Working Papers - Economics

# Platform Competition with Free Entry of Sellers

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Working Paper N. 22/2021

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# Platform Competition with Free Entry of Sellers

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December 2021

**Abstract**

We study platforms setting access prices and commissions on revenues of sellers engaged in monopolistic competition with free entry, as the app providers on the app stores of Apple and Android devices. Competition to attract buyers and sellers induces the platforms to redistribute all the revenues through lower access prices and set the optimal commission rates from the point of view of consumers, taking into account the pass-through on the prices of sellers, the elasticities of demand and surplus for their services and the elasticity of entry with respect to profitability. We discuss the role of heterogeneous sellers, substitutability between sellers's products and the introduction of platforms's products, as well as some limitations of the basic alignment of interest of platforms and consumers due to direct channels for sellers and consumer myopia.

Key words: Digital platforms, Third-party Sellers, Commissions, Entry.
JEL Code: L1, L4.

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[1]Florence School of Economics and Management, Florence and Charles River Associates, London. I am extremely grateful to Paolo Bertoletti, Gary Biglaiser, Cristina Caffarra, Greg Crawford, Dominik Fischer, Patrick Rey, Fiona Scott-Morton, Xavier Vives and in particular to Benno Buehler, Özlem Bedre-Defolie, Martin Peitz and participants at the Cresse Conference in Heraklion, Greece, for comments and discussions. Nevertheless any responsibility for the results is my own. I am currently consulting Apple and Amazon in ongoing antitrust investigations, but this work reflects only my own opinions. *Correspondence*: Federico Etro: Florence School of Economics and Management, Via delle Pandette 32, Florence, 50127. Phone: 055-2759603. Email: *federico.etro@unifi.it.*

# 1 Introduction

Digital platforms as app stores and online marketplaces attract third party sellers and set percentage commissions on their sales. They compete to attract customers and sellers monetizing on both sides as typical of two-sided marketplaces (Rochet and Tirole, 2003; Armstrong, 2006; Belleflamme and Peitz, 2019; Karle *et al.*, 2020; Jeon and Rey, 2021). Some of these platforms have been recently under antitrust investigation for exclusionary abuses, even if the underlying concerns appear more about exploitative abuses, associated with excessive commissions aimed at extracting sellers's profits or biasing their entry choices.[2] In this work we analyze the impact of these commissions on the endogenous entry of sellers and on the welfare of consumers that purchase their services and products, suggesting that some of these concerns may have been missplaced.

Our main application is to competing mobile application ecosystems, which feature hundreds of thousand apps. The distribution of sales and profits of app developers on these platforms is highly skewed, with a large number of small apps in the long tail (Carere, 2012; Bresnahan *et al.*, 2015). In a study of more than a million apps running on Apple's App Store, Google Play Store and Windows, Hyrynsalmi *et al.* (2016) found that 97% of the apps were single-homing, with 3% of the apps available on at least two platforms and only 0.14% on all the three platforms. However, about half of the superstar apps featured in top 100 applications' listings were already multihoming as well as raising a disproportionate part of revenues and profits.[3] In this environment, commissions should be set to balance the objectives of attracting new apps and buyers while monetizing on superstar apps.

An online marketplace such as Amazon sets commission rates on third party sales and customer fees on Prime customers, trading off the need to attract buyers and sellers from competing retailers and monetize on both sides. It hosts millions of third party sellers who face low entry costs, again with a highly skewed distribution of revenues and profits, and a long tail of small single-homing sellers (Hagiu and Wright, 2015; Masden and Vellodi, 2021): attracting these small sellers is important not only for the commission revenues they generate, but because they bring new customers to the marketplace.[4] Also the success

[2] See Acemoglu (2020) for a related analysis expressing concerns that go beyond antitrust and are about entry and directed technological change by sellers.

[3] As noticed by Bresnahan *et al.* (2015), "the largest costs of multi-platform supply are the marketing costs to reach a new population of users on the other platforms. Before even being considered for download by consumers, an app needs to attract the potential user's attention out of over 1 million apps on either Google Play store or iTunes App store." Technological costs are emphasized by a study for the European Commission (Duch-Brown, 2017): "Apps for different app stores have to be written using specific code libraries (Swift or Objective-C for iOS/Java for Android) using app store-specific Software Development Kits. Targeting multiple app stores involves significant effort in re-writing or modifying the apps so that they can be included in the corresponding app store." (p. 17).

[4] According to Duch-Brown (2017), multi-homing by sellers exhibits low/medium levels of adoption in online marketplaces compared to the high levels of ad-funded platforms for online advertising and social networks.

of e-book readers, video game consoles, media and cloud platforms relies both on the number of customers attracted and on the number and variety of books, games and content that they host, with crucial interactions between the two sides.

In recent work we have argued that device-funded platforms tend to internalize the interest of consumers in setting commissions on third party sales because their revenues are partly shifted back through lower access prices or better quality of the platform (Etro, 2021a,b). The mechanism was centred on the impact of the commissions on the prices of third party sellers, taking as given their number. Here we show that such a mechanism is strengthened under platform competition and with free entry of sellers, we characterize the determinants and the comparative statics of the equilibrium commission rates, and we discuss conditions under which they are set at efficient levels from the point of view of consumers or at suboptimal levels.

We model competition between two horizontally differentiated platforms (Armstrong, 2006) setting a uniform commission rate on third party sales: for instance, both Apple and Google have set commissions at 30% on in-app purchases without changes for over a decade (except for a reduction to 15% on renewed subscriptions and small apps). Contrary to most of the literature on two-sided platforms, we focus on long run decisions by sellers on entry on each platform, which takes place before the platforms set access prices for the buyers: in practice, manufacturers can change the prices of their devices in the short run when the set of available apps is largely pre-determined.[5] The sellers are engaged in monopolistic pricing facing homogeneous costs and independent demands in the benchmark model, but we also extend the analysis to heterogeneous costs and imperfect substitutability. The framework generates the simultaneous presence of multi-homing sellers making high profits and single-homing sellers making lower profits, and heterogeneity between sellers allows us to reproduce also the skewed distribution of profits and revenues, in line with the evidence on mobile apps (Bresnahan *et al.*, 2015; Hyrynsalmi *et al.*, 2016). The equilibrium commission rates are low when the pass-through on sellers's prices is high, when the demand faced by the sellers is highly elastic to price changes and the surplus they generate rather inelastic, and when the distribution of entry costs is concentrated on high costs so that entry is highly sensitive to profit changes. Platform competition strengthens the tendency to shift the commission revenues to consumers through lower access prices, generating equilibrium commission rates that maximize consumer welfare: similarly to models of competition in the utility space (Armstrong and Vickers, 2001), each platform makes the commitment on the commission that maximizes the joint surplus with its customers, because it can then extract part of it with the access price. In practice, competition for customers induces the platforms to

[5]The framework applies exactly to the "walled garden" of a device-funded platform such as Apple, but also an ad-funded platform as that of Android (or a manufacturer such as Samsung) can shift commission revenues to consumers through higher quality of app stores (or lower device prices). Even online marketplaces are rapidly expanding the use of customers' fees to access preferential conditions (and of course they invest in the quality of their services).

reduce excessive commission rates to attract more sellers, and therefore buyers, and to increase insufficient commission rates to shift all revenues to buyers through lower prices of the devices.

The alignment of incentives of platforms and customers holds also with heterogeneous sellers, imperfectly substitutable products and with the introduction of own products by the platforms (as apps by Apple and Google for their devices). However, it could break down in scenarios that are potentially relevant for the competition between mobile platforms. In particular, in the presence of direct channels for sellers that are not subject to the payment of commissions, consumers purchase through them neglecting the externality that they generate on all the other consumers (by inducing higher access prices): this forces the platforms to set commissions below the efficient level. Instead, in the presence of myopia in consumers' purchasing decisions, the platforms set commissions above the efficient level, though all the associated revenues are still redistributed to consumers (through lower access prices).

The analysis is relevant for debates concerning the commissions set by Apple on third-party apps (as in the US litigation with Epic Games) or the provision of Apple's apps competing with them (as in the EU case on music streaming services by Spotify). Our results are in contrast with the idea that a device-funded platform would harm consumers in the long run through excessive commissions aimed at foreclosing or biasing entry. As mentioned, there are circumstances where commissions can be set by a device-funded platform above the levels that maximize consumer welfare, whose consequences for sellers's entry should be evaluated on a case by case basis, but we need to stress that setting prices above the level that maximizes consumer welfare should not the relevant benchmark for antitrust intervention or regulation.

**Related literature** Our work belongs to the literature on two-sided platforms (Rochet and Tirole, 2003; Caillaud and Jullien, 2003; Armstrong, 2006; Hagiu, 2006) and is particularly related to studies on competition on one side of the platforms (Bresnahan *et al.*, 2015; Belleflamme and Toulemonde, 2016; Belleflamme and Peitz, 2019; Gautier *et al.*, 2021; Padilla *et al.*, 2021). However, here we examine a fully fledged form of monopolistic competition with free entry of sellers rather than relying on stylized entry decisions by sellers that generate externalities on buyers, and we allow the platforms to precommit only to the commission rate before entry of sellers takes place, and not to the access price for the buyers. Under similar assumptions and timing, recent works by Bertoletti (2021) and Jeon and Rey (2021) have actually emphasized the emergence of excessive commissions with negative consequences on entry of monopolistically competitive sellers and on consumer welfare.[6] In particular, Bertoletti (2021) has developed a novel representative agent model with monopolistic competition between sellers under platform-specific entry costs, where two

[6] Etro (2021a) emphasizes other sources of excessive commissions, such as vertical differentiation between two asymmetric platforms (namely a device-funded one and an ad-funded one), with sellers that differ across platforms.

perfectly substitutable platforms compete by setting commissions and quantities of devices *à la* Cournot, and consumers perceive devices and third party products as imperfect complements. These assumptions are not related with the mentioned cases of platform competition, but the analysis confirms the link between commission revenues and access prices, emphasizing the emergence of excessive commissions that reduce entry of sellers. Relying on competition *à la* Bertrand between differentiated platforms, we restore the alignment between equilibrium and optimal commissions from the point of view of consumers. Jeon and Rey (2021) have compared percentage commissions and wholesale prices set by device-funded platforms where sellers generate a stochastic surplus at no cost except for a fixed entry cost that is not platform-specific: in this environment the platforms always host all the sellers once they enter, and the authors show that they select percentage commissions and set them so high that, paradoxically, there is no entry at all. Departing from their assumptions, we show that platform competition can account for the simultaneous presence of singlehoming and multihoming sellers and more realistic levels of commission and entry rates. Moreover, we develop alternative mechanisms through which the commissions may not be efficient, which could work together with the other mentioned mechanisms.

