) can be rewritten as

$$\Omega^i = \left(\gamma^i - \widetilde{\gamma}^i \right) \left(\gamma^{-i} - \widetilde{\gamma}^{-i} \right) - \widehat{\gamma}^i \widehat{\gamma}^{-i}, \quad i = A, B. \tag{40}$$

Suppose, by contradition, that $\Omega^i > 0$. From (40), this implies that

$$\frac{\gamma^i - \widetilde{\gamma}^i}{\widehat{\gamma}^i} \frac{\gamma^{-i} - \widetilde{\gamma}^{-i}}{\widehat{\gamma}^{-i}} > 1, \quad i = A, B,$$

or

$$\left[\left(\frac{\partial\rho}{\partial t^i}\right)^2-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}\right]\left[\left(\frac{\partial\rho}{\partial t^{-i}}\right)^2-\frac{\widetilde{\gamma}^{-i}}{\widehat{\gamma}^{-i}}\right]>1,\quad i=A,B. \tag{41}$$

In (41), $\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}>0$, $i=A,B$, because Assumption 1 implies that $\widehat{\gamma}^i>0$, and $\widetilde{\gamma}^i>0$ by taking also into account (10). Further, from Assumptions 1 and 2 and (10), it follows that

$$\widetilde{\gamma}^i-\widehat{\gamma}^i=\left(K^i_{r^i}+D^i_{r^i}\right)\left(1+\frac{\partial\rho}{\partial t^i}\right)-\frac{dS_i}{d\rho}-\left(K^i_{r^i}+K^i_{s^i}\right)\frac{\partial\rho}{\partial t^i}-\left(K^i_{r^i}-K^i_{s^i}\right)+D^i_{s^i}<0,\quad i=A,B,$$

which implies that $\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}<1$. Thus, $-1<-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}<0$. In (41), notice that $\left[\left(\frac{\partial\rho}{\partial t^i}\right)^2-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}\right]$ must be negative because $\left[\left(\frac{\partial\rho}{\partial t^i}\right)^2-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}\right]=\frac{\gamma^i-\widetilde{\gamma}^i}{\widehat{\gamma}^i}<0$ given that $\widehat{\gamma}^i>0$ and $\gamma^i-\widetilde{\gamma}^i=\frac{\partial F^i}{\partial t^i}<0$ being the second order condition of the problem in (19). Further, $\left[\left(\frac{\partial\rho}{\partial t^i}\right)^2-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}\right]<0$ together with $-1<-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}<0$ and $0<\left(\frac{\partial\rho}{\partial t^i}\right)^2<1$ from (10), imply that $-1<\left[\left(\frac{\partial\rho}{\partial t^i}\right)^2-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}\right]<0$, $i=A,B$. Accordingly, $\left[\left(\frac{\partial\rho}{\partial t^i}\right)^2-\frac{\widetilde{\gamma}^i}{\widehat{\gamma}^i}\right]\left[\left(\frac{\partial\rho}{\partial t^{-i}}\right)^2-\frac{\widetilde{\gamma}^{-i}}{\widehat{\gamma}^{-i}}\right]<1$, $i=A,B$, which contradicts (41). Thus, we have proved that $\Omega^i<0$, $i=A,B$.□

Proof of Proposition 2.

First, let us prove that $\frac{\partial t^i}{\partial\alpha^i}>0$. To sign (25), from (24) and taking into account Assumption 1, it is easy to check that the first term in parenthesis is positive:

$$\frac{\partial F^i}{\partial t^{-i}}=\frac{\partial\rho}{\partial t^i}\frac{\partial\rho}{\partial t^{-i}}\left[\frac{dS_i}{d\rho}-\left(K^i_{r^i}+D^i_{r^i}\right)-\left(K^i_{s^i}+D^i_{s^i}\right)\right]>0,\quad i=A,B. \tag{42}$$