Other works have been focused on device-funded platforms and marketplaces. Gans (2012) was one of the first to emphasize the inefficiency of sellers's pricing on a device-funded platform and the link between commissions and the price of devices. Rey and Tirole (2019) have sketched a model of competition between device-funded platforms with single-homing apps to show that agreements between app providers on price caps would also benefit consumers in spite of the induced increase in the price of devices, but they have not analyzed commissions on apps. On antitrust policies for device-funded platforms see also Caffarra (2019), Choi and Jeon (2020), Teh (2020), Zennyo (2020), Ushakov (2021) and Casner and Teh (2021). On the expanding literature on marketplaces see Fu *et al.* (2021), Hervas-Drane and Shelegia (2021), Masden and Vellodi (2021), Hagiu *et al.* (2022), and, in particular, Anderson and Bedre-Defolie (2021), Zennyo (2021), Etro (2021c) and Shopova (2021), who have compared the commission rates set by pure and hybrid marketplaces on third party sellers. In our environment, we show that hybrid platforms may set either higher or lower commissions than pure platforms (respectively to divert demand toward their own products or expand demand of all sellers and attract more buyers), shifting again the generated revenues to consumers through lower access prices.

The rest of the paper is organized as follows. Section 2 presents the basic model and derives the main efficiency result. Section 3 extends the model to account for heterogeneity between sellers. Section 4 introduces imperfect substitutability between the products of the sellers. Section 5 discusses limitations of the baseline model. Section 6 concludes. The Appendix contains further details.

# 2 The Model

Two horizontally differentiated platforms $i = 1, 2$ compete for consumers and sellers *à la* Hotelling. Consumers are uniformly distributed in the unit interval with types $x \in [0, 1]$, and select one of the platforms at a cost $d > 0$ times the distance from the selected platform. Platforms are located at the extremes of the unit interval, in $l_1 = 0$ and $l_2 = 1$, produce devices at marginal cost $h > 0$, sell them at a price $P_i$, and host a (normalized) number $n_i \in [0, 1]$ of products by third-party sellers, earning commissions on their revenues at rate $\tau_i \in [0, 1]$.[7] Sellers are engaged in monopolistic competition, setting prices $p_i(\omega)$ for product $\omega$ on each platform $i = 1, 2$, and generating an incremental surplus function $v(p(\omega))$ positive, decreasing and convex in the price. Therefore, consumer's utility on platform $i$ is:

$$V(x) = \int_0^{n_i} v(p_i(\omega))d\omega - d\,|l_i - x| - P_i \tag{1}$$

and, by Roy's identity, a seller $\omega$ faces the demand:

$$q_i(\omega) = |v'(p_i(\omega))| \tag{2}$$

per consumer on platform $i$.[8]

Each seller bears an entry cost $f$ to operate on a platform and a marginal cost $c \geq 0$.[9] To capture uncertainty on sellers's profitability, each one of the two entry costs is independently and identically drawn from a known and common distribution $G(f)$ with support $[0, \bar{f}]$. As an example we will often employ a Pareto distribution:

$$G(f) = \left(\frac{f}{\bar{f}}\right)^{\kappa} \tag{3}$$

with shape parameter $\kappa > 0$.

The timing of the game is the following: 1) the platforms simultaneously set their commissions; 2) entry of sellers takes place on each platform; 3) the platforms simultaneously set the access prices and 4) the sellers set their prices under

[7] An alternative interpretation is that platforms invests in quality, enhancing the valuation of the platform. We exclude negative commissions because they would generate free riding incentives by the sellers (as in Choi and Jeon, 2021).

[8] The framework is inspired by the analysis of monopolistic competition under indirect additivity in Bertoletti and Etro (2017). This implies independent products, which simplifies the analysis of pricing and entry by the sellers. However, one can extend the framework to more general microfoundations implying substitutable products and strategic interactions between sellers, without major changes: an example of an alternative microfoundation based on Nocke and Schutz (2018) is considered in Section 4.

[9] For instance, native apps must be written (and constantly updated) in the specific programming language of each app store and satisfy the specific requirements and quality standards of the platform, which differentiates substantially costs and opportunity costs of entry on each platform. As noticed by Bresnahan *et al.* (2015), however, the main platform-specific entry costs are the marketing costs to reach a new population of users. Similar entry costs apply for third party sellers on online marketplaces. Notice that Bertoletti (2021) makes our same assumption in a deterministic environment, while Jeon and Rey (2021) assume a unique and uncertain cost for entry on both platforms, which leads to multihoming by each seller.

monopolistic competition. This captures the commitment that mobile platforms as those of Apple and Google have adopted on their commission rates on app subscriptions and in-app purchases when they introduced their app stores, and the fact that entry decisions by app developers are typically long term decisions based on expected profitability, while both the prices of devices and the prices of apps can be changed in the short term.[10]

As a preliminary remark, we should notice that tipping equilibria are possible in this environment due to network effects. Our interest, however, is about equilibria where both platforms are active and all consumers purchase a device, which requires large enough differentiation between the platforms. In this case, comparing the utility from the two platforms, the market share of platform $i$ can be determined as:

$$x_i = \frac{1}{2} + \frac{\int_0^{n_i} v(p_i(\omega))d\omega - \int_0^{n_j} v(p_j(\omega))d\omega - (P_i - P_j)}{2d} \tag{4}$$

which allows us to derive the expected profits of all players. We now solve the game by backward induction.

## 2.1 Sellers's prices

Under monopolistic competition each seller $\omega$ on platform $i = 1, 2$ sets the price to maximize profits:

$$\pi_i(\omega) = x_i[(1 - \tau_i)p_i(\omega) - c] \left|v'\left(p_i(\omega)\right)\right| \tag{5}$$

taking as given the market share of the platform $x_i$ (because with a large number of atomistic sellers each one has a negligible impact on the market share). With a positive marginal cost, the profit-maximizing price follows the same rule $p = p(\tau_i)$ on each platform satisfying:

$$p = \frac{\varepsilon(p)c}{(1 - \tau_i)(\varepsilon(p) - 1)} \tag{6}$$

where $\varepsilon(p) \equiv -\frac{v''(p)p}{v'(p)} > 1$ is the demand elasticity assumed larger than unity to have an interior solution. This implies that an increase of the commission rate is translated into higher prices because it affects the effective marginal cost.[11]

[10] This timing differs from the one usually adopted in models of platform competition, where entry of both sellers and buyers takes place after the platforms set the prices for both sides (see Hagiu, 2006, for a general analysis of timing issues). I am grateful to Martin Peitz for comments on this aspect.

[11] With a zero marginal cost, the profit-maximizing price would instead satisfy the rule $\varepsilon(p) = 1$, which is independent from the commission: in the absence of variable costs (as for apps providing software) the commission on revenues is a commission on profits which does not affect the profit-maximizing price. As noticed by Rey and Tirole (2019) in a related environment, the double marginalization problem on the platform implies that the sellers would agree on a price cap that would benefit consumers.

The pass-through elasticity of the price with respect to the marginal cost can be computed as:

$$\eta(p) = \frac{1}{1+\frac{\xi(p)}{\varepsilon(p)-1}} \geq 0$$

where $\xi(p) \equiv \frac{\varepsilon'(p)p}{\varepsilon(p)}$ is positive when the demand elasticity is increasing in the price, a property known as the Marshall's second law of demand. This property insures incomplete pass-through $\eta(p) < 1$, approaching full pass-through only under isoelastic demand (the case assumed by Bertoletti, 2021). More important, the impact of the commission on the price is:

$$\frac{\partial p(\tau_i)}{\partial \tau_i} = \frac{\eta(p(\tau_i))p(\tau_i)}{1-\tau_i} \tag{7}$$

which is therefore a positive function of the same pass-through elasticity. In practice, when a higher price exposes a seller to a lower and more elastic demand, a higher commission tends to have a smaller impact on the price of the seller.

Accordingly, the commission of each platform affects the surplus of its consumers $v(p(\tau_i))$ and the revenues of its sellers $r(\tau_i) = p(\tau_i)\,|v'(p(\tau_i))|$. Only with zero marginal cost the pass-through is null and surplus and revenues are independent from the commission, with fixed expected values, say $v$ and $r$ (the case assumed by Jeon and Rey, 2021).

## 2.2 Platforms's prices

Given the prices of the sellers, we can solve for the market shares of the two platforms and therefore the profits:

$$\Pi_i = \left[\frac{1}{2} + \frac{n_i v(p(\tau_i)) - n_j v(p(\tau_j)) - (P_i - P_j)}{2d}\right][P_i - h + \tau_i n_i r(\tau_i)] \tag{8}$$

where the revenues per user include the markup on the device and the commission revenues from all the sellers. Each platform $i$ independently sets its access price $P_i$ to maximize its profits. The Bertrand equilibrium prices can be easily derived as:

$$P_i = h + d + \frac{n_i v(p(\tau_i)) - n_j v(p(\tau_j)) - 2\tau_i n_i r(\tau_i) - \tau_j n_j r(\tau_j)}{3} \tag{9}$$

Each price is given by the sum of the marginal cost, a basic markup associated with platform differentiation, and an additional component that increases in the differential surplus relative to the rival platform and decreases in its commission revenues (partly shifted back to consumers to expand sales) and also in the commission revenues of the rival (because this strengthens competition under strategic complementarity). This implies the equilibrium market shares:

$$x_i = \frac{1}{2}\left[1 + \frac{n_i[v(p(\tau_i)) + \tau_i r(\tau_i)] - n_j[v(p(\tau_j)) + \tau_j r(\tau_j)]}{3d}\right]$$

which depend on the differential between surplus and commission revenues generated on the two platforms.

## 2.3 Free entry of sellers

Given all the equilibrium prices, we can express the gross profits of each seller on platform $i$ as:

$$\pi_i = \left(1 + \frac{\Omega_i}{3d}\right) \frac{(1-\tau_i)r(\tau_i) - c\left|v'\left(p(\tau_i)\right)\right|}{2}$$

where we defined the differential value:

$$\Omega_i = [v(p(\tau_i)) + \tau_i r(\tau_i)]n_i - [v(p(\tau_j)) + \tau_j r(\tau_j)]n_j \tag{10}$$

Sellers enter on each platform as long as their gross profits cover the fixed costs. The free entry system:

$$n_1 = G\left[\left(1 + \frac{\Omega_1}{3d}\right) \frac{(1-\tau_1)r(\tau_1) - c\left|v'\left(p(\tau_1)\right)\right|}{2}\right] \tag{11}$$

$$n_2 = G\left[\left(1 - \frac{\Omega_1}{3d}\right) \frac{(1-\tau_2)r(\tau_2) - c\left|v'\left(p(\tau_2)\right)\right|}{2}\right] \tag{12}$$

can be solved for the number of sellers active on platform $i$, say $n_i = n_i(\tau_i, \tau_j)$. As an example, let us consider zero marginal costs and a uniform distribution for the entry cost, with $r = v = d = \kappa = \bar{f} = 1$. Then, the free entry system can be solved for:

$$n_i(\tau_i, \tau_j) = \frac{(1-\tau_i)(2+\tau_j^2)}{4 + \tau_i^2 + \tau_j^2}$$

In this example, when a platform increases its commission, it is going to reduce the profitability of its sellers and therefore their number, which drives customers to the other platform, attracting more sellers to that platform.

The two free entry conditions (11)-(12) can be combined with (10) to determine the differential value as a function of the commission rates such that:

$$\begin{aligned} \Omega \;=\; & [v(p(\tau_i)) + \tau_i r(\tau_i)]G\left[\left(1 + \frac{\Omega}{3d}\right) \frac{(1-\tau_i)r(\tau_i) - c\left|v'\left(p(\tau_i)\right)\right|}{2}\right] \\ & -[v(p(\tau_j)) + \tau_j r(\tau_j)]G\left[\left(1 - \frac{\Omega}{3d}\right) \frac{(1-\tau_j)r(\tau_j) - c\left|v'\left(p(\tau_j)\right)\right|}{2}\right] \end{aligned} \tag{13}$$

This equation implicitly defines an equilibrium expression for $\Omega = \Omega(\tau_i, \tau_j)$ in function of the two commission rates, which satisfies $\Omega(\tau_i, \tau_j) + \Omega(\tau_j, \tau_i) = 0$ and $\Omega(\tau, \tau) = 0$ for any $\tau$. Since the slope of the left hand side of (13) with respect to $\Omega$ is unitary and the slope of the right hand side is positive, the solution is unique and stable if the latter is smaller than unity. In a symmetric equilibrium for a given commission $t$, this requires:

$$d > [v(p(\tau)) + \tau r(\tau)]\, G' \left|v'\left(p(\tau)\right)\right| \frac{(1-\tau)p(\tau) - c}{3}$$

and this stability condition is satisfied when the two platforms are differentiated enough. For instance, the earlier example provides a unique differential value for any commission rates:

$$\Omega(\tau_i, \tau_j) = \frac{3(\tau_j^2 - \tau_i^2)}{4 + \tau_i^2 + \tau_j^2}$$

which is clearly decreasing in $\tau_i$, increasing in $\tau_j$ and null for equal commissions. Stability holds for any symmetric equilibrium $\tau$ in the unit interval since $1 > (1 - \tau^2)/3$.

### 2.4 Platforms's commissions

In the initial stage each platform $i$ selects $\tau_i$ to maximize its profits, which can be expressed as follows:

$$\Pi_i = \frac{d}{2}\left[1 + \frac{\Omega(\tau_i, \tau_j)}{3d}\right]^2 \tag{14}$$

The platform internalizes the direct impact of its commission on its revenues and on the number of its sellers, but also the indirect impact on the number of sellers of the rival platform (which will be null in equilibrium). More formally, $\tau_1$ is set to maximize $\Omega(\tau_1, \tau_2)$, determining a best response function $\tau_1 = \hat{\tau}_1(\tau_2)$, and $\tau_2$ is set to minimize $\Omega(\tau_1, \tau_2)$, determining a best response function $\tau_2 = \hat{\tau}_2(\tau_1)$, and the symmetric Nash equilibrium must of course satisfy $\tau^e = \hat{\tau}_i(\tau^e)$.[12] For instance, our example provides $\hat{\tau}_i(\tau_j) = 0$ since $\Omega(\tau_i, \tau_j)$ is decreasing in $\tau_i$, and has only a symmetric equilibrium with $\tau^e = 0$ which satisfies the stability condition above.

In the general case, exploiting the envelope theorem, we can compute:

$$\begin{aligned}\frac{\partial\Omega(\tau_i, \tau_j)}{\partial\tau_i} \quad \propto \quad & [v'(p(\tau_i)) + \tau_i \left|v'(p(\tau_i))\right| + \tau_i p(\tau_i) v''(p(\tau_i))]p'(\tau_i)G(\pi_i) + \\ & + r(\tau_i)G(\pi_i) - [v(p(\tau_i)) + \tau_i r(\tau_i)]G'(\pi_i)\left(1 + \frac{\Omega}{3d}\right)\frac{r(\tau_i)}{2}\end{aligned}$$

The best response $\hat{\tau}_i(\tau_j)$ must equate the above expression to zero and the symmetric Nash equilibrium commission rate $\tau^e$ must also satisfy $\Omega(\tau^e, \tau^e) = 0$. Using the pass-through of the commission on the price (7), and manipulating the last expression, an interior Nash equilibrium must solve:

$$1 - \eta(p(\tau^e))\left(1 + \frac{\tau^e \varepsilon(p(\tau^e))}{1 - \tau^e}\right) = \frac{v(p(\tau^e)) + \tau^e r(\tau^e)}{2}\frac{G'[\pi(\tau^e)]}{G[\pi(\tau^e)]} \tag{15}$$

[12] The existence of a unique equilibrium requires the quasi-concavity of $\Omega(\tau_i, \tau_j)$ and the existence of a unique intersection of the best responses. The former condition is assumed here, while the latter can be verified in simple examples. For instance, in case of a Pareto distribution, the best response $\hat{\tau}_i$ is independent from $\tau_j$. Otherwise $\hat{\tau}_i(\tau_j)$ is non-monotonic, with a null derivative at the Nash equilibrium.

where the profits of the sellers are $\pi(\tau) = [(1-\tau)r(\tau) - c\,|v'(p(\tau))|]/2$. The equilibrium commission balances the marginal revenue and marginal cost. The former on the left hand side represents the marginal revenue per seller taking into account the pass-through rate of the commission on prices and the elasticity of demand of the sellers's products. The latter on the right hand side depends on the rate of reduction of the number of sellers due to a higher commission multiplied by the marginal benefits per seller: the more elastic is entry, the higher is the marginal cost of raising the commission.

In equilibrium the two platforms host the same number of sellers $n(\tau^e) = G(\pi(\tau^e))$ due to symmetry, but a fraction of them multihomes making large enough profits to be active on both platforms and the other sellers singlehome on the only platform where they manage to cover the entry cost. More precisely $n(\tau^e)^2$ sellers multihome, and $2n(\tau^e)[1-n(\tau^e)]$ singlehome, therefore depending on the nature of uncertainty on profits we can change the fraction of singlehoming firms. As mentioned, the evidence on mobile platforms (Hyrynsalmi *et al.*, 2016) suggests that a majority of sellers are singlehoming, and the model is consistent with this if the distribution is skewed enough to have $G(\pi(\tau^e)) < 2/3$. The distribution of profits depends on the distribution of the entry costs (and in the next section we will extend the model to obtain also heterogeneity in gross revenues).

The final equilibrium price of each device is:

$$P^e = h + d - \tau^e n(\tau^e) r(\tau^e)$$

emphasizing that all the commission revenues are shifted back to consumers, leaving the profits $\Pi_1 = \Pi_2 = d/2$, which depend only on the differentiation between platforms. This result differs from what found in case of a monopolistic platform or one competing with an ad-funded platform (Etro, 2021a), where only part of the commission revenues were shifted into lower access prices. Price competition and strategic complementarity lead the platforms to fully shift revenues to consumers in terms of lower access prices (or higher quality), independently from the level of differentiation.

## 2.5 Implications

To appreciate the implications of the equilibrium commission, it is useful to evaluate it in a few extreme cases. When entry is not sensitive to changes in profitability ($G' = 0$), the right hand side of (15) is null and the equilibrium commission boils down to:

$$\tau^e = \frac{1-\eta}{1-\eta+\varepsilon\eta} \tag{16}$$

which corresponds to the one emphasized in Etro (2021a) under homogeneous sellers. The commission is positive as long as $\eta < 1$, namely with an incomplete cost pass-through, and null under isoelastic demands associated with full pass-through (as in Bertoletti, 2021, under isoelastic demands and price competition). More important, a higher pass-through on sellers's prices and a more elastic demand of the products induce the platforms to set lower commission rates to limit the impact on final prices and purchases of the products.

In the simple case of zero marginal cost and zero pass-through ($\eta = 0$) we obtain $G[\pi(\tau^e)] = \frac{v+\tau^e r}{2} G'[\pi(\tau^e)]$ and $\tau^e$ increases in the fixed revenue $r$ and decreases in the fixed surplus $v$ generated by each seller. Under the Pareto distribution (3) we can explicitly derive it as:

$$\tau^e = \frac{r - \kappa v}{r(1+\kappa)} \tag{17}$$

which is positive assuming $r/v > \kappa$ and null in the example with a uniform distribution and $r = v = \kappa = 1$. When the distribution of the entry cost is more concentrated toward its upper bound ($\kappa$ is higher) the equilibrium commission is smaller because entry is more elastic with respect to profitability.

Let us finally consider the case where the commission affects both prices and entry. Assuming again the Pareto distribution (3), we can compute the equilibrium commission as:

$$\tau^e = \frac{(1-\eta)[r(\tau^e) - c\,|v'(p(\tau^e))|] - \kappa v(p(\tau^e))}{r(\tau^e)(1+\kappa)}$$

which generalizes the earlier formula taking into account sellers's costs and endogenous pricing by sellers. Using the pricing rule (6) and defining $\zeta(p) \equiv -\frac{v'(p)p}{v(p)} > 0$ as the elasticity of the surplus function with respect to the price, we can finally simplify the last expression for the equilibrium commission as follows:

$$\tau^e = \frac{1 - \eta - \frac{\varepsilon}{\zeta}\kappa}{1 - \eta + \varepsilon\eta + \varepsilon\kappa} \tag{18}$$

which generalizes (16). This equilibrium commission rate is still low when the pass-through elasticity is high and the demand is highly elastic to price changes, but now also when the surplus is rather inelastic to price changes (because the commission affects revenues more than surplus), and the cost distribution is concentrated on high costs (because the commission disincentivizes entry substantially).