By using (33), the second term in parenthesis obtains as

$$\frac{\partial F^{-i}}{\partial\alpha^i}=\frac{\partial\rho}{\partial t^{-i}}\frac{\partial\rho}{\partial t^i}\left[\frac{dS_{-i}}{d\rho}-\left(K^{-i}_{r^{-i}}+K^{-i}_{s^{-i}}\right)-\left(D^{-i}_{s^{-i}}+D^{-i}_{r^{-i}}\right)\right]-\frac{1}{2}\tau\frac{\partial\rho}{\partial t^{-i}}\left(D^i_{s^i}+D^i_{r^i}\right)<0,\quad i=A,B, \tag{43}$$

which is negative because of (10), and assumption 2. Further, the third term in parenthesis is negative since $\frac{\partial F^{-i}}{\partial t^{-i}}<0$ by the second order condition of the problem in (19). Finally, from (24) the last term in parenthesis can be rewritten as

$$\frac{\partial F^i}{\partial\alpha^i}=\Phi^i+\Psi^i,\quad i=A,B, \tag{44}$$

where

$$\Phi^i\equiv\frac{\partial\rho}{\partial t^i}\frac{\partial\rho}{\partial\alpha^i}\widehat{\gamma}^i+\frac{\partial\rho}{\partial t^i}\tau K^i_{s^i}<0, \tag{45}$$

and

$$\Psi^i\equiv\frac{1}{2}\tau\left[D^i_{s^i}\frac{\partial\rho}{\partial t^i}-D^i_{r^i}\left(\frac{\partial\rho}{\partial t^i}+1\right)\right],\quad i=A,B. \tag{46}$$

The term Φ^i is negative because of (10), Lemma 1, and $\widehat{\gamma}^i > 0$ as it was shown in the proof of Lemma 2 while the term Ψ^i is negative if $D^i_{r^i} > \left| \frac{D^i_{s^i}}{1+\frac{1}{\frac{\partial \rho}{\partial t^i}}} \right| \equiv \widetilde{D}^i_{s^i}$. Thus, if $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then $\frac{\partial F^i}{\partial \alpha^i} < 0$, and the difference in parenthesis in (25) is negative. Accordingly, from Lemma 2, if $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then $\frac{\partial t^i}{\partial \alpha^i} > 0$.

Let us now prove that $\frac{\partial t^{-i}}{\partial \alpha^i} > 0$. In (26), the first term in parenthesis is negative being $\frac{\partial F^i}{\partial t^i}$ the second order condition of the problem in (19), while from (43), the second term is negative. Further, from (34) the third term in parenthesis is positive, and as we have shown above if $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then the fourth term in parenthesis is negative, and the difference in parenthesis in (26) is positive. Thus, from Lemma 2, if $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then $\frac{\partial t^{-i}}{\partial \alpha^i} > 0$.□

Proof of Corollary 1.

In (28), first it is easy to check that $\frac{\partial \rho}{\partial \alpha^i} > 0$ from Lemma 1, and $\frac{\partial \rho}{\partial t^i} < 0$ from (10). Further, $\frac{\partial t^{-i}}{\partial \alpha^i} > 0$, $i = A, B$, if $D^i_{r^i} > \widetilde{D}^i_{s^i}$ from Proposition 2. This implies that if $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then $\frac{d\rho}{d\alpha^i} > 0$, $i = A, B$. Second, it is also immediate to check that if $D^i_{r^i} > \widetilde{D}^i_{s^i}$ then $\left|\frac{d\rho}{d\alpha^i}\right| < \left|\frac{\partial \rho}{\partial \alpha^i}\right|$ because $\sum_{j=A,B} \frac{\partial \rho}{\partial t^j} \frac{\partial t^j}{\partial \alpha^i} < 0$ in (28).□

Proof of Proposition 3.

At the Nash equilibrium, by taking into account (22), let us differentiate (33) and (24) with respect to τ:

$$\begin{aligned} \frac{\partial F^{-i}}{\partial \tau} + \frac{\partial F^{-i}}{\partial t^{-i}}\frac{dt^{-i}}{d\tau} + \frac{\partial F^{-i}}{\partial t^i}\frac{dt^i}{d\tau} &= 0, \\ \frac{\partial F^i}{\partial \tau} + \frac{\partial F^i}{\partial t^i}\frac{dt^i}{d\tau} + \frac{\partial F^i}{\partial t^{-i}}\frac{dt^{-i}}{d\tau} &= 0, \quad i = A, B. \end{aligned} \tag{47}$$

By solving this system of equations, it is easy to check that

$$\frac{dt^i}{d\tau} = \frac{1}{\Omega^i}\left(\frac{\partial F^i}{\partial t^{-i}}\frac{\partial F^{-i}}{\partial \tau} - \frac{\partial F^{-i}}{\partial t^{-i}}\frac{\partial F^i}{\partial \tau}\right), \quad i = A, B. \tag{48}$$