We should remind the reader that a formula as (18) is implicit. Explicit derivations for the equilibrium commission are available, for instance, under zero marginal cost with zero pass-through ($\eta = 0$) and a uniform distribution ($\kappa = 1$): then, a translated power surplus function $v(p) = \frac{(a-p)^{1+\gamma}}{1+\gamma}$ with substitutability parameter $\gamma \in (0, \infty)$ provides the equilibrium commission $\tau^e = \frac{1}{1+\gamma} \in (0, 1)$.[13]

### 2.5.1 Optimal commissions

Always focusing on scenarios with sufficient differentiation to exclude the optimality of tipping, a social planner maximizing consumer welfare would set

[13] Instead, in case of exponential subutilities $v(p) = e^{-p/\mu}$, where $\mu > 0$ parametrizes product differentiation for a loglinear demand, we obtain $\zeta(p) = \varepsilon(p)$, and the equilibrium commission is $\tau^e = 0$ always.

a common commission $\tau$ that splits consumers equally across platforms and maximizes the value generated on each platform. Clearly, the problem:

$$\max_{\tau} \left[v(p(\tau)) + \tau r(\tau)\right] G\left[\pi(\tau)\right]$$
$$s.v. \quad : \quad \pi(\tau) = \frac{(1-\tau)r(\tau) - c\left|v'\left(p(\tau)\right)\right|}{2}$$

has $\tau^e$ as its solution, confirming that the equilibrium is efficient and maximizes consumer welfare. The intuition derives from the fact that, if a commission is above the efficient level, the platform competes by reducing it to attract more sellers and, consequently, more customers from the rival platform. Instead, when the commission is below the efficient level, the platform competes by increasing its commission to expand revenues and reduce the price of its devices, to attract more customers and, consequently, more sellers from the rival platform. Similarly to competition on utility space and using two-part tariffs (Armstrong and Vickers, 2001), the platforms converge toward the commission rates that maximize consumers' utility with the expectation of extracting part of the surplus through the access fees.[14]

Finally, a zero commission would actually increase total users' welfare defined as including also the profits of the sellers, but in this model this would decrease the aggregate welfare of consumers in favour of inframarginal profits of sellers.[15] These results are relevant for the ongoing debate on platform regulation on both sides of the Atlantic, which appears to be driven by the presumption that commissions are excessive (as transaction fees on apps for payment services, while clearly these commissions are something different, namely prices to sell through the platforms): in the case of device-funded platforms, this may not be the case, because consumers benefit indirectly from the commissions on apps, through devices of lower price or higher quality.

### 2.5.2 Hybrid platforms

In our environment, the platforms have also an incentive to introduce their own products, as apps by Apple and Google or private label products by Amazon, which can affect the choice of the commissions. To sketch the idea, suppose that each platform replaces $m$ products by sellers with its own products. Also the profits generated on these products will be shifted to consumers through lower access prices. However, in this framework the platforms would price their own products at marginal cost to maximize the total value created (as in Etro, 2021a). Taking into account that the effective number of active entrants is re-

[14] The commission is also efficient from the point of view of the platforms, maximizing their joint profits *ex ante*, which confirms the alignment of interests of device-funded platforms and consumers emphasized in related contexts (Buehler, 2015).

[15] Departing from the covered market assumption may affect these results, on one side introducing further distortions arising from the market power of the platforms and on the other side shifting toward sellers part of the benefits of larger demand (generated by higher commissions and lower access prices).

duced by $m$ on each platform, the equilibrium relation (15) is amended as follows:[16]

$$1-\eta\left(1+\frac{\tau^e\varepsilon}{1-\tau^e}\right)=\frac{v+\tau^e r}{2}\frac{G'(\pi)}{G(\pi)-m}$$

As long as entry of sellers is elastic, this implies an equilibrium commission that decreases in $m$. In practice, hybrid platforms would find it convenient to reduce commissions to attract new sellers, so as to increase revenues on the extensive margin (rather than on the intensive one) and attract buyers as well.

The equilibrium commission remains the optimal one for consumers. Moreover, the introduction of products by a platform is profitable only if it increases the total surplus created on the platform and shared with consumers through lower prices and more variety, which excludes forms of harmful foreclosure in this setup (as in Etro, 2021a). This is in contrast with results obtained in a model with saturated demand by Padilla *et al.* (2021) for the simple reason that in that work the commissions are assumed to be null: to the extent that the platforms can monetize on third party sellers, they have incentives to introduce their own products giving up to commission revenues only when this creates additional gains to be shared with consumers. We will explore further the role of hybrid platforms after augmenting the model with imperfect substitutability between products, which is a more realistic assumption when considering competiton between products by the platforms and third party sellers.

# 3 Heterogeneous Sellers

Until now we have considered identical products by sellers. To account for the wide differences in sales and revenues obtained by different apps on app stores and sellers on marketplaces, we now extend the basic model to heterogeneity between sellers in an efficiency parameter. We condition entry to the payment of a fixed cost on each platform in the spirit of selection models *à la* Melitz (2003). Then, prices, revenues and market shares are endogenously differentiated across sellers, and only sellers that are efficient enough become operative on at least one platform, with some of them operative on both platforms (as suggested by Bresnahan *et al.*, 2015).

We assume that the sellers are heterogeneous in the marginal cost, though analogous results would emerge with heterogeneity in a preference parameter. Accordingly, the cost of each seller $c$ is now drawn from a distribution $\tilde{G}(c)$ on $[0,\bar{c}]$ for each platform. The fixed cost to be operative on a platform is $F>0$.

[16] Assuming that platforms's products are provided at cost $\bar{c}$ and price $\bar{p}_i$, generating a surplus $\bar{v}(\bar{p}_i)$, the differential value (13) becomes:

$$\begin{aligned}\Omega_i &= [v(p(\tau_i))+\tau_i r(\tau_i)](G(\pi_i)-m)-[v(p(\tau_j))+\tau_j r(\tau_j)](G(\pi_j)-m)+\\ &+m\left[\bar{v}(\bar{p}_i)+(\bar{p}_i-\bar{c})\left|\bar{v}'(\bar{p}_i)\right|-\bar{v}(\bar{p}_j)-(\bar{p}_j-\bar{c})\left|\bar{v}'(\bar{p}_j)\right|\right]\end{aligned}$$

which implies marginal cost pricing $\bar{p}_i=\bar{p}_j=\bar{c}$ independently from the commissions and generates the equilibrium commissions in the text.

Given the commission rate $\tau_i$ on platform $i$, the most efficient sellers in $[0, \hat{c}_i]$ are operative and set a price $p_i(c)$ depending on the efficiency parameter.

Consumer utility from a device of platform $i = 1, 2$ can be expressed as follows:

$$V(x) = \int_0^{\hat{c}_i} v(p_i(c))d\tilde{G}(c) - P_i - d\,|l_i - x| \tag{19}$$

where the surplus depends on the set of active sellers. Comparing the utility of the two platforms, the market share of platform $i$ can be determined as:

$$x_i = \frac{1}{2} + \frac{\int_0^{\hat{c}_i} v(p_i(c))d\tilde{G}(c) - \int_0^{\hat{c}_j} v(p_j(c))d\tilde{G}(c) - (P_i - P_j)}{2d} \tag{20}$$

In the final stage the gross profits of each seller $c$ active on platform $i$ are:

$$\pi_i(c) = x_i\left[(1-\tau_i)p_i(c) - c\right]|v'(p_i(c))|$$

implying monopolistic competition prices $p(c) = p(c, \tau_i)$ defined in function of the marginal cost as:

$$p(c) = \frac{\varepsilon(p(c))c}{(1-\tau_i)(\varepsilon(p(c)) - 1)} \tag{21}$$

The pass-through elasticity is defined by a function $\eta(p(c))$ which now increases in the marginal cost as long as the demand elasticity increases in the price.

Each platform $i$ sets the price of its device to maximize profits:

$$\Pi_i = x_i\left(P_i + \tau_i \int_0^{\hat{c}_i} p(c, \tau_i)\,|v'(p(c, \tau_i))|\,d\tilde{G}(c) - h\right)$$

The Bertrand equilibrium prices can be derived as before, and they are always decreasing in the commission revenues generated on each platform. This delivers the equilibrium market shares:

$$x_i = \frac{1}{2}\left(1 + \frac{\Omega(\hat{c}_i, \hat{c}_j)}{3d}\right) \tag{22}$$

where we defined the differential value of the platforms as:

$$\begin{aligned}\Omega(\hat{c}_i, \hat{c}_j) \quad = \quad & \int_0^{\hat{c}_i} \left[v(p(c, \tau_i)) + \tau_i p(c, \tau_i)\,|v'(p(c, \tau_i))|\right] d\tilde{G}(c) + \\ & - \int_0^{\hat{c}_j} \left[v(p(c, \tau_j)) + \tau_j p(c, \tau_j)\,|v'(p(c, \tau_j))|\right] d\tilde{G}(c)\end{aligned}$$

Accordingly, the profits of the sellers can be rewritten in function of the cut-offs as:

$$\pi_i(c) = \frac{1}{2}\left(1 + \frac{\Omega(\hat{c}_i, \hat{c}_j)}{3d}\right)\left[(1-\tau_i)p(c, \tau_i) - c\right]|v'(p(c, \tau_i))|$$

Entry takes place on each platform for sellers with marginal cost below cut-offs satisfying the following zero profit conditions:

$$\left(1+\frac{\Omega(\hat{c}_1,\hat{c}_2)}{3d}\right)\frac{[p(\hat{c}_1,\tau_1)(1-\tau_1)-\hat{c}_1]\,|v'(p(\hat{c}_1,\tau_1))|}{2} \quad = \quad F$$
$$\left(1-\frac{\Omega(\hat{c}_1,\hat{c}_2)}{3d}\right)\frac{[p(\hat{c}_2,\tau_2)(1-\tau_2)-\hat{c}_2]\,|v'(p(\hat{c}_2,\tau_2))|}{2} \quad = \quad F$$

The two zero profit conditions are interdependent, and their system defines now the two cut-offs $\hat{c}_i(\tau_i,\tau_j)$ in function of the two commission rates (rather than the two numbers of sellers on each platform as in the benchmark model). Typically, a higher commission by a platform induces a selection effect on that platform, namely reducing the cut-off marginal cost and limiting access to a more restricted selection of efficient sellers, while an opposite effect takes place on the rival platform.

In the initial stage the commissions are set to maximize platforms' profits:

$$\Pi_i=\frac{d}{2}\left(1+\frac{\Omega(\hat{c}_i(\tau_i,\tau_j),\hat{c}_j(\tau_j,\tau_i))}{3d}\right)^2$$

which depend on the two commission rates through the cut-offs on the two platforms. In Appendix A we derive the implicit expression for the symmetric Nash equilibrium commission as follows:

$$t^e=\frac{1-\mathbb{E}[\eta]-\frac{\hat{\varepsilon}}{\hat{\zeta}}\hat{\kappa}}{1-\mathbb{E}[\eta]+\mathbb{E}[\eta\varepsilon]+\hat{\varepsilon}\hat{\kappa}} \tag{23}$$

where the expectations of the elasticities are weighted with market shares, and the elasticities $\hat{\varepsilon}$ and $\hat{\zeta}$, as well as the term $\hat{\kappa}$ defined in Appendix A, apply all to the marginal sellers. Compared to the commission of the baseline model (18), the relevant elasticities of pass-through and demand are replaced with their weighted average values across sellers, and the additional terms account for the selection effects induced by the commissions on the entry of sellers on their platforms.

Also in this more general framework, the equilibrium commission corresponds to the solution of the problem of maximization of consumer welfare:

$$\max_{\tau}\int_0^{\hat{c}}\left[v(p(c,\tau))+\tau p(c,\tau)\,|v'(p(c,\tau))|\right]d\tilde{G}(c) \tag{24}$$
$$s.v. \quad : \quad \frac{[p(\hat{c},\tau)(1-\tau)-\hat{c}]\,|v'(p(\hat{c},\tau))|}{2}=F$$

confirming our main result for which platform competition with endogenous entry of sellers on each platform leads to efficient choices from the point of view of consumers (see Appendix A). While in the benchmark model with heterogeneity in entry costs the commission could achieve the price and the number of sellers that maximized consumer welfare, in this framework with heterogeneity in marginal costs the commission achieves the price distribution and the set of sellers that maximize consumer welfare. The advantage of this framework for quantitative applications is that it allows one to reproduce heterogeneous pricing across sellers and, most of all, the highly skewed distribution of sellers' sales and revenues (as well as profits) which emerges on both appstores and online marketplaces (Carere, 2012; Hagiu and Wright, 2015), where the leading sellers account for most of the revenues and also of the commission revenues.

We should remark that in practice the platforms would gain from adopting a more flexible monetization policy than the uniform commission assumed here. In particular, the elimination of any commission on sellers that are not active in equilibrium would allow some of them to enter and generate additional benefits. This is consistent with the zero commission applied by Apple and Google on app developers with revenues below a certain threshold, as well as with promotions for small sellers implemented by most marketplaces. Moreover, a platform would ideally change the commission rate across sellers's types, reducing it in case of higher pass-through or more elastic demand, and for sellers whose entry is more elastic with respect to profitability. This is consistent with the fact that the mobile platforms of Apple and Google collect high commission revenues from digital apps provided at zero marginal cost and largely used on devices, concede reduced commission rates for automatic renewal of subscriptions whose demand is typically more elastic (music streaming), allow the circumvention of the commission through web subscriptions for apps providing content at high marginal cost (the reader rule for music streaming) or largely used on other platforms (the multi-platform rule for games), and, finally, they entirely waive the commission for apps that provide physical products outside devices, whose demand is much more elastic (ride-hailing or food delivery). The differentiation of commission rates is also consistent with the policies adopted by Amazon across different product categories, with commission rates depending on demand elasticity and often reduced to attract entry of sellers in selected sectors (for instance fashion and luxury items).

As in the benchmark model, hybrid platforms may introduce own products reducing commission rates on third party sellers to incentivize further their entry. However, heterogeneity between sellers generates an additional incentive for the platforms to imitate the most profitable products, which creates static benefits from lower prices on one side and dynamic losses from lower investment on the other side. Masden and Vellodi (2021) discuss further the welfare implications of this trade-off.

# 4 Substitutability between sellers's products

The main microfoundation employed until now generated independent demands for the products of the sellers. In practice, apps on appstores compete for users's time on devices and they are therefore substitutes from the point of view of consumers. Similarly, different products and services sold on marketplaces compete to attract online spending by consumers and are therefore to some extent substitutable, as assumed in analysis of online marketplaces by Zennyo (2021) and Anderson and Bedre-Defolie (2021). We now extend our framework to account for these forms of substitutability.

We consider the simplest version of quasi-linear preferences that provides demand functions depending on an aggregator of all prices. This is based on the logarithmic transformation employed by Nocke and Schutz (2018), but analogous results would emerge with alternative (concave) transformations (or more general preferences delivering demand systems depending on a single aggregator). The indirect utility:

$$V(x) = \log\left(\int_0^{n_i} v(p_i(\omega))d\omega\right) - d\,|l_i - x| - P_i \tag{25}$$

implies, by Roy's identity, that each seller $\omega$ faces the demand:

$$q_i(\omega) = \frac{|v'(p_i(\omega))|}{D_i}$$

per consumer on platform $i$, where $D_i \equiv \int_0^{n_i} v(p_i(k))dk$ is the additive price aggregator that determines both consumer welfare and demand functions. Notice that in the case of exponential subutilities $v(p) = e^{-p/\mu}$ the model is isomorphic to one based on a Logit foundation (Zennyo, 2021; Anderson and Bedre-Defolie, 2021), but more general versions are encompassed in this microfoundation. We assume a constant marginal cost $c \geq 0$ and a fixed entry cost $F$ for each seller, eliminating uncertainty for simplicity.[17]

Under monopolistic competition each seller $\omega$ on platform $i = 1, 2$ sets the price to maximize gross profits:

$$\pi_i(\omega) = \frac{x_i[(1-\tau_i)p_i(\omega) - c]\,|v'(p_i(\omega))|}{D_i}$$

taking market share and aggregator as given, according to the same price rule $p = p(\tau_i)$ as in (6). Entry of sellers increases the aggregator reducing the gross profits until they match the fixed cost. Accordingly, free entry pins down the value of the aggregator on each platform $i = 1, 2$:

$$D_i = \frac{x_i[(1-\tau_i)p(\tau_i) - c]\,|v'(p(\tau_i))|}{F} \tag{26}$$

or, analogously, the number of sellers $n_i = D_i/v(p(\tau_i))$.

Given the equilibrium pricing and entry, we can derive the Bertrand equilibrium in the prices of the platforms. This allows us to compute the associated market shares and a new expression for the equilibrium differential value:

$$\Omega = \log\frac{D(\tau_i;\Omega)}{D(\tau_j;-\Omega)} + \frac{\tau_i n_i p(\tau_i)\,|v'(p(\tau_i))|}{D(\tau_i;\Omega)} - \frac{\tau_j n_j p(\tau_j)\,|v'(p(\tau_j))|}{D(\tau_j;-\Omega)}$$

[17] This avoids the division between singlehoming and multihoming sellers, but uncertainty on the entry cost could be added as in the baseline model. We could also generalize preferences as $V(x) = \frac{(D_i+H)^\sigma}{\sigma} - d\,|l_i - x| - P_i$ with $\sigma \in (0,1)$ and $H \geqslant 0$, nesting loglinear preferences (25) for $\sigma \to 0$ and the baseline preferences (1) for $\sigma \to 1$ (see also Etro, 2021c).

where the price aggregator satisfies the equilibrium free profit condition:

$$D(\tau_i;\Omega) = \left(1 + \frac{\Omega}{3d}\right) \frac{[(1-\tau_i)p(\tau_i) - c]\,|v'(p(\tau_i))|}{2F} \tag{27}$$

which is decreasing in the commission rate: in practice, a higher commission reduces profitability and therefore entry of sellers, which reduces consumer welfare. Using the equilibrium numbers of sellers we can rewrite the equilibrium expression for the differential value as:

$$\Omega = \log \frac{D(\tau_i;\Omega)}{D(\tau_j;-\Omega)} + \tau_i \zeta(p(\tau_i)) - \tau_j \zeta(p(\tau_j))$$

Each platform sets the commission rate to maximize its own profits and analogous steps to those of the benchmark model deliver a symmetric Nash equilibrium with $\Omega = 0$ and a commission $\tau^e$ that satisfies:

$$\zeta(p(\tau^e)) + \tau^e \zeta'(p(\tau^e))p'(\tau^e) = \frac{|D_\tau(\tau^e;0)|}{D(\tau^e;0)} \tag{28}$$

assuming an interior solution in equilibrium. The left hand side is the marginal impact of the commission on revenues, and the right hand side is the relative change of the value of the aggregator due to reduced entry of sellers.

To better relate this model with our benchmark model, we can develop the formula (28) using the slope of the surplus elasticity:

$$\zeta'(p(\tau)) = \frac{\zeta(p(\tau))}{p(\tau)} \left[1 + \zeta(p(\tau)) - \varepsilon(p(\tau))\right], \tag{29}$$

the impact of the commission on the aggregator (27) and the pass-through of the commission rate on sellers's prices (7). This provides a neater expression for the equilibrium commission rate:

$$\tau^e = \frac{1 - \frac{\varepsilon}{\zeta}}{1 - \eta\,(1 + \zeta - \varepsilon)} \tag{30}$$

which is positive for $\zeta > \varepsilon$. If it is also in the unit interval, the commission is decreasing in the demand elasticity $\varepsilon$ and in the pass-through elasticity $\eta$, and increasing in the surplus elasticity $\zeta$, confirming earlier comparative statics. An explicit derivation is easily obtained under zero pass-through, when a linear demand confirms a commission of 50% and a loglinear demand confirms a zero commission.[18]

[18] Under the subutility $v(p) = e^{-p/\mu}$ sellers set the price $p(\tau) = \frac{c}{1-\tau} + \mu$, the mass of sellers on each platform is $n(\tau) = \frac{1-\tau}{2F}$, and the commission maximizes $\log[n(\tau)v(p(\tau))] + \tau p(\tau)/\mu \propto \log(1-\tau) + \tau$, which provides a zero commission. Of course, if the platforms cannot set optimal access prices for consumers, as probably in case of online marketplaces, the commissions are higher as the main tool of monetization.

In this model, the equilibrium commission remains the same as the one maximizing consumer surplus taking as given the entry and pricing processes, namely solving the problem:

$$\max_{\tau} \log D(\tau, 0) + \tau\zeta(p(\tau))$$

Again, it is the framework where platforms compete in the utility space under the covered market assumption which delivers commitments on the commissions that are efficient for consumers taking as given the price distortions. Moreover, since here there are no supracompetitive profits under free entry, the equilibrium commission is also the one that maximizes the total users's welfare, that is the surplus of consumers and sellers.