From (42), the first term in parenthesis is positive, and the third term is negative by the second order condition of the problem in (19). From (24), the second term in parenthesis obtains as (an equivalent expression obtains for the fourth term in parenthesis, *mutatis mutandis*)

$$\frac{\partial F^i}{\partial \tau} = \frac{\partial \rho}{\partial t^i}\left[\frac{\partial S_i}{\partial \rho}\frac{\partial \rho}{\partial \tau} - (K^i_{r^i} + D^i_{r^i})\frac{\partial \rho}{\partial \tau} - (K^i_{s^i} + D^i_{s^i})\left(\frac{\partial \rho}{\partial \tau} + 1 - \alpha^i\right)\right] + \tag{49}$$

$$+\frac{1}{2}\left\{(1-\alpha^i)\left[D^i_{s^i}\frac{\partial \rho}{\partial t^i} + D^i_{r^i}\left(\frac{\partial \rho}{\partial t^i} + 1\right)\right] + (1-\alpha^{-i})\left[(D^{-i}_{s^{-i}} + D^{-i}_{r^{-i}})\frac{\partial \rho}{\partial t^i}\right]\right\}, \quad i = A, B. \tag{50}$$

From (12), and (10) if $\frac{\partial \rho}{\partial \tau} < -(1-\alpha^i)$ then $\frac{\partial F^i}{\partial \tau} > 0$, $i = A, B$. Thus, if $\frac{\partial \rho}{\partial \tau} < -(1-\alpha^i)$ and $\Omega^i \gtrless 0$ then $\frac{dt^i}{d\tau} \gtrless 0, i = A, B$.□

Proof of Proposition 4.

By following the same procedure underlying proposition 1, since equation (30) implicitly defines the reaction function $\tau = \tau(t^i, t^{-i}\alpha^i, \alpha^{-i})$, $\; i = A, B$, we can evaluate how a change in α^i affects the federal tax rate, i.e. $\frac{\partial \tau}{\partial \alpha^i} = -\frac{\partial G / \partial \alpha^i}{\partial G / \partial \tau}$. By assuming that $\partial G / \partial \tau < 0$ by the second order condition of the problem in (29), then $sign\ \frac{\partial \tau}{\partial \alpha^i} = sign\ \frac{\partial G}{\partial \alpha^i}$, $i = A, B$. Thus, from (30), it is easy to check that

$$\frac{\partial G}{\partial \alpha^i} = \Lambda \left[\frac{d\rho}{d\tau} \frac{dS_i}{d\rho} - \sum_{j=A,B} \frac{d\rho}{d\tau} \left(K^j_{r^j} - D^j_{r^j} \right) \right] - \Delta \sum_{j=A,B} \frac{d\rho}{d\tau} \left(K^j_{s^j} - D^j_{s^j} \right) + \tag{51}$$

$$+ \Theta^i \sum_{j=A,B} \left(\frac{\partial t^j}{\partial \alpha^j} K^j_{r^j} - \tau D^j_{r^j} \right) + \Gamma^i \sum_{j=A,B} \left(\frac{\partial t^j}{\partial \alpha^j} K^j_{s^j} - \tau D^j_{s^j} \right), \quad i = A, B, \tag{52}$$

where

$$\Lambda \equiv \sum_{j=A,B} \frac{\partial \rho}{\partial t^j} \frac{\partial t^j}{\partial \tau} + 1,$$

$$\Delta \equiv \sum_{j=A,B} \frac{\partial \rho}{\partial t^j} \frac{\partial t^j}{\partial \tau} + 1 - \tau.$$

Notice that condition i) $1 - \alpha^i < \left| \frac{d\rho}{d\tau} \right| < \frac{\partial t^i}{\partial \tau}$ implies $\Theta^i > 0$ and $\Gamma^i < 0$ while condition ii) $1 - \tau < \left| \sum_{j=A,B} \frac{\partial \rho}{\partial t^j} \frac{\partial t^j}{\partial \tau} \right| < 1$ implies $\Lambda > 0$ and $\Delta < 0$. Accordingly, in (51), it is easy to check that $\frac{\partial \tau}{\partial \alpha^i} < 0$ because of Assumptions 1 and 2, and (31).$\square$

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