The model can be extended to heterogeneous sellers with uncertain costs as in the previous section, restoring the selection of the most profitable ones and generating a skewed distribution of revenues and profits. Then, the equilibrium commission would maximize consumer welfare while sellers would benefit from a lower commission. This suggests that the conflict between the interest of platforms and sellers concerning the commission rate is essentially a battle for rents on the division of the inframarginal profits obtained by sellers. In the absence of supracompetitive profits, the commission is simply set at the efficient level that maximizes total welfare, while in the presence of inframarginal sellers obtaining positive profits the commission is set at the level that maximizes consumer welfare.

In Appendix B we extend this analysis to the case of hybrid platforms that provide their own products, as in Zennyo (2021), Anderson and Bedre-Defolie (2021), Etro (2021c) and Shopova (2021). We show that the introduction of products by the platforms generates two contrasting effects on the equilibrium commissions. On one side, the platforms want to increase commission rates to shift demand toward their own products (the effect of Anderson and Bedre-Defolie, 2021) and on the other side, the platforms want to reduce commission rates to attract more sellers and expand commission revenues on the extensive margin while attracting also more buyers (the effect we already emphasized in the benchmark model of Section 2.5). Either effect can prevail depending on the shape of the demand functions, or the two effects can compensate each other, as in Zennyo (2021). Remarkably, the commissions set by Apple and Google have been essentially constant over time, also after the introduction of successful apps by each platforms, except for reductions introduced for special categories of purchases. And also the commissions set by Amazon on different product categories have been quite stable over time, also after the introduction of private label products by Amazon. Last, we remark that the device-funded nature of the platforms implies once again that in our framework the revenues generated from both commissions and own products are shifted to consumers through lower access fees.

# 5 Breaking the alignment

In this Section we extend the baseline model in ways that can break the alignment of interests between platforms and consumers. First, we consider a direct channel through which sellers provide their own products, which can reduce the equilibrium commission below the optimal level, and then we consider consumers's myopia in the evaluation of the products purchased on the platform, which instead increases the equilibrium commission above the optimal level.[19]

Another mechanisms that breaks the alignment has been introduced by Jeon and Rey (2021) considering a unique entry cost that allows sellers to be active on both platforms in the absence of platform-specific costs: this leads to excessive commissions and too little entry of sellers because the platforms neglect the negative externality that high commissions generate on sellers's investments.[20] Similarly, in our framework we could add a preliminary stage where a common pool of potential sellers is created through investments driven by the expected profitability on both platforms, and then sellers enter each platform if they can cover platform-specific fixed costs (in the spirit of Melitz, 2003). This would lead to excessive commissions because the platforms would ignore the impact on the investment stage (which does not affect the differential value of the platforms) and would internalize only the impact on the fraction of selected sellers. However, the commission levels would be still moderated by positive pass-through and high demand elasticity and, in general, by platform competition: the reduction of a commission by one platform would increase the total number of sellers, but it would also attract more of them to the platform that has reduced the commission, creating the incentives to set low commissions.[21] The practical relevance of this argument for excessive commissions is limited when platform-specific profitability and platform-specific costs are important for the entry decision of sellers, and when the preliminary investment of perspective sellers is also driven by profitability on third-party platforms.

## 5.1 Direct channels for sellers

Some sellers, for instance music apps and games for app stores or large sellers for online marketplaces, are typically available on other platforms and consumers can usually purchase their products also through a direct channel (as their own website accessed through desktop computers). In such a case, a platform can only collect commission revenues in case of purchases through the same

[19] I am extremely grateful to Benno Buehler for inspiring and discussing these extensions. Related sources of excessive commissions analyzed elsewhere emerge when consumers differ in the valuation of the quality of the platforms' services and one of them is an ad-funded platform, or when the marginal consumer valuation of transactions with additional sellers differs from the average one.

[20] This inefficiency could materialize if mobile platforms are constrained to host competing app stores, so that any app could be automatically available on all platforms (a possible case being the Chinese market for Android). An additional effect of opening platforms to third party app stores would be a reduction of investments in app curation.

[21] Similar effects would emerge if there was a correlation between platform-specific entry costs.

platform. Since consumers typically enjoy access through a single marketplace or app store, they can be available to pay higher prices there, while sellers are available to allow this to the extent that they can obtain higher revenues net of the payment of commissions.[22] Both aspects, however, generate a constraint on the commission rates that platforms can set.

In our baseline model, let us assume that payments for services through the direct channel generate nuisance costs which reduce the surplus from usage on the platforms to $\lambda v(p)$ for $\lambda \in (0, 1)$. If a seller decides to make available payments only through the direct channel, it raises profits:

$$\tilde{\pi}_i = x_i[p(0) - c]\lambda \left|v'\left(p(0)\right)\right|$$

from consumers of platform $i$, where the profit-maximizing price $p(0)$ corresponds to the one in the absence of commissions. Then, the platform can only raise positive commission revenues if $\tilde{\pi}_i \leq \pi_i(\tau)$, that is for a commission rate below a threshold $\bar{\tau}$, which, using equilibrium pricing, can be expressed by the following condition:

$$\bar{\tau} = 1 - \lambda \frac{\zeta(p(0))\varepsilon(p(\bar{\tau}))v\left(p(0)\right)}{\zeta(p(\bar{\tau}))\varepsilon(p(0))v(p(\bar{\tau}))} \tag{31}$$

We should also check that consumers prefer access through the platform, namely that $v\left(p(\bar{\tau})\right) > \lambda v\left(p(0)\right)$, a condition which is here assumed satisfied.[23] Whenever $\bar{\tau} < \tau^e$, both platforms set commission rates at the upper bound and consumers purchase through the platforms. This limits the commission revenues, increases the equilibrium access prices above the unconstrained equilibrium and induces excess entry of sellers, which breaks the alignment between the interest of platforms and consumers creating consumer harm. The inefficiency is due to a negative externality between consumers, who have an incentive to purchase cheaper services through the direct channel ignoring that this reduces commission revenues and increases the prices of devices for all consumers. In such a case anti-circumvention rules aimed at avoiding the constraint on commission rates can be actually efficient, raising revenues that are entirely shifted to consumers through lower prices of the devices.[24]

[22] For instance, Spotify gave up to subscriptions through the Apple app store to avoid the associated commission. Alternative strategies for sellers facing heterogeneous consumers include setting much higher prices on the platform compared to the direct channel, to attract customers with high willingness to pay or high nuisance costs from purchasing through the direct channel, and adopting freemium models to monetize on ads from customers with low willingness to pay.

[23] For the prominent family of power surplus functions, nesting linear, loglinear, isoelastic and other demands (see Etro, 2021b), the definition of the threshold simplifies to $\bar{\tau} = 1 - \lambda v(p(0))/v(p(\bar{\tau}))$, which has a unique root in the unit interval.

[24] We have also explored the case where consumers are heterogeneous in nuisance costs from using the direct channel. Customers with low nuisance costs purchase through the direct channel, generating a negative externality on the other consumers through an increase of the price of devices. Also in this case anti-steering rules may allow the platform to increase commission revenues and make the average consumer better off.

A similar outcome emerges in case of entry costs that are not platform-specific. As shown by Jeon and Rey (2021) this scenario leads to an unconstrained equilibrium with excessive commissions (because the platforms do not internalize the impact of their choices on sellers). However, the ability of sellers to monetize also through a direct channel introduces a binding constraint on the commission rates, leading to ambiguous welfare consequences (because consumers do not internalize the impact of their choices on other consumers): if the constraint is strong enough, the commissions would be set below the efficient level from the point of view of consumers. We should also remark that equilibria with insufficient entry generate a natural incentive for the platforms to offer their own products with exclusivity (as own apps on app stores or private label products on marketplaces), enhancing the benefits of hybrid platforms.

## 5.2 Myopic consumers

Behavioral economics has widely shown that consumers are often engaged in impulsive behavior and suffer of myopia in complex decisions (see, for instance, Thaler *et al.*, 1997). Consumers who purchase a durable good and subsequently purchase complement products in an aftermarket may tend to underestimate the impact of future purchases of secondary products on the value of the primary good in the beforemarket, leading to opportunistic behavior by suppliers with market power on both goods. For instance, in the competition between mobile application ecosystems, it has been argued that devices compete in a beforemarket, apps' services are purchased in an aftermarket, and the platforms may exploit consumers's myopia in setting commissions. A well-known constraint on this kind of opportunistic behavior is competition in the beforemarket, which shifts all the aftermarket revenues into lower access prices. We now evaluate the impact of this mechanism in our duopoly model by introducing some form of consumers' myopia on the *ex ante* valuation of the benefits obtained from purchases of services.

Let us assume that, at the time of purchasing a device, consumers perceive only a fraction $\theta \in [0,1]$ of the surplus generated from future purchases of sellers's products (for instance, if this is zero they do not perceive any surplus from future app purchases and simply buy the cheapest device). In our basic model the pricing by the sellers is not affected, but the Bertrand equilibrium prices of the platforms are not given by (9), but by:

$$P_i = h + d + \frac{\theta[n_i v(p(\tau_i)) - n_j v(p(\tau_j))] - 2\tau_i n_i r(\tau_i) - \tau_j n_j r(\tau_j)}{3}$$

emphasizing that the differential surplus is underestimated, but the link between commission revenues and prices of devices remains the same as under full rationality. Myopia reduces the strength of the network effects that relate entry of sellers and demand for each platform. Extending the earlier analysis we can amend the formula for the equilibrium commission (15) into:

$$1 - \eta\left(1 + \frac{\tau^e \varepsilon + \theta - 1}{1 - \tau^e}\right) = \frac{\theta v + \tau^e r}{2} \frac{G'(\pi)}{G(\pi)} \tag{32}$$

with $\partial\tau^e/\partial\theta < 0$: myopia increases the equilibrium commission because it increases the marginal revenues on sellers on the left hand side and reduces the marginal costs of lost entry on the right hand side.

The commission rate that maximizes (paternalistically) consumer welfare *ex post* remains the same as in the benchmark model, therefore limited rationality of consumers pushes toward excessive commissions. The intuition is that the platforms put a lower weight on the surplus generated by the sellers due to the fact that the same consumers put a lower weight on that in evaluating different platforms. However, consumers's myopia does not affect the commission rate compared to the one emerging under perfectly competitive platforms, it does not affect the pass-through of commission revenues into lower access prices (since this remains full as under perfectly competitive platforms), and it does not even affect the profits of the platforms, which remain determined by their differentiation in the beforemarket. Accordingly, the main consequence of myopia is an inefficient reduction of the prices in the beforemarket associated with insufficient entry of sellers in the aftermarket (independently from the strength of competition). This is a scenario where platforms could create further gains for consumers by committing to lower commission rates (and by introducing own products which bypass the mentioned inefficiency).

# 6 Conclusion

We have extended the analysis of competition between two-sided platforms to the case of monopolistic competition between third party sellers and free entry on each platform. This framework applies in particular to device-funded or subscription-based platforms, and tends to generate an efficient allocation from the point of view of consumers, with commission rates depending on the elasticities of pass-through, demand, surplus and entry of sellers. These results are in contrast with the idea that a device-funded platform would systematically harm consumers in the long run through excessive commissions aimed at foreclosing entry, which may be relevant for the ongoing debate on platform regulation. At least, our works suggest that different business models, as those of device-funded platforms and ad-funded platforms, generate different implications for regulation of conditions for third party sellers.

We have also discussed extensions that lead to departures from the basic alignment of interest between platforms and consumers. For instance, consumer myopia or externalities in entry decisions can generate commissions above the levels that maximize consumer welfare. This suggests that to judge whether platforms set inefficient commissions requires specific investigations on the particular market conditions where they operate. Nevertheless, we should also stress that setting prices (in this case commissions) above the level that maximizes consumer welfare is not the relevant threshold for antitrust intervention or for the adoption of more pervasive forms of platform regulation.

# References

Acemoglu, Daron, 2020, Antitrust alone won't fix the innovation problem, Project Syndicate.

Alfaro, Martìn and David Lander, 2021, Restricting entry without aggressive pricing, *Research in Economics*, 75, 4, 305-319.

Anderson, Simon and Özlem Bedre-Defolie, 2021, Hybrid platform model, CEPR DP 16243.

Anderson, Simon, Nisvan Erkal and Daniel Piccinin, 2020, Aggregative games and oligopoly theory: short-run and long-run analysis, *RAND Journal of Economics*, 51, 2, 470-495.

Armstrong, Mark, 2006, Competition in two-sided markets, *The RAND Journal of Economics*, 37, 3, 668-691.

Armstrong, Mark and John Vickers, 2001, Competitive price discrimination, *The RAND Journal of Economics*, 32, 4, 579-605.

Belleflamme, Paul and Martin Peitz, 2019, Managing competition on a two-sided platform, *Journal of Economics & Management Science*, 28, 1, 5-22.

Belleflamme, Paul and Eric Toulemonde, 2016, Who benefits from increased competition among sellers on B2C platforms?, *Research in Economics*, 70, 4, 741-751.

Bertoletti, Paolo, 2021, A preference-based model of competition among platforms, DEMS WP 468, University of Milan, Bicocca.

Bertoletti, Paolo and Federico Etro, 2016, Preferences, entry and market structure, *RAND Journal of Economics*, 47, 4, 792-821.

Bertoletti, Paolo and Federico Etro, 2017, Monopolistic competition when income matters, *The Economic Journal*, 127, 603, 1217-1243.

Bresnahan, Tim, Joe Orsini and Pai-Ling Yin, 2015, Demand heterogeneity, inframarginal multihoming, and platform market stability: Mobile apps, mimeo, Stanford University.

Buehler, Benno, 2015, Do international roaming alliances harm consumers?, *Journal of Industrial Economics*, 63, 4, 642-672.

Caffarra, Caffarra, 2019, "Follow the Money" - Mapping issues with digital platforms into actionable theories of harm, *Antitrust Case Laws e-Bulletin*, e-Competitions Special Issue Platforms, August 29.

Caillaud, Bernard and Bruno Jullien, 2003, Chicken & egg: Competition among intermediation service providers, *RAND Journal of Economics*, 34, 2, 309-328.

Carare, Octavian, 2012, The impact of bestseller rank on demand: Evidence from the app market, *International Economic Review*, 53, 3, 717-742.

Casner, Ben and Tat-How Teh, 2021, Membership vs discovery platforms, mimeo, National University of Singapore.

Choi, Jay Pil and Doh-Shin Jeon, 2020, Two-sided platforms and biases in technology adoption, mimeo, Toulouse School of Economics.

Choi, Jay Pil and Doh-Shin Jeon, 2021, A leverage theory of tying in two-sided markets with non-negative price constraints, *American Economic Journal: Microeconomics*, 13, 1, 283-337.

Duch-Brown, Néstor, 2017, The competitive landscape of online platforms, JRC Digital Economy WP 4, European Commission, Brussels

Etro, Federico, 2011, Endogenous market structures and contract theory: Delegation, principal-agent contracts, screening, franchising and tying, *European Economic Review*, 55, 4, 463-479.

Etro, Federico, 2021a, Device-funded vs ad-funded platforms, *International Journal of Industrial Organization*, 75, 102711.

Etro, Federico, 2021b, Product selection in online marketplaces, *Journal of Economics & Management Strategy*, 30, 3, 614-37.

Etro, Federico, 2021c, Hybrid marketplaces with free entry of sellers, WP 21, DISEI, University of Florence.

Fu, Xingyu, Ying-Ju Chen, Guillermo Gallego and Pin Gao, 2021, Policy implementation in the presence of platform-retailer conflict, mimeo, Hong Kong University.

Gans, Joshua, 2012, Mobile application pricing, *Information Economics and Policy*, 24, 52-59.

Gautier, Axel, Leonardo Madio and Shiva Shekhar, 2021, Platform duality and network externalities, mimeo, University of Liége.

Hagiu, Andrei, 2006, Pricing and Commitment by Two-Sided Platforms, *The RAND Journal of Economics*, 37, 3, 720-37.

Hagiu, Andrei, Tat-How Teh and Julian Wright, 2022, Should platforms be allowed to sell on their own marketplaces?, *The RAND Journal of Economics*, forthcoming.

Hagiu, Andrei and Julian Wright, 2015, Marketplace or reseller?, *Management Science*, 61, 1, 184-203.

Hervas-Drane, Andres and Sandro Shelegia, 2021, Retailer-led marketplaces, mimeo, Universitat Pompeu Fabra, Barcelona.

Hyrynsalmi, Sami, Arho Suominen and Matti Mäntymäki, 2016, The influence of developer multi-homing on competition between software ecosystems, *Journal of Systems and Software*, 111, 119-127.

Jeon, Doh-Shin and Patrick Rey, 2021, Platform competition, *ad valorem* commissions and app development, mimeo, Toulouse School of Economics.

Karle Heiko, Peitz, Martin and Markus Reisinger, 2020, Segmentation versus agglomeration: competition between platforms with competitive sellers, *Journal of Political Economy*, 128, 6, 2329-74.

Masden, Erik and Nikhil Vellodi, 2021, Insider imitation, mimeo, New York University.

Melitz, Marc, 2003, The impact of trade on intra-industry reallocations and aggregate industry productivity, *Econometrica*, 71, 6, 1695-725.

Nocke, Volker and Nicolas Schutz, 2018, Multiproduct-firm oligopoly: An aggregative games approach, *Econometrica*, 86, 2, 523-557.

Padilla, Jorge, Joe Perkins and Salvatore Piccolo, 2021, Self-preferencing in markets with vertically-integrated gatekeeper platforms, *Journal of Industrial Economics*, forthcoming.

Rey, Patrick and Jean Tirole, 2019, Price caps as welfare enhancing coopetition, *Journal of Political Economy*, 127, 6, 3018-69.

Rochet, Jean-Charles and Jean Tirole, 2003, Platform competition in two-sided markets, *Journal of the European Economic Association*, 1, 4, 990-1029.

Shopova, Radostina, 2021, Private Labels in Marketplaces, Vienna Economics WP 2104.

Teh, Tat-How, 2020, Platform governance, mimeo, National University of Singapore.

Thaler, Richard, Amos Tversky, Daniel Kahneman and Alan Schwartz, 1997, The effect of myopia and loss aversion on risk taking: An experimental test, *Quarterly Journal of Economics*, 112, 2, 647-661.

Ushakov, Nikolay, 2021, *Essays in Applied Microeconomics*, Ph.D Thesis, Michigan State University.

Zennyo, Yusuke, 2020, Cross-Market Platform Competition in Mobile App Economy, Japan Fair Trade Commission CPRC Discussion Paper 83.

Zennyo, Yusuke, 2021, Platform encroachment and own-content bias, *Journal of Industrial Economics*, forthcoming.

# Appendix A: Heterogeneous sellers

In this Appendix we solve for the equilibrium commission of the model with heterogeneous sellers of Section 3. The solution of the free entry system can be characterized after rewriting it as follows:

$$\begin{aligned}\left(1+\frac{\Omega}{3d}\right)\frac{\left[p(\hat{c}_1,\tau_1)(1-\tau_1)-\hat{c}_1\right]\left|v'(p(\hat{c}_1,\tau_1))\right|}{2} &= F\\ \left(1-\frac{\Omega}{3d}\right)\frac{\left[p(\hat{c}_2,\tau_2)(1-\tau_2)-\hat{c}_2\right]\left|v'(p(\hat{c}_2,\tau_2))\right|}{2} &= F\end{aligned}$$

which now defines two cut-off functions $\hat{c}_i(\tau_i,\Omega)$ depending on the own commission rate and the value $\Omega$ which satisfies:

$$\begin{aligned}\Omega &= \int_0^{\hat{c}_i(\tau_i,\Omega)}\left[v(p(c,\tau_i))+\tau_i p(c,\tau_i)\left|v'(p(c,\tau_i))\right|\right]d\tilde{G}(c)+\\ &\quad -\int_0^{\hat{c}_j(\tau_i,\Omega)}\left[v(p(c,\tau_j))+\tau_j p(c,\tau_j)\left|v'(p(c,\tau_j))\right|\right]d\tilde{G}(c)\end{aligned}$$

The FOCs for the choice of $\tau_i$ can be expressed as follows:

$$\int_0^{\hat{c}_i(\tau_i,\Omega)}\left[v'(p(c,\tau_i))-\tau_i v'(p(c,\tau_i))-\tau_i p(c,\tau_i)v''(p(c,\tau_i))\right]\frac{\partial p(c,\tau_i)}{\partial\tau_i}d\tilde{G}(c)+$$

$$-\int_0^{\hat{c}_i(\tau_i,\Omega)}p(c,\tau_i)v'(p(c,\tau_i))d\tilde{G}(c)-p(\hat{c}_i(\tau_i,\Omega),\tau_i)v(p(\hat{c}_i(\tau_i,\Omega)),t_i)\tilde{G}'(\hat{c}_i(\tau_i,\Omega))$$

$$-\frac{d\hat{c}_i(\tau_i,\Omega)}{d\tau_i}\tau_i p(\hat{c}_i(\tau_i,\Omega),\tau_i)v'(p(\hat{c}_i(\tau_i,\Omega)),\tau_i)\tilde{G}'(\hat{c}_i(\tau_i,\Omega))=0$$

The FOCs for the choice of $\tau_j$ can be derived analogously.

In a symmetric equilibrium we must have $\Omega=0$ and $\hat{c}_1(\tau,0)=\hat{c}_2(\tau,0)=\hat{c}$. The impact of the commission can be computed as:

$$\frac{\partial p(c,\tau_i)}{\partial\tau_i}=\frac{\eta(p(c,\tau_i))p(c,\tau_i)}{1-\tau_i}$$

which depends on the pass-through elasticity of the specific seller. Moreover, total differentiation of the free entry system above with respect to $\hat{c}_i$ and $\tau_i$ for a given $\Omega=0$ implies:

$$\frac{d\hat{c}_i(\tau_i,0)}{d\tau_i}=-p(\hat{c}_i,\tau_i)$$

Using these equilibrium results, we can obtain the Nash equilibrium condition:

$$\int_0^{\hat{c}}\left[v'(p(c,\tau^e))-\tau^e v'(p(c,\tau^e))-\tau^e p(c,\tau^e)v''(p(c,\tau^e))\right]\frac{\eta(p(c,\tau^e))p(c,\tau^e)}{1-\tau^e}d\tilde{G}(c)=$$

$$=\int_0^{\hat{c}}p(c,\tau^e)v'(p(c,\tau^e))dG(c)+p(\hat{c},\tau^e)\left[v(p(\hat{c},\tau^e))-\tau^e p(\hat{c},\tau^e)v'(p(\hat{c},\tau^e))\right]\tilde{G}'(\hat{c})$$

Standard manipulations allow us to rewrite this implicit expression for the equilibrium commission rate as:

$$\tau^e = \frac{1 - \mathbb{E}[\eta] - \frac{\hat{\varepsilon}}{\hat{\zeta}}\hat{\kappa}}{1 - \mathbb{E}[\eta] + \mathbb{E}[\eta\varepsilon] + \hat{\varepsilon}\hat{\kappa}}$$

where:

$$\mathbb{E}[x] = \int_0^{\hat{c}} \varpi(c)x(c)d\tilde{G}(c) \quad \text{with } \varpi(c) = \frac{p(c)\left|v'(p(c))\right|}{\int_0^{\hat{c}} p(s)\left|v'(p(s))\right| d\tilde{G}(s)},$$

the elasticities $\hat{\varepsilon} = \varepsilon(p(\hat{c}, \tau^e))$ and $\hat{\zeta} = \zeta(p(\hat{c}, \tau^e))$ apply to the marginal sellers, and we defined:

$$\hat{\kappa} = \frac{\varpi(\hat{c})F}{2\left|v'(p(\hat{c}, \tau^e))\right|}$$

again referred to the marginal seller.

The social planner problem (24) implies the optimality condition:

$$\int_0^{\hat{c}} \left[v'(p(c,\tau)) - \tau v'(p(c,\tau)) - \tau p(c,\tau)v''(p(c,\tau))\right] \frac{\partial p(c,\tau)}{\partial \tau} d\tilde{G}(c) =$$

$$= \int_0^{\hat{c}} p(c,\tau)v'(p(c,\tau))d\tilde{G}(c) + \left[v(p(\hat{c},\tau)) - \tau p(\hat{c},\tau)v'(p(\hat{c},\tau))\right]\tilde{G}'(\hat{c})\frac{d\hat{c}}{d\tau}$$

Since total differentiation of the constraint implies $d\hat{c}/d\tau = -p(\hat{c}, \tau)$, using this and the equilibrium pass-through we can recover the same condition as the one for the equilibrium commission rate.

# Appendix B: Hybrid platforms

In this Appendix we extend the model with imperfectly substitutable products of Section 4 to the case where the platforms provide their own products. The purpose is to show that this may lead to either higher or lower commission rates, but still set in line with the interest of consumers.

Zennyo (2021) and Anderson and Bedre-Defolie (2021) have recently analyzed a marketplace with free entry of sellers that introduces an exogenous set of products and can replace some of the imperfectly substitutable products offered by other sellers. Their aim is to verify the impact of such a strategy on the commission level, on the endogenous number of products and on consumer welfare, and a main result within their Logit microfoundation of the demand system is that an hybrid platform increases a percentage commission and reduces entry and consumer welfare.

We can extend our analysis to competition between hybrid platforms in the same spirit. Let us assume that each platform $i$ provides $m$ own products at cost $\bar{c}$ and price $\bar{p}_i$, generating a surplus function $\bar{v}(\bar{p}_i)$, which can differ from the one of the sellers. For a given commission, the platform's provision does not affect monopolistic competition pricing by sellers and does not even affect the value of the aggregator:

$$D_i = (n_i - m)v(p(\tau_i)) + m\bar{v}(\bar{p}_i)$$

in the free entry equilibrium, which is still given by (26), a well known neutrality property for this class of games with a leader and endogenous entry of followers.[25] Taking into account that third party products generate commission revenues and own products generate direct profits, we can amend the expression for the differential value of the platforms as:

$$\begin{aligned}\Omega &= \log \frac{D(\tau_i;\Omega)}{D(\tau_j;-\Omega)} + \tau_i\zeta(p(\tau_i)) - \tau_j\zeta(p(\tau_j)) + \\ &\quad + \left[\frac{\Delta(\tau_i,\bar{p}_i)}{D(\tau_i,\Omega)} - \frac{\Delta(\tau_j,\bar{p}_j)}{D(\tau_j,-\Omega)}\right]\end{aligned}$$

where $\Delta(\tau,\bar{p}) \equiv m[(\bar{p}-\bar{c})\,|\bar{v}'(\bar{p})| - \tau\zeta(p(\tau))\bar{v}(\bar{p})]$ refers to the profits generated by the products of platform $i$ net of the lost commission revenues.

Each platform selects the prices of its own products taking as given the price aggregator, since this is indeed fixed under free entry, but taking into account the opportunity cost of losing commission revenues on sellers's products, which increases prices above marginal costs. Maximizing the differential value with respect to prices of platform $i$, that is maximizing $\Delta(\tau_i,\bar{p}_i)$ with respect to $\bar{p}_i$, provides price rules $\bar{p} = \bar{p}(\tau_i)$ satisfying:

$$\bar{p} = \frac{\bar{\varepsilon}(\bar{p})\bar{c}}{\bar{\varepsilon}(\bar{p}) - 1 - \tau_i\zeta(p(\tau_i))}$$

[25] For a related statement of the neutrality property see Etro (2011) and Anderson *et al.* (2020). For an extension to strategies of the leader affecting the followers see Alfaro and Lander (2021).

where $\bar{\varepsilon}(p) = -\frac{\bar{v}''(p)p}{\bar{v}'(p)}$ is the demand elasticity for the platforms's products. One can verify that the markup is the same as that of the sellers under identical isoelastic surplus functions, but otherwise, it can be either higher or lower.

Each platform sets its commission rate taking into account the additional effects on the price aggregator and on the opportunity cost of losing commission revenues (by the envelope theorem the effect on the price of own products is null). The former mechanism (analogous to what found by Anderson and Bedre-Defolie, 2021) pushes toward higher commissions to shift demand toward the platform's products, but the latter (analogous to what found in our benchmark model) pushes toward lower commissions because these are applied to a smaller number of sellers. The Nash equilibrium commission for the hybrid platforms satisfies:

$$\zeta(p(\tau^e)) + \tau^e \zeta'(p(\tau^e))p'(\tau^e) = \frac{|D_\tau(\tau^e;0)|}{D(\tau^e;0)} \left[\frac{D(\tau^e;0) - \Delta(\tau^e)}{D(\tau^e;0) - m\bar{v}(\bar{p}(\tau^e))}\right]$$

where, using the prices of the platforms's products, we can express $\Delta(\tau) = \Delta(\tau, \bar{p}(\tau))$ as:

$$\Delta(\tau) = m\bar{v}(\bar{p}(\tau)) \left[\frac{\bar{\zeta}(\bar{p}(\tau))}{\bar{\varepsilon}(\bar{p}(\tau))}[1 + \tau\zeta(p(\tau))] - \tau\zeta(p(\tau))\right] > 0$$

with $\bar{\zeta}(p) \equiv -\frac{\bar{v}'(p)p}{\bar{v}(p)}$ as the surplus elasticity for the platforms's products. The new formula for the commission of hybrid platforms extends (28) with a new term in squared parenthesis on the right hand side, which affects the marginal cost of reducing entry of sellers. On the one side, demand is shifted to the products of the platforms generating additional profits, which induces a substitution effect that pushes for higher commissions, especially when $\Delta$ is large. On the other side, some sellers are replaced by products of the platforms, and it becomes more convenient to attract other sellers with lower commissions to expand revenues on the extensive margin and try to attract purchases by new customers, a sort of extensive margin effect that is large when $\bar{v}$ is large.

Either effect can prevail depending on the underlying preferences for the platforms's products, therefore the commissions can either increase or decrease with the introduction of platforms's products. In particular, one can verify that $\partial\tau^e/\partial m \gtrless 0$ if $\Delta(\tau)/m \gtrless \bar{v}(\bar{p}(\tau))$ which is equivalent to:

$$\partial\tau^e/\partial m \gtrless 0 \quad \text{if} \quad \bar{\varepsilon}(\bar{p}(\tau)) \lessgtr \bar{\zeta}(\bar{p}(\tau))$$

so the introduction of own products leads to a reduction of commission rates when their demand is more elastic than their surplus.[26] We should remark that in the absence of access prices, the commissions would be set at higher levels as the main form of monetization, but again it can be shown that, depending on the underlying microfoundation, hybrid marketplaces could either increase or

[26] The equilibrium commission does not change in the special case of exponential subutilities for which $\bar{\zeta}(p) = \bar{\varepsilon}(p)$.

reduce commissions compared to pure marketplaces, generating an ambiguous impact on welfare (see Etro, 2021c).

In our framework the equilibrium commission remains the welfare maximizing one, and the introduction of a product by a platform is profitable only if it increases the total surplus created and shared with consumers. Overall, our model suggests that there is no basis for a ban of the dual mode of hybrid platforms, which resonates well with findings obtained in different frameworks by Shopova (2021), Hagiu *et al.* (2022) and others.